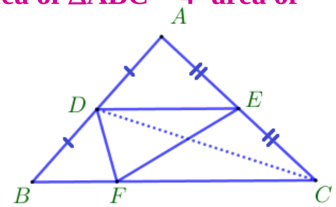
Important Extra Theorems (Area of Triangle and Quadrilaterals)

1. In ΔABC; D and E are the mid-points of sides AB and AC respectively and F is any point on BC. Prove that area of ΔABC = 4×area of ΔDEF.



Solution:

Given: In ΔABC; D and E are the mid-points of sides AB and AC respectively and F is any point on BC

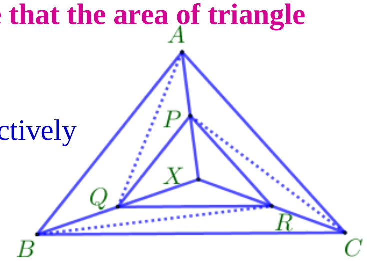
To prove: Area of ΔABC = 4×Area of ΔDEF.

Construction: D and C are joined.

Proof:

S.N.	Statements	S.N.	Reasons
1.	DE//BC	1.	In ΔABC; DE joins the mid-points of AB and AC respectively.
2.	ΔDEF = ΔDEC	2.	Both are standing on the same base DE and between DE//BC.
3.	$\Delta DEC = \frac{1}{2}\Delta ADC$	3.	The median DE bisects ΔDEC.
4.	$\Delta ADC = \frac{1}{2}\Delta ABC$	4.	The median DC bisects ΔABC.
5.	$\Delta DEF = \frac{1}{2} \times \frac{1}{2}\Delta ABC \therefore \Delta ABC = 4\Delta DEF$	5.	From statements (2), (3) and (4).
			Proved

2. X is any point within a triangle ABC. If P, Q and R are the mid-points of AX, BX and CX respectively. Prove that the area of triangle ABC is 4 times the area of triangle PQR.



Solution:

Given: X is any point within a triangle ABC. If P, Q and R are the mid-points of AX, BX and CX respectively

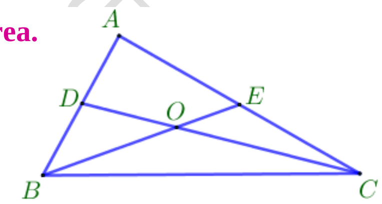
To prove: Area of ΔABC = 4×Area of ΔPQR.

Construction: A to Q, B to R and C to P are joined.

Proof:

S.N.	Statements	S.N.	Reasons
1.	$\Delta PXQ = \frac{1}{2}\Delta AXQ$ and $\Delta AQX = \frac{1}{2}\Delta AXB$	1.	The median PQ bisects ΔAXQ and the median AQ bisects ΔAXB.
2.	$\Delta PXQ = \frac{1}{2} \times \frac{1}{2}\Delta AXB = \frac{1}{4}\Delta AXB$	2.	From statement (1)
3.	$\Delta QXR = \frac{1}{4}\Delta BXC$ and $\Delta PXR = \frac{1}{4}\Delta AXC$	3.	Same as above process.
4.	$\Delta PXQ + \Delta QXR + \Delta PXR = \frac{1}{4}(\Delta AXB + \Delta BXC + \Delta AXC)$	4.	Adding statements (2) and (3).
5.	$\Delta PQR = \frac{1}{4}\Delta ABC \quad \therefore \Delta ABC = 4\,\Delta PQR$	5.	From statement (4), by whole part axiom.
			Proved

3. In ΔABC; medians BE and CD intersect at O. Prove that ΔBOC and quadrilateral ADOE are equal in area.



Solution:

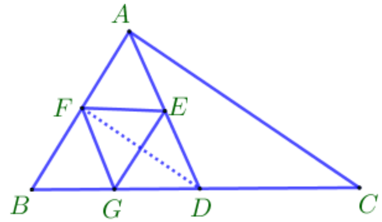
Given: In ΔABC; medians BE and CD intersect at O.

To prove: Area of ΔBOC = Area of quadrilateral ADOE

Proof:

S.N.	Statements	S.N.	Reasons
1.	ΔABE = $\frac{1}{2}$ΔABC and ΔBCD = $\frac{1}{2}$ΔABC	1.	The medians BE and CD bisect ΔABC.
2.	ΔABE = ΔBCD	2.	From statement (1)
3.	ΔABE = Quad. ADOE + ΔBOD and ΔBCD = ΔBOC + ΔBOD	3.	By whole part axiom.
4.	Quad. ADOE + ΔBOD = ΔBOC + ΔBOD ∴ Quad. ADOE = ΔBOC	4.	From statements (2) and (3).
			Proved

4. In ΔABC; D is mid-point of side BC, E is the mid-point of AD, F is mid-point of AB and G is any point on BD. Prove that ΔABC = 8ΔEFG.



Solution:

Given: In ΔABC; D is mid-point of side BC, E is the mid-point of AD, F is mid-point of AB and G is any point on BD

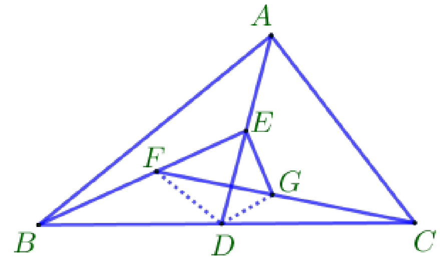
To prove: ΔABC = 8ΔEFG.

Construction: F and D are joined.

Proof:

S.N.	Statements	S.N.	Reasons
1.	FE//BD	1.	In ΔABD; FE joins the mid-points of AB and AD respectively.
2.	ΔEFG = ΔEFD	2.	Both are standing on the same base FE and between FE//BD.
3.	ΔFED = $\frac{1}{2}$ΔAFD	3.	The median FE bisects ΔAFD.
4.	ΔAFD = $\frac{1}{2}$ΔABD	4.	The median FD bisects ΔABD.
5.	ΔABD = $\frac{1}{2}$ΔABC	5.	The median AD bisects ΔABC.
6.	ΔEFG = $\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}$ΔABC ∴ΔABC = 8ΔEFG	6.	From statements (2), (3), (4) and (5)
			Proved

5. In triangle ABC; D is mid-point of side BC, E is the mid-point of AD, F is mid-point of BE and G is the mid-point of FC. Prove that the area of triangle ABC is equal to 8 times the area of triangle EFG.



Solution:

Given: In ΔABC; D is mid-point of side BC, E is the mid-point of AD, F is mid-point of BE and G is the mid- point of FC.

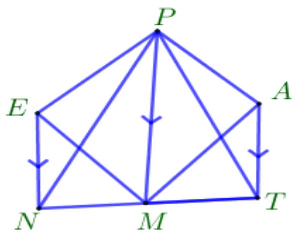
To prove: $\Delta ABC = 8\Delta EFG$.

Construction: F, D and D, G are joined.

Proof:

S.N.	Statements	S.N.	Reasons
1.	BF//DG	1.	In ΔBFC; DG joins the mid-points of BC and FC respectively.
2.	$\Delta EFG = \Delta EFD$	2.	Both are standing on the same base FE and between BE//DG.
3.	$\Delta FED = \frac{1}{2}\Delta BED$	3.	The median FD bisects ΔBED.
4.	$\Delta BED = \frac{1}{2}\Delta BAD$	4.	The median BE bisects ΔBAD.
5.	$\Delta BAD = \frac{1}{2}\Delta ABC$	5.	The median AD bisects ΔABC.
6.	$\Delta EFG = \frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\Delta ABC \therefore \Delta ABC = 8\Delta EFG$	6.	From statements (2), (3), (4) and (5)
			Proved

6. In a pentagon PENTA; M is any point on side NT so that EN//PM//AT. Prove that the area of triangle NPT is equal to the area of quadrilateral PEMA.



Solution:

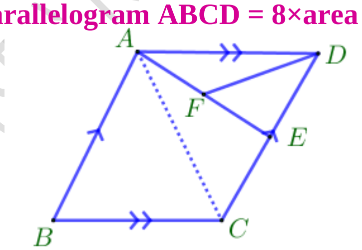
Given: In pentagon PENTA; M is any point on side NT so that EN//PM//AT

To prove: Area of ΔNPT = Area of quadrilateral PEMA.

Proof:

S.N.	Statements	S.N.	Reasons
1.	$\Delta PNM = \Delta PEM$	1.	Both are standing on the same base PM and between EN//PM.
2.	$\Delta PMT = \Delta PAM$	2.	Both are standing on the same base PM and between AT//PM.
3.	$\Delta PNM + \Delta PMT = \Delta PEM + \Delta PAM$	3.	Adding statements (1) and (2).
4.	$\Delta NPT = PEMA$	4.	From statement (3), by whole part axiom.
			Proved

7. In parallelogram ABCD; E is the mid-point of side CD and F is the mid-point of AE. Prove that area of parallelogram ABCD = 8×area of ΔAFD.



Solution:

Given: In parallelogram ABCD; E is the mid-point of side CD and F is the mid-point of AE

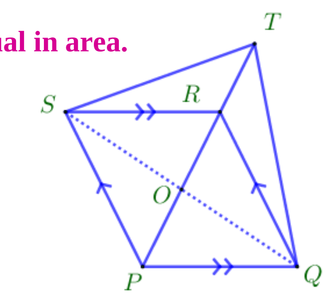
To prove: Area of parallelogram ABCD = 8×Area of ΔAFD.

Construction: A and C are joined.

Proof:

S.N.	Statements	S.N.	Reasons
1.	$\Delta AFD = \frac{1}{2}\Delta AED$	1.	The median FD bisects the ΔAED.
2.	$\Delta AED = \frac{1}{2}\Delta ACD$	2.	The median AE bisects the ΔACD.
3.	$\Delta ACD = \frac{1}{2}$ ▱ABCD	3.	The diagonal AC bisects the parallelogram ABCD.
4.	$\Delta AFD = \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}$ ▱ABCD ∴ ▱ABCD = 8ΔAFD	4.	From statements (1), (2) and (3).
			Proved

8. In parallelogram PQRS; diagonal PR is produced to the point T. Prove that the ΔRST and ΔRQT are equal in area.



Solution:

Given: In parallelogram PQRS; diagonal PR is produced to the point T.

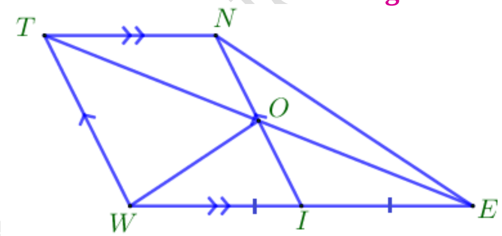
To prove: Area of ΔRST = Area of ΔRQT

Construction: S and Q are joined so that the diagonals PR and SQ intersect at O.

Proof:

S.N.	Statements	S.N.	Reasons
1.	O is the mid-point of SQ.	1.	The diagonals of parallelogram bisect each other.
2.	ΔSOT = ΔQOT	2.	The median OT bisects the ΔSQT.
3.	ΔSOR = ΔQOR	3.	The median OR bisects the ΔSQR.
4.	ΔSOT – ΔSOR = ΔQOT– ΔQOR	4.	Subtracting statement (2) from statement (1)
5.	ΔRST = ΔRQT	5.	From statement (4)
			Proved

9. In parallelogram TWIN; side WI is produced to E such that WI = IE. TE and NI intersect at O. Prove that the area of triangle TWO is equal to twice the area of triangle ONE.



Solution:

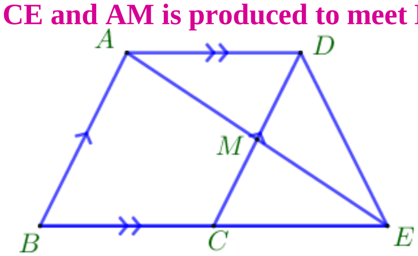
Given: In parallelogram TWIN; WI = IE. TE and NI intersect at O.

To prove: ΔTWO = 2ΔONE

Proof:

S.N.	Statements	S.N.	Reasons
1.	ΔTWO = $\frac{1}{2}$ ▱TWIN	1.	Both are standing on same base TW and between NI//TW.
2.	ΔTEN = $\frac{1}{2}$ ▱TWIN	2.	Both are standing on same base TW and between NI//TW.
3.	ΔTWO = ΔTEN	3.	From statements (1) and (2)
3.	O is the mid-point of TE.	3.	In ΔTWE; OI//TW and WI = IE.
4.	ΔONE = $\frac{1}{2}$ ΔTEN	4.	The median NO bisects ΔTEN.
5.	ΔONE = $\frac{1}{2}$ ΔTWO ∴ ΔTWO = 2ΔONE	5.	From statements (2) and (4)
			Proved

10. In parallelogram ABCD; M is the mid-point of side CD. If side BC is produced to E such that BC = CE and AM is produced to meet BE at E. Prove that the area of parallelogram ABCD is four times the area of ΔDME.



Solution:

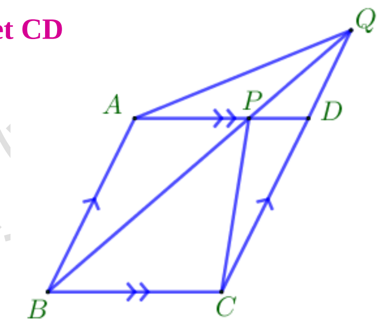
Given: In parallelogram ABCD; M is the mid-point of side CD. If side BC is produced to E such that BC = CE and AM is produced to meet BE at E

To prove: Area of ▱ABCD = 4 Area of ΔDME

Proof:

S.N.	Statements	S.N.	Reasons
1.	M is the mid-point of AE.	1.	In ΔABE; BC = CE and CM//BA.
2.	ΔDME = $\frac{1}{2}$ΔDAE	2.	The median DM bisects ΔDAE.
3.	ΔDAE = $\frac{1}{2}$ ▱ABCD	3.	Both are standing on same base AD and between AD//BE.
4.	ΔDME = $\frac{1}{2} \times \frac{1}{2}$ ▱ABCD ∴ ▱ABCD = 4ΔDME	4.	From statements (2) and (3)
			Proved

11. In quadrilateral ABCD; AB//CD and BC//AD. P is any point on the side AD and BP is produced to meet CD produced at Q. Prove that the area of triangles APQ and PDC are equal.



Solution:

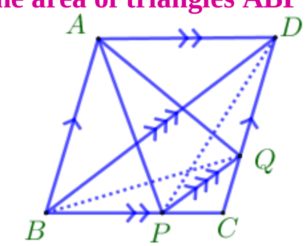
Given: In quadrilateral ABCD; AB//CD and BC//AD. P is any point on the side AD and BP is produced to meet CD produced at Q.

To prove: Area of ΔAPQ = Area of ΔPDC

Proof:

S.N.	Statements	S.N.	Reasons
1.	ΔPBC = $\frac{1}{2}$ ▱ABCD	1.	Both are standing on same base BC and between AD//BC.
2.	ΔPAB + ΔPDC = $\frac{1}{2}$ ▱ABCD	2.	Remaining parts of parallelogram ABCD.
3.	ΔABQ = $\frac{1}{2}$ ▱ABCD	3.	Both are standing on same base AB and between AB//QC.
4.	ΔPAB + ΔPDC = ΔABQ	4.	From statements (2) and (3)
5.	ΔPAB + ΔPDC = ΔAPQ + ΔPAB ∴ΔPDC = ΔAPQ	5.	ΔABQ = ΔAPQ + ΔPAB
			Proved

12. In parallelogram ABCD; P and Q are any points on BC and CD respectively such that BD//PQ. Prove that the area of triangles ABP and AQD are equal.



Solution:

Given: In parallelogram ABCD; P and Q are the points on BC and CD respectively such that BD//PQ.

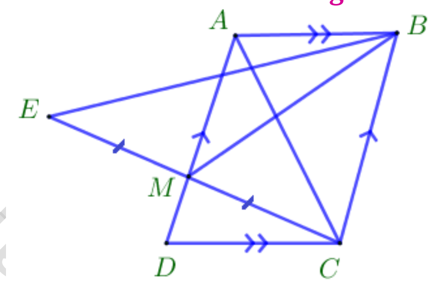
To prove: Area of ΔABP = Area of ΔAQD

Construction: B, Q and P, D are joined.

Proof:

S.N.	Statements	S.N.	Reasons
1.	ΔABP = ΔDBP	1.	Both are standing on same base BP and between AD//BP.
2.	ΔDBP = ΔDBQ	2.	Both are standing on same base BD and between PQ//BD.
3.	ΔDBQ = ΔAQD	3.	Both are standing on same base QD and between AB//QD.
4.	ΔABP = ΔAQD	4.	From statements (1), (2) and (3)
			Proved

13. In parallelogram ABCD; M is any point on side AD. CM is produced to E such that CM = ME. Prove that the area of triangles BEM and ADC are equal.



Solution:

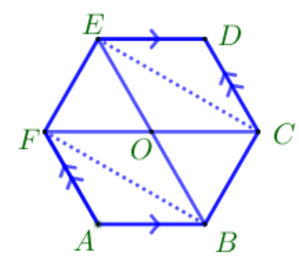
Given: In parallelogram ABCD; M is any point on side AD. CM is produced to E such that CM = ME.

To prove: Area of ΔBEM = Area of ΔADC

Proof:

S.N.	Statements	S.N.	Reasons
1.	ΔBEM = ΔBMC	1.	The median BM bisects the ΔBEC.
2.	ΔBMC = $\frac{1}{2}$ ▱ABCD	2.	Both are standing on same base BC and between AD//BC.
3.	ΔADC = $\frac{1}{2}$ ▱ABCD	3.	The diagonal AC bisects the parallelogram ABCD.
4.	ΔBEM= ΔADC	4.	From statements (1), (2) and (3)
			Proved

14. In a hexagon ABCDEF; AB//FC//ED and AF//BE//CD. If the diagonals BE and CF intersect at O such that parallelograms ABOF and OCDE are equal in area, prove that BC//FE.



Solution:

Given: In a hexagon ABCDEF; AB//FC//ED and AF//BE//CD. The diagonals BE and CF intersect at O. Area of parallelograms ABOF = Area of parallelogram OCDE.

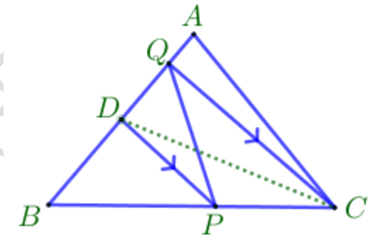
To prove: BC//FE

Construction: B, F and C, E are joined.

Proof:

S.N.	Statements	S.N.	Reasons
1.	▱ABOF = ▱OCDE	1.	Given
2.	ΔBOF = $\frac{1}{2}$▱ABOF and ΔCOE = $\frac{1}{2}$▱OCDE	2.	The diagonals bisect the parallelograms.
3.	ΔBOF = ΔCOE	3.	From statements (1) and (2)
4.	ΔFBC = ΔEBC	4.	Adding ΔBOC in statement (3)
5.	BC//FE	5.	From statement (4), triangles on the same base BC have equal areas.
			Proved

15. In ΔABC; D is the mid-point of side AB. If P is any point on BC and Q is any point on AD such that CQ//PD, prove that the area of ΔBPQ is half of the area of ΔABC.



Solution:

Given: In ΔABC; D is the mid-point of side AB. P is any point on BC and Q is any point on AD such that CQ//PD.

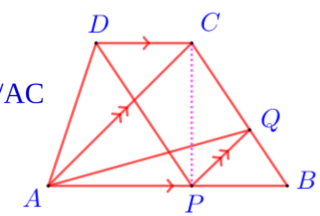
To prove: Area of ΔBPQ = $\frac{1}{2}$ Area of ΔABC

Construction: D and C are joined.

Proof:

S.N.	Statements	S.N.	Reasons
1.	ΔQDP = ΔCDP	1.	Both are standing on the same base DP and between QC//DP.
2.	ΔQDP +ΔBPD = ΔCDP+ΔBPD	2.	Adding ΔBPD on both sides of statement (1)
3.	ΔBPQ = ΔBDC	3.	By whole part axiom
4.	ΔBDC = $\frac{1}{2}$ ΔABC	4.	The median DC bisects the ΔABC.
5.	ΔBPQ = $\frac{1}{2}$ ΔABC	5.	From statements (3) and (4).
			Proved

16. In a trapezium ABCD, AB//CD, the point P and the point Q lie on the side AB and BC respectively. If AC//PQ, prove that the area of ΔADP = area of ΔACQ. (N-PABSAN 2078)



Solution:

Given: In trapezium ABCD; AB//CD. Points P and Q lie on the sides AB and BC respectively. PQ//AC

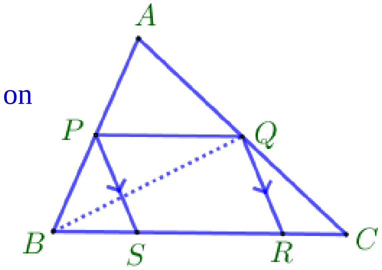
To prove: Area of ΔADP = Area of ΔACQ.

Construction: C and P are joined.

Proof:

S.N.	Statements	S. N.	Reasons
1.	ΔADP = ΔACP	1.	Both are standing on the same base AP and between DC//AP.
2.	ΔACP = ΔACQ	2.	Both are standing on the same base AC and between AC//PQ.
3.	ΔADP = ΔACQ	3.	From statements (1) and (2).
			Proved

17. In ΔABC; P and Q are the mid-points of sides AB and AC respectively. If S and R are the points on BC so that PS//QR, prove that the area of ΔABC is double of area of quadrilateral PQRS.



Solution:

Given: In ΔABC; P and Q are the mid-points of sides AB and AC respectively, S and R are the points on BC so that PS//QR.

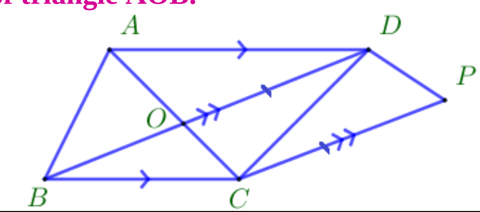
To prove: Area of ΔABC = 2×Area of quadrilateral PQRS.

Construction: B and Q are joined.

Proof:

S.N.	Statements	S.N.	Reasons
1.	PQ//BC	1.	In ΔABC; PQ joins the mid-points P and Q of AB and AC respectively.
2.	ΔPBQ $= \frac{1}{2}$▱PQRS	2.	Both are standing on the same base PQ and between PQ//BC.
3.	ΔPBQ $= \frac{1}{2}$ΔABQ	3.	The median QP bisects the ΔABQ.
4.	ΔABQ = ▱PQRS	4.	From statements (2) and (3).
5.	ΔABQ $= \frac{1}{2}$ ΔABC	5.	The median BQ bisects the ΔABC.
6.	▱PQRS $= \frac{1}{2}$ ΔABC ∴ΔABC = 2▱PQRS	6.	From statements (4) and (5).
			Proved

18. In a quadrilateral ABCD; AD//BC and diagonals AC and BD intersect at O. From the vertex C, CP//BD is drawn such that OD = CP and a quadrilateral OCPD is formed. Prove that the area of quadrilateral OCPD is twice the area of triangle AOB.



Solution:

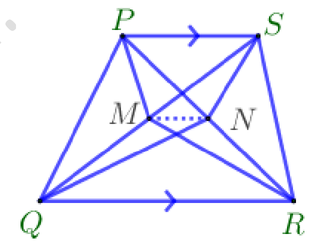
Given: In a quadrilateral ABCD; AD//BC and diagonals AC and BD intersect at O. From the vertex C, CP//BD is drawn such that OD = CP.

To prove: Area of quadrilateral OCPD = 2×Area of ΔAOB.

Proof:

S.N.	Statements	S.N.	Reasons
1.	ΔBAD = ΔCAD	1.	Both are standing on the same base AD and between AD//BC.
2.	ΔAOB = ΔCOD	2.	Subtracting ΔAOD from both sides of statement (1).
3.	OC//DP and OC = DP	3.	OD//CP and OD = CP
4.	ΔCOD $= \frac{1}{2}$▱OCPD	4.	The diagonal CD bisects the parallelogram OCPD.
5.	ΔAOB $= \frac{1}{2}$▱OCPD ∴▱OCPD = 2ΔAOB	5.	From statements (2) and (4)
			Proved

19. In trapezium PQRS, M and N are the middle points of diagonals QS and PR respectively. Prove that the area of triangle PMR is equal to the area of triangle SNQ.



Solution:

Given: In trapezium PQRS, M and N are the middle points of diagonals QS and PR respectively.

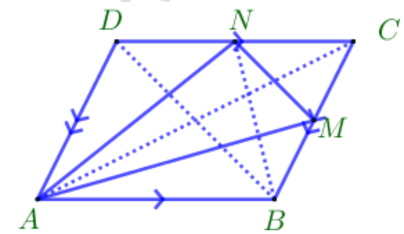
To prove: Area of ΔPMR= Area of ΔSNQ.

Construction: M and N are joined.

Proof:

S.N.	Statements	S.N.	Reasons
1.	ΔPQR = ΔSQR	1.	Both are standing on the same base QR and between PS//QR.
2.	ΔPQR = 2ΔNQR and ΔSQR = 2ΔMQR	2.	The median QN bisects ΔPQR and the median MR bisects ΔSQR.
3.	ΔNQR = ΔMQR	3.	From statements (1) and (2).
4.	MN//QR	4.	From statement (3).
5.	ΔPMN = ΔSMN	5.	Both are standing on the same base MN and between PS//MN.
6.	ΔMNR = ΔMNQ	6.	Both are standing on the same base MN and between QR//MN.
7.	ΔPMN + ΔMNR = ΔSMN + ΔMNQ	7.	Adding statements (5) and (6)
8.	ΔPMR= ΔSNQ	8.	By whole part axiom.
			Proved

20. M and N are the mid-points of sides BC and CD respectively of a parallelogram ABCD. Prove that 3 times of area of parallelogram ABCD is equal to 8 times the area of triangle MAN.



Solution:

Given: In parallelogram ABCD, M and N are the middle points of sides BC and CD respectively

To prove: 3×Area of ▱ABCD= 8×Area of ΔMAN.

Construction: B to D and N and A to C are joined.

Proof:

S.N.	Statements	S.N.	Reasons
1.	ΔNAD = $\frac{1}{2}$ΔCAD and ΔABM = $\frac{1}{2}$ΔABC	1.	The median AN bisects ΔADC and the median AM bisects ΔABC.
2.	ΔADC = ΔABC = $\frac{1}{2}$▱ABCD	2.	The diagonal AC bisects ▱ABCD.
3.	ΔNAD = ΔABM = $\frac{1}{2}\times\frac{1}{2}$▱ABCD = $\frac{1}{4}$▱ABCD	3.	From statements (1) and (2).
4.	ΔMNC = $\frac{1}{2}$ΔBNC and ΔBNC = $\frac{1}{2}$ΔBDC	4.	The median MN bisects ΔBNC and the median BN bisects ΔBCD.
5.	ΔBCD = $\frac{1}{2}$▱ABCD	5.	The diagonal BD bisects ▱ABCD.
6.	ΔMNC = $\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}$▱ABCD = $\frac{1}{8}$▱ABCD	6.	Both are standing on the same base MN and between QR//MN.
7.	ΔMAN + ΔNAD + ΔABM + ΔMNC = ▱ABCD	7.	By whole part axiom.
8.	ΔMAN + $\frac{1}{4}$▱ABCD + $\frac{1}{4}$▱ABCD + $\frac{1}{8}$▱ABCD = ▱ABCD or, ΔMAN + $\frac{5}{8}$▱ABCD = ▱ABCD or, ΔMAN = ▱ABCD - $\frac{5}{8}$▱ABCD or, ΔMAN = $\frac{3}{8}$▱ABCD ∴3▱ABCD = 8 ΔMAN	8.	From statements (3), (6) and (7)

Proved