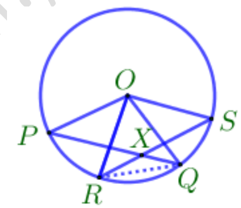
Important Extra Theorems (Circle)

1. ***In a circle with centre O; two chords PQ and RS intersect at a point X. Prove that ∠POR + ∠QOS = 2∠PXR.***



Solution:

Given: O is the centre of circle. The chords PQ and RS intersect at a point X.

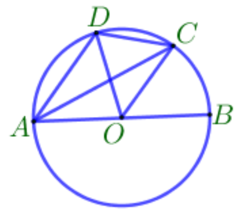
To prove: ∠POR + ∠QOS = 2∠PXR

Construction: R and Q are joined.

Proof:

S.N.	Statements	S.N.	Reasons
1.	$\angle PQR = \frac{1}{2}\angle POR$	1.	The inscribed angle is half of the central angle standing on the same arc PR.
2.	$\angle QRS = \frac{1}{2}\angle QOS$	2.	The inscribed angle is half of the central angle standing on the same arc QS.
3.	∠PXR = ∠PQR + ∠QRS	3.	The exterior angle of ΔRXQ is equal to the sum of two opposite interior angles.
4.	$\angle PXR = \frac{1}{2}\angle POR + \frac{1}{2}\angle QOS$ $\therefore \angle POR + \angle QOS = 2\angle PXR$	4.	From statements (1), (2) and (3)
			Proved

2. ***In a circle centered at O; AB is a diameter. C and D are two points on the circumference on the same side of AB such $\widehat{BC} = \widehat{CD}$. Prove that area of ΔAOC = area of ΔCOD.***



Solution:

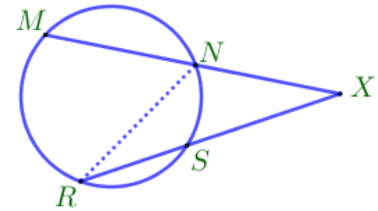
Given: O is the centre of circle, AB is the diameter and $\widehat{BC} = \widehat{CD}$.

To prove: Area of ΔAOC = Area of ΔCOD.

Proof:

S.N.	Statements	S.N.	Reasons
1.	$\widehat{BD} = 2\widehat{BC}$	1.	$\widehat{BC} = \widehat{CD}$
2.	$\angle BAD = \frac{1}{2}\ \widehat{BD}^o$	2.	Relation between the inscribed angle and its opposite arc.
3.	$\angle BAD = \frac{1}{2} \times 2\widehat{BC}^o = \widehat{BC}^o$	3.	From statements (1) and (2)
4.	$\angle BOC = \widehat{BC}^o$	4.	Relation between the centre angle and its opposite arc.
5.	∠BAD = ∠BOC	5.	From statements (3) and (4)
6.	AD//OC	6.	From statement (5), the corresponding angles are equal.
7.	Area of ΔAOC = Area of ΔCOD	7.	Both are standing on the same base OC and between AD//OC.
			Proved

3. *In a circle; if the chords MN and RS intersect at an external point X. Prove that* $\angle MXR = \frac{1}{2}(\widehat{MR}^o - \widehat{NS}^o)$.



Solution:

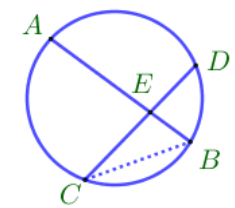
Given: Two chords MN and RS intersect at an external point X.

To prove: $\angle MXR = \frac{1}{2}(\widehat{MR}^o - \widehat{NS}^o)$

Proof:

S.N.	Statements	S.N.	Reasons
1.	$\angle MNR = \frac{1}{2}\widehat{MR}^o$ and $\angle NRX = \frac{1}{2}\widehat{NS}^o$	1.	Relation between the inscribed angles and their opposite arcs.
2.	$\angle MNR = \angle NRX + \angle MXR$	2.	The exterior angle of ΔNRX is equal to the sum of two opposite interior angles.
3.	$\frac{1}{2}\widehat{MR}^o = \frac{1}{2}\widehat{NS}^o + \angle MXR$	3.	From statements (1) and (2).
4.	$\angle MXR = \frac{1}{2}\widehat{MR}^o - \frac{1}{2}\widehat{NS}^o = \frac{1}{2}(\widehat{MR}^o - \widehat{NS}^o)$	4.	From statement (3).
			Proved

4. *In a circle; if the chords AB and CD intersect at an internal point E. Prove that* $\angle AEC = \frac{1}{2}(\widehat{AC} + \widehat{BD})$.



Solution:

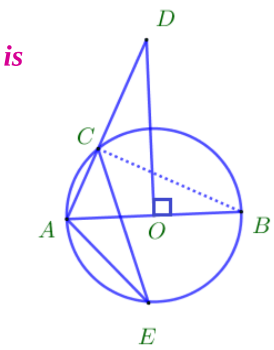
Given: Two chords AB and CD intersect at an internal point E.

To prove: $\angle AEC = \frac{1}{2}(\widehat{AC}^o + \widehat{BD}^o)$

Proof:

S.N.	Statements	S.N.	Reasons
1.	$\angle ABC = \frac{1}{2}\widehat{AC}^o$ and $\angle BCD = \frac{1}{2}\widehat{BD}^o$	1.	Relation between the inscribed angles and their opposite arcs.
2.	$\angle AEC = \angle ABC + \angle BCD$	2.	The exterior angle of ΔNRX is equal to the sum of two opposite interior angles.
3.	$\angle AEC = \frac{1}{2}\widehat{AC}^o + \frac{1}{2}\widehat{BD}^o = \frac{1}{2}(\widehat{AC}^o + \widehat{BD}^o)$	3.	From statements (1) and (2).
			Proved

5. ***O is the centre of a circle, AB is a diameter. D is an external point of circle such that DO⊥AB. AD cuts the circle at C and E is any point on the circumference. Prove that ∠AEC = ∠ODA.***



Solution:

Given: O is the centre of a circle, AB is a diameter. D is an external point of circle such that DO⊥AB. AD cuts the circle at C and E is any point on the circumference.

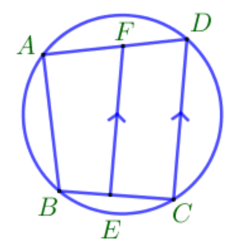
To prove: ∠AEC = ∠ODA

Construction: B and C are joined.

Proof:

S.N.	Statements	S.N.	Reasons
1.	∠ACB = 90^0	1.	The angle in semi-circle is always a right angle.
2.	In ΔABC and ΔADO (i) ∠ACB = ∠AOD (ii) ∠CAB = ∠DAO (iii) ∠ABC = ∠ODA	2.	(i) Both are right angles (ii) Common angle (iii) Remaining angles
3.	∠ABC = ∠AEC	3.	Botha are standing on the same arc AC.
4.	∠AEC = ∠ODA	4.	From statements 2, (iii) and (3)
			Proved

6. ***In a cyclic quadrilateral ABCD; E and F are the points on BC and AD respectively so that CD//EF. Prove that ABEF is also a cyclic quadrilateral.***



Solution:

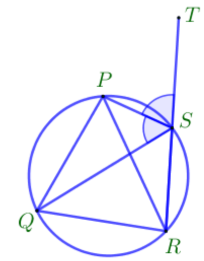
Given: In a cyclic quadrilateral ABCD; E and F are the points on BC and AD respectively so that CD//EF.

To prove: ABEF is also a cyclic quadrilateral.

Proof:

S.N.	Statements	S.N.	Reasons
1.	∠AFE = ∠ADC	1.	EF//CD and corresponding angles.
2.	∠ABC + ∠ADC = 180^0	2.	The opposite angles of cyclic quadrilateral are supplementary.
3.	∠ABC + ∠AFE = 180^0	3.	Adding statements (1) and (2).
4.	ABEF is also a cyclic quadrilateral	4.	From statement (3)
			Proved

7. ***PQRS is a cyclic quadrilateral. RS is produced to T. If PS is an angular bisector of ∠QST, prove that ΔPQR is an isosceles triangle.***



Solution:

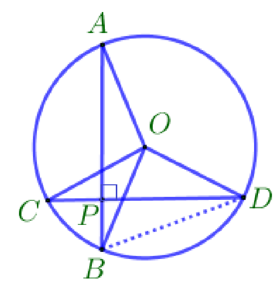
Given: PQRS is a cyclic quadrilateral. RS is produced to T and ∠PSQ= ∠PST.

To prove: ΔPQR is an isosceles triangle

Proof:

S.N.	Statements	S.N.	Reasons
1.	∠PRQ = ∠PSQ	1.	The inscribed angles standing on the same arc are equal.
2.	∠PQR = ∠PST	2.	The exterior angle of cyclic quad. is equal to opposite interior angle.
3.	∠PSQ= ∠PST	3.	Given
4.	∠PQR= ∠PRQ	4.	From statements (1), (2) and (3).
5.	ΔPQR is an isosceles triangle	5.	From statement (4)
			Proved

8. ***O is the centre of a circle. Two chords AB and CD intersect perpendicularly at P. Prove that the angles AOD and BOC are supplementary.***



Solution:

Given: O is the centre of circle. Chords AB ⊥ CD.

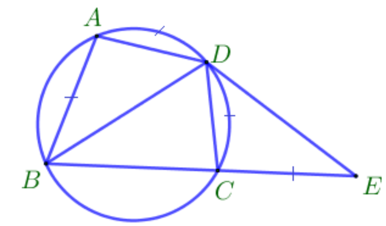
To prove: $\angle AOD + \angle BOC = 180^0$

Construction: B and D are joined.

Proof:

S.N.	Statements	S.N.	Reasons
1.	$\angle ABD = \frac{1}{2}\angle AOD$	1.	The inscribed angle is half of the central angle on the same arc AD.
2.	$\angle BDC = \frac{1}{2}\angle BOC$	2.	The inscribed angle is half of the central angle on the same arc AD.
3.	∠ABD + ∠BDC = ∠APC	3.	The exterior angle of ΔBPD is equal to the sum of two opposite interior angles.
4.	$\frac{1}{2}\angle AOD + \frac{1}{2}\angle BOC = 90^0$ $\therefore \angle AOD + \angle BOC = 180^0$	4.	From statements (1), (2) and (3), $\angle APC = 90^0$
			Proved

9. ***The side BC of a cyclic quadrilateral ABCD is extended to E so that AB = CE and $\overset{\frown}{AD} = \overset{\frown}{CD}$. Prove that BED is an isosceles triangle.***



Solution:

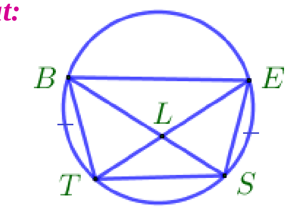
Given: The side BC of a cyclic quadrilateral ABCD is extended to E so that AB = CE and $\overset{\frown}{AD} = \overset{\frown}{CD}$

To prove: BED is an isosceles triangle

Proof:

S.N.	Statements	S.N.	Reasons
1.	In ΔBAD and ΔCDE (i) AB = CE (S) (ii) ∠BAD = ∠DCE (A) (iii) AD = CD (S)	1.	(i) Given (ii) The exterior angle of cyclic quad is equal to opposite interior angle (iii) Chords corresponding to equal arcs are equal.
2.	ΔBAD ≅ ΔCDE	2.	By S.A.S. axiom
3.	BD = DE	3.	The corresponding sides of congruent triangles are equal.
3.	BED is an isosceles triangle	3.	From statement (3)
			Proved

10. ***The points B, E, S and T are concyclic such that arc BT = arc SE. If the chords BS and ET intersect at the point L, prove that: (i) Area of ΔBLT = Area of ΔSEL (ii) chord BS = chord ET.***



Solution:

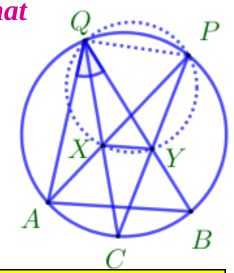
Given: Arc BT = Arc SE, Chords BS and ET intersect at the point L.

To prove: (i) Area of ΔBLT = Area of ΔSEL (ii) chord BS = chord ET.

Proof:

S.N.	Statements	S.N.	Reasons
1.	BE//TS	1.	Arc BT = Arc SE
2.	ΔBTS = ΔETS	2.	Both are standing on the same base TS and between BE//TS.
3.	ΔBLT = ΔSEL	3.	Subtracting ΔLTS from both sides of statement (2).
4.	Arc BT + Arc TS = Arc SE + Arc TS	4.	Adding Arc TS on both sides of Arc BT = Arc SE
5.	Arc BTS = Arc TSE	5.	By whole part axiom.
6.	Chord BS = Chord ET	6.	From statement (5), chords corresponding to equal arcs.
			Proved

11. ***In a circle; chords AP and CQ intersect at an internal point X and the chords CP and BQ intersect at an internal point Y so that CQ is the bisector of ∠AQB. Prove that XY//AB.***



Solution:

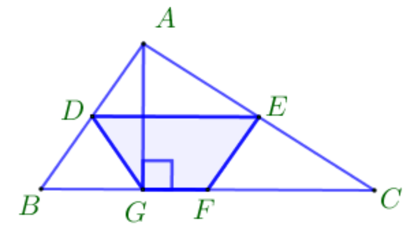
Given: In a circle; chords AP and CQ intersect at a point X and the chords CP and BQ intersect at a point Y and ∠AQC = ∠BQC

To prove: XY//AB

Proof:

S.N.	Statements	S.N.	Reasons
1.	∠AQC = ∠BQC	1.	Given
2.	∠AQC = ∠APC	2.	The inscribed angles standing on the same arc AC.
3.	∠BQC = ∠APC	3.	From statements (1) and (2).
4.	Points X, Y, P and Q are concyclic.	4.	From statements (3), segment XY subtends equal inscribed angles.
5.	∠QPA = ∠QYX	5.	The inscribed angles standing on the same arc QX.
6.	∠QPA = ∠QBA	6.	The inscribed angles standing on the same arc QA.
7.	∠QYX = ∠QBA	7.	From statements (5) and (6)
8.	XY//AB	8.	From statement (7), corresponding angles are equal.
			Proved

12. ***In ΔABC; D, E and F are the mid-points of sides AB, AC and BC respectively and G is any point on BC so that AG⊥BC. Prove that DEFG is a cyclic quadrilateral.***



Solution:

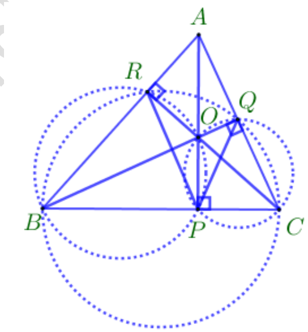
Given: In ΔABC; D, E and F are the mid-points of sides AB, AC and BC respectively and G is any point on BC so that AG⊥BC.

To prove: DEFG is a cyclic quadrilateral.

Proof:

S.N.	Statements	S.N.	Reasons
1.	DE//BC and EF//AB	1.	In ΔABC; DE and EF join the mid-points of sides AB, AC and BC respectively.
2.	DEFB is a parallelogram	2.	From statement (1).
3.	∠DEF = ∠DBF	3.	The opposite angles of parallelogram DEFB are equal.
4.	BD = GD = AD	4.	The mid-point of hypotenuse of a right angled triangle is equidistance from its each vertex.
5.	∠DGB = ∠DBG i.e., ∠DGB =∠DBF	5.	The base angles of isosceles ΔDBG.
6.	∠BEF = ∠DGB	6.	From statements (3) and (5)
7.	DEFG is a cyclic quadrilateral	7.	From statement (6); the exterior is equal to opposite interior angle of quadrilateral DEFG.
			Proved

13. ***In ΔABC; the altitudes AP, BQ and CR intersect at O. Prove that ∠OPQ = ∠OPR.***



Solution:

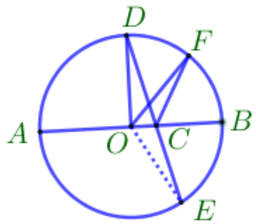
Given: In ΔABC; AP⊥BC, BQ⊥AC and CR⊥AB. AP, BQ and CR intersect at O.

To prove: ∠OPQ = ∠OPR

Proof:

S.N.	Statements	S.N.	Reasons
1.	∠BRO +∠BPO = 180^0, ∠OPC +∠OQC = 180^0, ∠BRC = ∠BQC	1.	From figure.
2.	BPOR, POQC and BCQR are cyclic quadrilaterals	2.	From statement (1)
3.	∠RBO =∠OPR, ∠OPQ =∠OCQ, ∠RBO =∠OCQ	3.	Pair of inscribed angles on the arcs RO, OQ and RQ respectively.
4.	∠OPQ = ∠OPR	4.	From statement (3)
			Proved

14. ***In a circle; O is the centre and AB is the diameter. C is any point on the diameter AB. Chord DE passes through the point C and F is on the miner arc BD so that CE = CF. Prove that ∠ODC and ∠OFC are equal.***



Solution:

Given: O is the centre of circle. AB is the diameter. CE = CF.

To prove: ∠ODC = ∠OFC

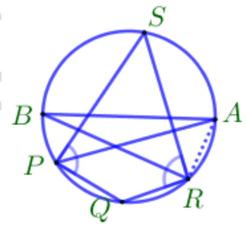
Construction: O and E are joined.

Proof:

S.N.	Statements	S.N.	Reasons
1.	In ΔCOF and ΔCOE (i) OF = OE (S) (ii) OC = OC (S) (iii) CF = CE (S)	1.	 (i) Radii (ii) Common side (iii) Given
2.	ΔCOF≅ ΔCOE	2.	By S.S.S. axiom
3.	∠OFC = ∠OEC	3.	The corresponding angles of congruent triangles.
4.	∠ODE = ∠OED i.e., ∠ODC = ∠OEC	4.	OD = OE and base angles of isosceles ΔODE.
5.	∠ODC = ∠OFC	5.	From statements (3) and (4)
			Proved

15. ***PQRS is a cyclic quadrilateral. If the bisectors of ∠QPS and ∠QRS meet the circle at the points A and B respectively then prove that AB is the diameter of the circle.***

Solution:



Given: PQRS is a cyclic quadrilateral. ∠QPA = ∠SPA and ∠QRB = ∠SRB.

To prove: AB is the diameter.

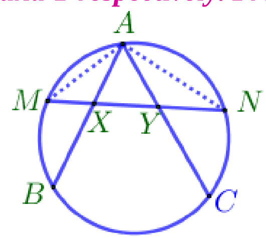
Construction: A and R are joined

Proof:

S.N.	Statements	S.N.	Reasons
1.	∠QPS = 2∠SPA and ∠QRS = 2∠SRB	1.	Given
2.	∠QPS +∠QRS = 180^0	2.	The opposite angles of cyclic quadrilateral PQRS.
3.	2∠SPA + 2∠SRB = 180^0 ∴∠SPA + ∠SRB = 90^0	3.	From statements (1) and (2)
4.	∠SPA=∠SRA	4.	The inscribed angles standing on the same arc SA.
5.	∠SRA + ∠SRB = 90^0	5.	From statements (3) and (4)
6.	∠BRA = 90^0	6.	By whole part axiom
7.	AB is a diameter	7.	From statement (6)
			Proved

16. ***M and N are the mid-points of arc AB and arc AC respectively. The chords AB and AC cut off the chord MN at X and Y respectively. Prove that AX = AY.***

Solution:



Given: M and N are the mid-points of arc AB and arc AC respectively. The chords AB and AC cut off the chord MN at X and Y respectively.

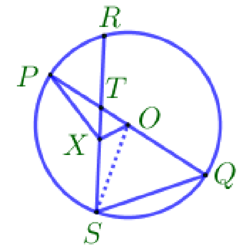
To prove: AX = AY.

Construction: M and N are joined to A.

Proof:

S.N.	Statements	S.N.	Reasons
1.	∠MAB =∠ANM	1.	Arc AM = Arc BM.
2.	∠AMN = ∠NAC	2.	Arc AN = Arc CN.
3.	∠MAB + ∠AMN =∠ANM +∠NAC	3.	Adding statements (1) and (2).
4.	∠AXY =∠AYX	4.	From (3), the ext. angle of triangle is equal to the sum of two opposite interior angles.
5.	AX = AY	5.	From statement (4)
			Proved

17. ***PQ is a diameter of circle with centre O. The chord RS intersects the diameter PQ internally at a point T. If X is a point on RS such that PX = SX, prove that OX//QS.***



Solution:

Given: O is the centre of circle. PQ is a diameter. The chord RS intersects the diameter PQ internally at a point T. X is a point on RS such that PX = SX

To prove: OX//QS

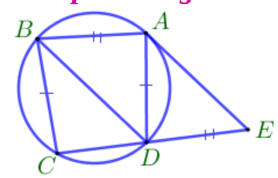
Construction: O and S are joined.

Proof:

S.N.	Statements	S.N.	Reasons
1.	In ΔPOX and ΔSOX (i) OP = OS (S) (ii) OX= OX (S) (iii) PX = SX (S)	1.	(i) Radii (ii) Common side (iii) Given
2.	ΔPOX ≅ ΔSOX	2.	By S.S.S. axiom
3.	∠POX = ∠SOX	3.	The corresponding angles of congruent triangles are equal.
4.	∠POS = 2∠POX	4.	From statement (3).
5.	∠POS = 2∠PQS	5.	The inscribed angles standing on the same arc PS are equal.
6.	∠POX = ∠PQS	6.	From statements (4) and (5).
7.	OX//QS	7.	From statement (6); corresponding angles are equal

Proved

18. ***In a cyclic quadrilateral ABCD; BC = AD and the side CD is produced to E such that AB = ED. Prove that ABDE is a parallelogram.***



Solution:

Given: In a cyclic quadrilateral ABCD; BC = AD and the side CD is produced to E such that AB = ED

To prove: ABDE is a parallelogram

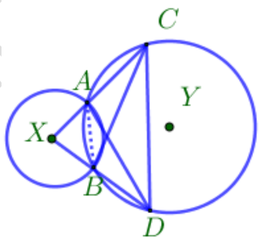
Proof:

S.N.	Statements	S.N.	Reasons
1.	Arc BC = Arc AD	1.	Chord BC = Chord AD
2.	BA//CD i.e., BA//CE	2.	From statement (1)
3.	BA = DE	3.	Given
4.	BD//AE and BD = AE	4.	BA//DE and BA = DE
5.	ABDE is a parallelogram	5.	From statements (3) and (4)

Proved

19. *X and Y are the centers of two circles which intersect at A and B. XA and XB are produced to meet another circle at C and D respectively.*

Prove that (i) Area of ΔABC = Area of ΔABD (ii) Arc AC = Arc BD



Solution:

Given: X and Y are the centers of two circles which intersect at A and B. XA and XB are produced to meet another circle at C and D respectively.

To prove: (i) Area of ΔABC = Area of ΔABD (ii) Arc AC = Arc BD

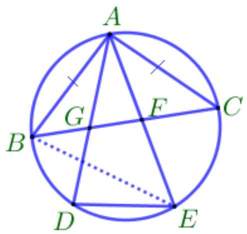
Construction: A and B are joined.

Proof:

S.N.	Statements	S.N.	Reasons
1.	AX = BX	1.	Radii of the circle centered at X.
2.	∠XAB = ∠XBA	2.	The base angles of isosceles ΔABX.
3.	∠XAB = ∠BDC	3.	The exterior angle of cyclic quadrilateral is equal to its opposite interior angle.
4.	∠XBA = ∠BDC	4.	From statements (3) and (4).
5.	AB//CD	5.	The corresponding angles are equal.
6.	ΔABC = ΔABD.	6.	Both are standing on the same base CD and between AB//CD.
7.	Arc AC = Arc BD	7.	From statement (5)
			Proved

20. *In a circle; the chords AB and AC are equal. Chords AD and AE cut the chord BC at the points G and F respectively.*

Prove that DEFG is a cyclic quadrilateral.



Solution:

Given: Chords AB = AC. AD and AE cut the chord BC at the points G and F respectively

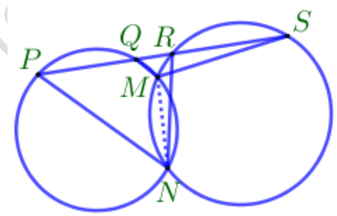
To prove: DEFG is acyclic quadrilateral.

Construction: B and E are joined.

Proof:

S.N.	Statements	S.N.	Reasons
1.	∠ABC = ∠ACB	1.	AB = AC
2.	∠ACB = ∠AEB	2.	The inscribed angles standing on the same arc AB are equal.
3.	∠ABC = ∠AEB	3.	From statements (1) and (2).
4.	∠BAD = ∠BED	4.	The inscribed angles standing on the same arc BD are equal.
5.	∠BAD + ∠ABC + ∠AGD = 180^0	5.	The sum of interior angles of triangle.
6.	∠BED + ∠AEB + ∠DGC = 180^0	6.	From statements (3), (4) and (5) and ∠AGD = ∠DGC
7.	∠AED + ∠DGC = 180^0	7.	∠BED + ∠AEB = ∠AED, by whole part axiom.
8.	DEFG is a cyclic quadrilateral	8.	From statement (7)
			Proved

21. ***Two circles intersect at the points M and N. The chord PQ produced of the first circle cuts the second circle at the points R and S. Prove that ∠QMS and ∠PNR are supplementary.***



Solution:

Given: Two circles intersect at the points M and N, the chord PQ produced of the first circle cuts the second circle at the points R and S

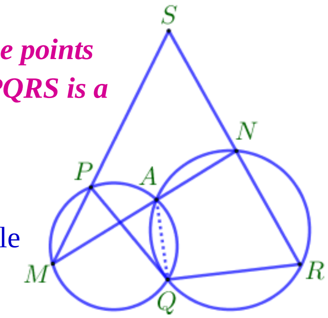
To prove: $\angle QMS + \angle PNR = 180^0$

Construction: M and N are joined.

Proof:

S.N.	Statements	S.N.	Reasons
1.	∠PNM = ∠MQS	1.	The exterior angle of cyclic quadrilateral is equal to its opposite interior angle.
2.	∠MNR = ∠QSM	2.	The inscribed angles standing on the same arc MR are equal.
3.	$\angle QMS + \angle MQS + \angle QSM = 180^0$	3.	The sum of interior angles of triangle.
4.	$\angle QMS + \angle PNM + \angle MNR = 180^0$	4.	From statements (1), (2) and (3)
5.	$\angle QMS + \angle PNR = 180^0$	5.	∠PNM + ∠MNR = ∠PNR
			Proved

22. ***Two circles intersect at the points A and Q. The chord MA produced of the first circle meets the second circle at the points N. The chord MP of the first circle and the chord RN of the second circle meet at an external point S. Prove that PQRS is a cyclic quadrilateral.***



Solution:

Given: Two circles intersect at the points A and Q. The chord MA produced of the first circle meets the second circle at the points N. The chord MP of the first circle and the chord RN of the second circle meet at an external point S.

To prove: PQRS is acyclic quadrilateral.

Construction: A and Q are joined.

Proof:

S.N.	Statements	S.N.	Reasons
1.	∠MPQ = ∠MAQ	1.	The inscribed angles standing on the same arc AB are equal.
2.	∠MAQ = ∠QRS	2.	The exterior angle of cyclic quadrilateral is equal to its opposite interior angle.
3.	∠MPQ = ∠QRS	3.	From statements (1) and (2).
4.	PQRS is a cyclic quadrilateral	4.	The exterior angle quadrilateral PQRS is equal to its opposite interior angle.
			Proved