UNIT-1 ALGEBRA

1.1 FUNCTION

Knowledge (K) Based Problems

1. (a) Define constant function with an example.
(b) State identity function with an example.

2. (a) Define composite function.
(b) What do you mean by inverse function?
(c) Under what condition, the inverse of a function is possible?

3. (a) Define trigonometric function. **[SEE MODEL-2076]**
(b) Find the maximum and minimum values of f (x) = sinx.
(c) Find the maximum and minimum values of f (x) = cosx.

4. (a) What is the period of the function f (x) = sinx?
(b) Write the period of the function f (x) = cosx.
(c) What is the period of the function f (x) = tanx?

Understanding (U) Based Problems

1. (a) If f = {(a, 1), (b, 2), (c, 3)} and g = {(1, p), (2, q), (3, r)}, show the function gof in an arrow diagram and write it in ordered pair form. *Ans: {(a, p), (b, q), (c, r)}*
(b) If f = {(1, 3), (0, 0), (-1, -3)} & g = {(0, 2), (-3, -1), (3, 5)}, write gof in ordered pair form by representing in a mapping diagram. *Ans:{(1, 5), (0, 2), (-1, -1)}*

2. (a) If f = {(-1, 1), (0, 2), (1, 3)} & gof = {(-1, 1), (0, 4), (1, 9)}, find the function g in ordered pair form by using an arrow diagram. *Ans: g = {(1, 1), (2, 4), (3, 9)}*
(b) If g = {(p, 1), (q, 2), (r, 3)} and fog = {(p, 4), (q, 4), (r, 4)}, find the function f in ordered pair form by the help of mapping diagram. *Ans: f = {(1, 4), (2, 4), (3, 4)}*

3. (a) If f(x) = 2x – 1 and g (x) = 4x, find the value of gof (x). *Ans:8x – 4* **[SEE MODEL-2076]**
(b) If f(x) = 3x + 2 and g (x) = 2x + 1, find fog (x). *Ans:6x + 5*

4. (a) If f(x) = x + 2, x∈ R and g (x) = 3x – 1, x∈ R; find (gof) (5). *Ans: 20*
(b) If g(x) = 4x + 15, x∈ R and h (x) = 3 – x, x∈ R; find (gof) (5). *Ans: 7*

5. (a) If f(x) = 6x + 7, calculate the value of ff (-1). *Ans:13*
(b) If g (x) = 3x + 2, find the value of gg (3). *Ans:35*

6. (a) If $g(x) = \frac{x+1}{2}$ and h(x) = 2x – 1are two functions, prove that hog (x) is an identity function.
(b) If g(x) =3x + 4, $h(x) = \frac{x-4}{3}$ are two functions, prove that hog (x) is an identity function.

7. (a) If f (x) = 3x and g (x) = x + 2, find fog(x) = 18, find the value of x. *Ans: 4*
(b) If f: x →3x + b and ff (2) = 12, find the value of b. *Ans:-3/2*
(c) If g: R →R: g: x →4x – p and gg (4) + 1 = 0, find the value of p. *Ans:13*

8. (a) If f (x) = 3x - 2 and fog (x) = 6x - 2, find g(x). *Ans: 2x*
(b) If f (x) = 5x - 6 and fog (x) = 10x – 1, find g(x). *Ans2x + 1*

9. (a) If g(x) = 2x + 2 and (fog)(x) = 8x + 13, find f (x) *Ans4x + 5*
(b) Let g: R →R: g (x) = 4 - x and (fog) (x) = 11 – 2x then find f (x). *Ans: 2x + 3*

10. (a) If f (x + 3) = 2x + 1, evaluate f (3). *Ans: 1*
(b) If f (2x + 1) = 4x + 5, evaluate f (1). *Ans: 5*

11. (a) Find $f^{-1}(x)$ if f (x) = 4x + 5. **[SEE MODEL-2076]** *Ans (x -5) /4*
(b) If f (x) = 8x – 3 is a one to one onto function, find $f^{-1}(x)$. *Ans: (x + 3)/8*

12. (a) If f (x) = 2x – 3 is a one to one onto function, find the value of f^{-1} (5). *Ans:4*
(b) If g(x) = 4x – 3 is a one to one onto function, find the value of g^{1} (7). *Ans: 5/2*

13. (a) If $f^{-1}(x) = 4x - 3$, find the value of f (5). *Ans: 2*

(b) If $g^{-1}(x) = \frac{2x + 1}{3}$, find the value of g (7). *Ans: 11*

14. (a) If f (x) = 3x, g (x) = x, find $(gof)^{-1}(x)$. *Ans: x / 3*

(b) If f (x) = 2x, g (x) = 5x – 1, find $(gof)^{-1}(x)$. *Ans: (x + 1) /10*

Application (A) Based Problems

1. (a) If f: N→N: f(x) = 2x and g: N→R: g (x) = 3x + 4 are two functions, find the values of (fog)(4) and (gof)(3). Can (gof) (-1) be defined? Ans: 32, 22, No

(b) If f: N→N: f(x) = 3x+1 and g: N→R: g (x) = 4x -3 are given functions, find the values of (fog) (6) and (gof) (9). Can (gof) (10) be defined? Ans: 64, 109, Yes

2. (a) If f(x) = 4x + 5 and (fog)(x) = 8x + 13, find the value of x such that (gof)(x) = 28. *Ans: 2*

(b) If f: R→R: f(x) = 3x + 2, (fog) (x) = 6x +17 and (gof) (x) = 3 find the value of x. *Ans: -1*

3. (a) f: R→R, g:R→R-{0} are two one to one and onto functions. If f(x) = x + 1 and $g(x) = \frac{3-x}{x}$, $x \neq 0$, what is the value of $(f^{-1}og^{-1})(2)$? *Ans: 0*

(b) If 3.f (x) = 4x + 5and g(x) = 5x – 4, find the value of $(f^{-1}og^{-1})(1)$. *Ans: -1/2*

(c) If f (x) = 2x + 1& g(x) = 2x – 1, find the value of $g^{-1}(3)$ and (fog)(-1). *Ans:2,- 5*

4. (a) If f (x) = 3x + 4 and g(x) = 2(x + 1), prove that (fog) = (gof) and find the value of $f^{-1}(2)$. *Ans:-2/3*

(b) If g (x) = 2x – 1 and h (x) = 3x - 2, prove that (goh)(x) = (hog) (x) and find $h^{-1}(4)$. *Ans: 2*

5. (a) If f (x) = 3x + a and fof (6) = 10, find the value of 'a' and $f^{-1}(4)$. *Ans:–11, 5*

(b) If f (x) = 2x + p, fof (2) = 2f (2) + 1, find the value of 'p' and $f^{-1}(3)$. *Ans: 1, 1*

(c) If $f(x) = \frac{a}{3-x}$, $g(x) = 11 - bx^2$, fof (5) = $\frac{4}{5}$ and fog (2) = $\frac{1}{2}$, find 'a' and 'b'. *Ans: 4, 4*

6. (a) If f (x) = 2x + 7 is a one to one and onto function, find f (x + 2) and $f^{-1}(x + 2)$. *Ans: 2x+11,(x – 5) / 2*

(b) If g(x) = 3x + 4, find g (2x – 1) and $g^{-1}(2x - 1)$. *Ans: 6x + 1, (2x – 5) / 3*

7. (a) If f (x) = 2x – 1, $g(x) = \frac{4x + 3}{5}$ and $fog^{-1}(x) = 5$, find the value of x. *Ans: 3*

(b) If f (x) = 3x – 7, $g(x) = \frac{5x + 2}{3}$ and $g^{-1}f(x) = 8$, find the value of x. *Ans: 7*

(c) If f (x) = 4x + 7, g (x) = 3x – 5 and $fog^{-1}(x) = 15$, find the value of x. *Ans: 1*

8. (a) If f (x) = 4x – 17, $g(x) = \frac{2x + 8}{5}$ and $ff(x) = g^{-1}(x)$, find the value of x. *Ans: 6*

(b) If f: R→R: $f(x) = \frac{3x + 10}{2}$ and f: R→R: g (x) =3x – 5 and gg (x) = $f^{-1}(x)$, find the value of x. *Ans: 2*

9. (a) If f (x) = 2x + 1, g (x) = px – 1 and $f^{-1}(3) = g^{-1}(4)$, find the value of p. *Ans : 5*

(b) If f (x) = 5x– 9, g (x) = 2x + k and $f^{-1}(6) = g^{-1}(9)$, find the value of k. *Ans: 3*

Higher Ability (HA) Based Problems

1. (a) If $f(x) = \frac{3x + 11}{x-3}$, $x \neq 3$; $g(x) = \frac{x - 3}{2}$ and $f^{-1}(x) = go\,f(x)$, find the value of x. *Ans: -2, 5*

(b) If $f(x) = \frac{x - 2}{2x + 1}$, $x \neq \frac{1}{2}$ and $g(x) = \frac{1}{x}$, $x \neq 0$ and $f(x) = g^{-1}(x)$, find the value of x. *Ans: ±1*

(c) If $f(x) = x^2 - 2x$, g (x) = 2x + 3 and $fog^{-1}(x) = 3$ then find the value of x. *Ans: 1, 9*

2. (a) If $g(x) = \frac{x}{2x - 3}$ and $g(x) = g^{-1}(x)$, find the value of x. *Ans: 0, 2*

(b) If $h(x) = \frac{x}{3 + 2x}$ and $h(x) = h^{-1}(x)$, find the value of x. *Ans:-1, 0*

3. (a) If $f(x) = \frac{2x + 1}{3}$, $g(x) = \frac{3x + 1}{4}$ and $gof^{-1}(x)$ is an identity function, find the value of x. *Ans: 1*

(b) For $f(x) = \frac{2x + 5}{8}$ and g (x) = 3x – 4, if $(fog)^{-1}(x)$ is an identity function, find the value of x.*Ans: - 3/2*

4. The number of food kept in a refregirator is N (T) = $20T^2 - 80T + 500$ ($2 \le T \le 14$), where T denotes the temperature and T (t) = 4t + 2 ($0 \le t \le 3$), where t represents the time in hour.

(i) Find (NoT)(t) *Ans: $320T^2 + 420$*

(ii) How many bacteria may be in the food afer 2 hours? *Ans: 1700*

(iii) After how long time, the number of bscterias may be 3300? *Ans: 3 hrs*

1.2 POLYNOMIAL

Knowledge (K) Based Problems

1. (a) What is the remainder of polynomial f (x) when it is divided by (x – 3)?

(b) What is the remainder of polynomial f (x) when it is divided by (x + 2)?

2. (a) If (x –3) is the factor of a polynomial f (x) , what is the value of f (3)?

(b) For what value of f (c), will (x – c) be a factor of the polynomial f (x)?

3. (a) In a polynomial f (x), if f (1) = 0, find one of the linear factors of f (x).

(b) In a polynomial f (x), if f (- 2) = 0, find one of the linear factors of f (x).

4. (a) State remainder theorem. (b) Define factor theorem.

Understanding (U) Based Problem

1. (a) If divisor d (x) is x – 1, the quotient is $x^2 - 8x - 1$and the remainder is – 4. What is the original polynomial? *Ans: $x^3 - 9x^2 + 7x - 3$*

(b) A polynomial p(x) is divided by x – 1 and the quotient is $x^2 + x + 1$ with the remainder 5, what is the original polynomial p(x)? *Ans: $x^3 + 4$*

2. Find the quotient and remainder by using synthetic division method.

(a) $(2x^3 - 3x^2 + x + 2) \div (x - 2)$ *Ans: $2x^2 + x + 3$, 8* (b) $(x^3 + 5x^2 + x - 1) \div (x + 2)$ *Ans: $x^2 + 3x - 5$, 4*

3. Find the quotient and remainder by using synthetic division method.

(a) $(4x^3 + 2x^2 - 4x + 9) \div (2x - 1)$ *Ans: $2x^2 + 2x - 1$, 8*

(b) $(3x^3 + x^2 + x + 1) \div (3x - 2)$ *Ans: $x^2 + x + 1$, 3*

4. (a) If $2x^3 - 7x^2 + x + 10 = (x - 1)$ Q (x) + R, find Q (x) and R. *Ans: $2x^2 - 5x - 4$, 6*

(b) If $3x^3 + 4x^2 + 5x - 6 = (x + 2)$ Q (x) + R, find Q (x) and R. *Ans: $3x^2 - 2x + 9$, -24*

5. (a) If p(x) = $4x^3 + 5x^2 - 6x + 7$ is a dividend and (x + 2) a divisor, what is the remainder? *Ans: 7*

(b) Find the remainder when $4x^3 + 7x^2 - 3x + 2$ is divided by (x + 2. *Ans: 4*

(c) Find the remainder when f (x) = $2x^3 + 3x^2 - 4x + 10$ is divided (2x – 1). *Ans: 9*

6. (a) If $x^4 + 3x^2 - 10x + p$ leaves the remainder 15 when it is divided by (x - 2), find the value of p.

(b) If $x^3 + 6x^2 + kx + 10$ leaves the remainder 4 when it is divided by x + 2, find the value of k.

(c) If $2x^3 + 3x^2 - kx + 4$ leaves the remainder 16 when it is divided by (x + 2), find the value of k.

Ans: (a) 7 (b) 11 (c) 8

7. (a) Given that the polynomial f(x) = $2x^4 - 3x^3 + 6x + k$. If f (1) = 0, find the value of k. *Ans: –5*

(b) For a polynomial f(x) = $x^3 - 9x^2 + (k + 1)x - 7$ so that f (7) = 0 what is the value of k? *Ans: 14*

8. (a) Show that (x + 2) is a factor of the polynomial $x^3 - 3x^2 - 4x + 12$.

(b) State factor theorem. Show that (x + 1) is a factor of $6x^3 + 7x^2 - x - 2$.

9. (a) Is (x – 3) a factor of $x^3 + 7x^2 - 2x - 27$? *Ans: Yes*

(b) Is (x + 2) a factor of the polynomial $3x^3 + 13x^2 - 16$? *Ans: No*

10. (a) If (x – 2) is a factor of $x^3 - 2ax^2 + ax + 10$, find the value of k. *Ans:3*

(b) If (x + 3) is a factor of $x^3 - (k - 1)x^2 + kx + 54$, find the value of k. *Ans: 3*

(c) If (x – p) is a factor of $x^3 - px^2 - 4x + p + 12$, find the value of p. *Ans: 4*

11. (a) What should be added to $x^3 - 5x^2 + x - 6$ to make a polynomial having a factor x – 2? *Ans: 16*

(b) What should be subtracted from g (x) = $3x^3 + x^2 + 2x + 7$ to make x + 1 a factor? *Ans: 3*

Application (A) Based Problems

1. (a) Factorize: $x^3 - 4x^2 - 7x + 10$ *Ans: (x- 1)(x + 2) (x -5)*

(b) Factorize: $3x^3 - 13x^2 + 16$ *Ans: (x- 1)(x-4) (4x-3)*

2. (a) Solve: $x^3 - 3x^2 - 4x + 12 = 0$ **[SEE Model-2076]** *Ans:– 2, 2, 3*

(b) Solve: $8x^3 - 2x^2 - 5x - 1 = 0$ *Ans:1, -1/4, -1/2*

3. (a) Solve: $2x^3 + 6 = 3x^2 + 11$ *Ans: -2, 3, 1/2*

(b) Solve: $3y^3 + 10 = 7y(2y + 1)$ *Ans: 1, 5, 2/3*

4. (a) Solve: $3x^3 - 13x^2 + 16 = 0$ *Ans:-1, 4, 4/3*

(b) Solve: $y^3 = 7y^2 - 36$ *Ans:-2, 3, 6*

5. (a) Solve: $y^3 - 19y - 30 = 0$ *Ans: – 3, – 2, 5*

(b) Solve: $x^3 - 3x - 2 = 0$ *Ans: -1, -1, 2*

6. (a) Solve: $(x+1)(x^2 - 5x + 10) - 12 = 0$ *Ans: 1, 1, 2*

(b) Solve: $(x-2)(x^2 - 6x + 7) - 2 = 0$ *Ans:1, 3, 4*

7. (a) Solve: $y = x^3 - 4x^2 + x + 8$ and $y = 2$ *Ans: – 1, 2, 3*

(b) Solve: $y = x^3 - 3x^2 - 10x + 30$ and $y = 6$ *Ans:-3, 2,4*

8. (a) A polynomial $x^3 + px^2 + qx + 6$ has $x - 2$ as a factor and leaves a remainder 3 when divided by $x - 3$, find the values of p and q.   *Ans:- 3,-1*

(b) The polynomial $3x^3 + 2x^2 - bx + a$ is exactly divisible by $(x - 1)$ and leaves a remainder 10 when divided by $(x + 4)$. Find the values of a and b.   *Ans: 30, 35*

9. (a) If $(x - 1)$ and $(x + 2)$ are the factors of $x^3 - 2mx^2 - 7x + n$, find the values of m and n. *Ans: 2, 10*

(b) If $x^3 + ax^2 - bx - 6$ has the factors $(x + 1)$ and $(x - 2)$, find the values of a and b. *Ans: 2, 5*

1.3 SEQUENCE AND SERIES

Knowledge (K) Based Problems

1. (a) Define arithmetic sequence.

(b) What do you mean by arithmetic mean?

2. (a) What is the arithmetic mean between the numbers 'a' and 'b'? **[SEE MODEL-2076]**

(b) If there are 'n' arithmetic means between the numbers 'a' and 'b', what will be the common difference (d)?

3. (a) If an AS has n terms, first term (a) and last term (l), write the formula of calculating the sum of the series.

(b) If an AS with n terms has the first term (a) and common difference (d), what is the sum of the series?

4. (a) Define geometric sequence.

(b) What do you mean by geometric mean?

5. (a) Write down the formula for finding the nth term (t_n) when first term (a), common ratio (r) and number of terms (n) of a geometric sequence are given.

(b) What is the geometric mean between the numbers 'a' and 'b'?

6. (a) If there are 'n' Geometric means between the two numbers 'a' and 'b', find the formula to find the common ratio(r).

(b) Find the formula for finding the last mean (m_n) of a GP.

7. (a) If a geometric series has n terms, first term (a) and last term (l), write the formula of calculating the sum of the series.

(b) If a geometric series has n terms, first term (a) and common ratio (r), write the formula to find the sum of the series.

Understanding (U) Based Problems

ARITHMETIC SEQUENCE/SERIES

1. (a) Examine whether 7, 10, 13, 16, .., is an arithmetic sequence or not.

(b) Examine whether 2, 8, 14, 20, , .., is an arithmetic sequence or not.

2. (a) Find the 7th term of an AP whose first term is 3 & common difference 4.   *Ans:27*

(b) Find the 10^{th} term of an arithmetic sequence 7, 11, 15, …   *Ans: 43*

3. (a) If the fifth term of an AP with first term 2 is 14, find the common difference. *Ans: 3*
   (b) The common difference of an AP is 5 and its tenth term is 40, find its first term. *Ans: -5*
4. (a) The first and the last terms of an AP having common difference 3 are 5 and 80 respectively, find the number of terms. *Ans: 26*
   (b) If the nth term of the series 84 + 78 + 72 + ….is 0, find the value of n. *Ans: 15*
5. (a) How many terms are there in AP: 7 + 12 + 17 + …+ 67? *Ans: 13*
   (b) How many terms are there in AP: 3, 6, 9, ..., 111? *Ans: 37*
6. (a) Which term of the series 2 + 5 + 8 + .. is 56? *Ans: 19^{th}*
   (b) Which term of the sequence 6, 9, 12, 15… is 66? *Ans :21^{st}*
7. (a) Is 23 a term of the series 3 + 7 + 9 + …? *Ans: Yes*
   (b) Is 35 a term of an AS 6 + 10 + 14 + …? *Ans : No*
8. (a) Find the numbers between 1 and 100 which are divisible by 3. *Ans : 33*
   (b) Find the numbers between 100 and 200 which are divisible by 5. *Ans: 19*
9. (a) If x + 2, 4x + 3 and x+4 are in an A.P., find the value of x. *Ans: 0*
   (b) If k, 4k – 9, 3k – 2 is an A.P., find the value of k. *Ans :4*
10. (a) If 6, p, q, 18 are in an AP, find the values of p and q. *Ans: 10, 14*
   (b) If 5, x, y, z, 21 are in an AP, find the values of x, y and z. *Ans: 9, 13, 17*
11. (a) If the arithmetic mean of the numbers 22 and 5x is 31, find the value of x. *Ans: 8*
   (b) Find the AM between the numbers x + 10 and 50 is 55, find the value of x. *Ans: 50*
12. (a) Find the sum of the series 2 + 4 + 6 + ….20 terms. *Ans: 420*
   (b) Find the sum of the series: 3 + 7 + 11 + 15 + ….10 terms. *Ans: 210*
13. (a) Find the sum of the series 5 + 7 + …+ 99 *Ans: 2496*
   (b) Find the sum of the series 8 + 13 + 18 + ….+ 78 *Ans: 645*
14. (a) The first and second terms of an AP are 5 and 8 respectively; find the sum of its first 10 terms.
   (b) The first and second terms of an AP are 6 and 11 respectively; find the sum of its first 20 terms.
   Ans: (a) 185 (b) 1070
15. (a) An arithmetic series has 20 terms and the last term is 50. If the first term is 4, find the sum of the series. *Ans: 540*
   (b) An arithmetic series has 30 terms and the 30^{th} term is 65. If the first term is 5, find the sum of the series. *Ans: 1050*
16. (a) The sum of the first 11 terms of an AP is 1320; find the 6th term. *Ans: 120*
   (b) If the sum of first 25 terms of an AP adds up to 5000; find the 13th term. *Ans: 200*
17. (a) If the 3^{rd} term of an arithmetic series is 13, find the sum of first 5 terms. *Ans: 65*
   (b) The 11th term of an AS is 10; find the sum of first 21terms. *Ans: 210*
18. (a) The sum of first 10 terms of an AP is 190 and common difference is 4, find its first term. *Ans: 2*
   (b) The sum of first 20 terms of an AP is 820 and common difference is 4, find its first term. *Ans: 3*
19. (a) Find the number of terms of an AS whose first term is 6, last term is 54 and the sum of the terms is 300. *Ans:10*
   (b) Find the number of terms of an AP whose first term is 13, last term is 87 and the sum of the terms is 400. *Ans:8*

GEOMETRIC SEQUENCE/SERIES

1. (a) Is 3, 6, 12, .. a geometric sequence? Give reason. Also, find the sixth term of the sequence: *Ans:96*
   (b) Is 2, 6, 18, a geometric sequence? Give reason. Also, find the sixth erm. *Ans:486*
2. (a) The first and the second term of a GP are 9 and 18 respectively. What is the fifth term? *Ans:144*
   (b) The first and the second term of a geometric sequence are 32 and 8 respectively. What is the sixth term? *Ans:1/32*
3. (a) In a GP, the common ratio is 2 and the 8th term is 256, find the first term. *Ans:2*
   (b) If the sixth term of a GS with common ratio 3 is 810, find the first term. *Ans:5*
4. (a) If the fourth term of a GP with first term 2 is 54, find the common ratio. *Ans:3*
   (b) The first term of a GS is 5. If its fifth term is 80, find the common ratio. *Ans: ±2*
5. (a) How many terms are there in the sequence 2 + 4 + 8 +.. + 256? *Ans:8*

(b) How many terms are there in the geometric series $\frac{1}{4} + \frac{1}{2} + 1 + 2 + .. + 64$ *Ans:9*

6. (a) Is 486 a term of a GP 2, 6, 18, 54, …,? *Ans:Yes*

(b) Is 768 a term of a GP 3, 6, 12, 24, …? *Ans:Yes*

7. (a) Which term of GP 2, 4, 8, 18, .. is 256? *Ans:8^{th}*

(b) Which term of GP 5, -10, 20, -40, …. is 320? *Ans:7^{th}*

8. (a) If x - 1, x + 2, x + 8 are in a GS, find the value of x. *Ans:4*

(b) If x – 1, x + 1, 3x – 1 are in a GS, find the value of x. *Ans:3*

9. (a) If the third term of a G.P. is 2, find the product of first five terms. *Ans: 32*

(b) If the third term of a G.P. is 3, find the product of first five terms. *Ans: 243*

10. (a) If 7, x, y, 56 form a geometric sequence, find the values of x and y. *Ans: 14, 28*

(b) Find the values of x, y and z from the given GP: $\frac{1}{8}$, x, y, 2 *Ans:1/4, 1/2, 1*

11. (a) If the arithmetic mean between 2 and x is 5, find the geometric mean. *Ans:4*

(b) If the arithmetic mean between 16 and x is 10, find the geometric mean. *Ans:8*

12. (a) Find the sum of the series 1 + 3 + 9 + …up to 6 terms *Ans: 364*

(b) Find the sum of the series 256 + 128 + 64+ …up to 8 terms. *Ans:510*

13. (a) Find the sum of the series 2 + 6 + 18 + …..+ 486. *Ans:728*

(b) Find the sum of the series 5 + 15 + 45+ …..+ 1245 *Ans: 1865*

Application (A) Based Problems

1. (a) If the nth rterems of an AP 7, 12, 17, … is same as the ntrh terem of an AP 27, 30, 33 …, finds the value of n. *Ans: 11*

(b) If the nth rterems of an AP 63, 65, 67, … is same as the ntrh terem of an AP 3, 10, 17, …, finds the value of n. *Ans: 13*

2. (a) If 6 times the 6 tems of an arithmetic sequence is equal to 9 times the 9^{th} term, show that the its 15^{th} term is zero.

(b) If 7 times the 7 tems of an arithmetic sequence is equal to 11 times the 11^{th} term, show that the its 18^{th} term is zero.

3. (a) If fifth and tenth terms of arithmetic sequence are 14 and 29 respectively.Find the first term and the common diiferece. Also, find the 17^{th} term. *Ans: 2, 3, 50*

(b) If third and nineth terms of arithmetic sequence are 20 and 5 respectively. Find the first term and the common diiferece. Also, find the 15^{th} term. *Ans: 25, -2/5, -10*

4. (a) If the fifth and tenth terms of an arithmetic sequence are 17 and 42 respectively, find the sequence. *Ans:-3, 2, 7, 12, …*

(b) The fourth and sixth terms of an arithmetic sequence are 24 and 16 respectively, find the sequence. *Ans:36, 32, 28 , …*

5. (a) If the third and eleventh terms of an arithmetic series are 8 and -8 respectively, find the sum of the first seven terms of the series. *Ans:42*

(b) If the 4^{th} and 15^{th} terms of an arithmetic series are 11 and 44 respectively, find the sum of its first 20 terms. *Ans:610*

6. (a) If the sixth and nineth terms of an arithmetic sequence are 52 and 76 respectively, which term is 100? Find it. *Ans:12^{th} term*

(b) If the 5th and 10th terms of an arithmetic sequence are 30 and 55 respectively, which term is 80? Find it. *Ans: 15th term*

7. (a) The sum of first eight terms of an arithmetic series is 180 and its fifth term is five times of the first term, find the sum of the first 10 terms. *Ans:275*

(b) The sum of first ten terms of an arithmetic progression is 50 and its fifth term is treble of the second term. Calculate the sum of the first twenty terms. *Ans:200*

8. (a) If the sum of first seven terms of an arithmetic series is 14 and the sum of the first ten terms is 125 then find the ninth term of the series. *Ans:37*

(b) If the sum of first four terms of an arithmetic series is 26 and the sum of the first eight terms is 100 then find the eleventh term of the series. Ans:32

9. (a) The sum of first 4 terms of arithmetic series is 26 and the sum of thirst 6 terms is 57, find the sum of first 10 terms. *Ans:155*

(b) The sum of first 9 terms of arithmetic series is 72 and the sum of thirst 17 terms is 289, find the sum of first 25 terms. *Ans:650*

10. (a) In an arithmetic series, if the sum of first 10 terms is equal to that of first 20 tems, find the sum of first 30 terms. *Ans:0*

(b) In an arithmetic series, if the sum of first 25 terms is equal to that of first 35 tems, find the sum of first 60 terms. *Ans:0*

11. (a) Three terms in an arithmetic progression have sum 21 and product 315. Find the terms. *Ans:9, 7, 5 or, 5, 7, 9*

(b) The sum of three consecutive terms in arithmetic series is 30 and their product is 750, find these terms. *Ans:5, 10, 15 or, 15, 10, 5*

12. (a) The first term and the last term of an arithmetic series are 8 and 128 respectively. If the sum of all terms of the series is 1700, find the number of terms and the common difference of the series. *Ans:25, 5*

(b) The first term and the last term of an arithmetic series are -24 and 72 respectively. If the sum of all terms of the series is 600, find the number of terms and the common difference of the series. *Ans:25, 4*

13. (a) There are n arithmetic means between 7 and 77. If the ratio of first mean to the last mean is 1:6, then find the number of arithmetic means. *Ans:13*

(b) There are n arithmetic means between 4 and 24. If the ratio of third mean to the last mean is 4:5, then find the number of terms of the series. *Ans:4*

14. (a) The sum of three consecutive terms in GP is 62 and their product is 1000, find the terms. *Ans:2, 10, 50 or 50, 10, 2*

(b) The sum of three consecutive terms in GP is 7 and their product is 8, find the terms. *Ans: 1, 2, 4 or 4, 2, 1*

15. (a) Insert 4 geometric means between 2/3and 162. *Ans:2, 6, 18, 54*

(b) Insert 3 geometric means between 1/9 and 9. *Ans:1/3, 1, 3*

16. (a) There are some geometric means between 1/2and 16. If the third mean be 4, find the numbers of means. *Ans:4*

(b) Find the numbers of geometric means inserted between 1 and 64 in which the ratio of first mean to the last mean is 1:16. *Ans:5*

17. (a) The second, fourth and ninth terms of an arithmetic progression are in geometric progression. Calculate the common ratio of the geometric progression. *Ans:5/2*

(b) In a geometric series, if the sixth term is 16 times the second term and the sum of the first seven terms is 127/4, find positive common ratio and the first term. *Ans:2, 1/4*

18. (a) The sum of first four terms is 40 and the sum of the first two terms is 4 of a geometric series whose common ratio is positive, find the sum of first 8 terms the series. *Ans:3280*

(b) The sum of first four terms is 45 and the sum of the first eight terms is 765 of a geometric series whose common ratio is positive, find the sum of first 6 terms the series. *Ans:189*

19. (a) If the arithmetic mean and geometric mean of two numbers are 5 and 4 respectively, find the numbers. *Ans:2 and 8 or 8 and 2*

(b) If the arithmetic mean of two numbers is 25 and their geometric mesn is 24, find the numbers. *Ans18 and 32 or 32 and 18*

Higher Ability (HA) Based Problems

1. (a) Three numbers whose sum is 21 are in AP. If 1, 3 and 10 are added to them respectively; then the numbers are in G.P. Fid the numbers. *Ans:4, 7, 10 or 19, 7, -5*

(b) Three numbers whose sum is 24 are in AP. If 1, 6 and 18 are added to them respectively; then the numbers are in G.P. Fid the numbers. *Ans:6, 8, 10 or 27, 8, -11*

2. (a) Three numbers whose sum is 7 are in GP. If 1, 3 and 4 are added to them respectively; then the numbers are in A.P. Fid the numbers. *Ans:1, 2, 4 or 4, 2, 1*

(b) Three numbers whose sum is 26are in GP. If 5, 9 and 5 are added to them respectively; then the numbers are in A.P. Fid the numbers. *Ans: 2, 6, 18 or 18, 6, 2*

3. (a) A taxi-meter shows Rs 14 at the time of stating and then runs up by Rs 36 for each additional kilometer travelled during 6:00 am to 9:00 pm. If Mr Chaudhary travelled a journey of 7 km from Tripureshwor to Bhaktapur, how much should be paid for journey? Also, how long can be travelled by taxi for Rs. 554? *Ans: Rs 266, 15km*

(b) A firm produced 2500 pair of shoes in its first year. If it increased its production by a constant number every year and produced 17500 pair of shoes at the end of the fifth yerar, find the increased number of pair of shoes in each year. *Ans: 500*

4. (a) A contrsctor on construction job specifies a penalty for delay of completion beyond a certain date as Rs 200 for the first day, Rs 250 for the second day, Rs 300 for the third day and so on. The penalty for each successing day being Rs 50 more than that of the preceeding day. How much money the contractor has to pay as penalty, if he has delayed the work by 30 days? **[SEE MODEL-2076]** *Ans:Rs 27750*

(b) A contrsctor on construction job specifies a penalty for delay of completion beyond a certain date as Rs 400 for the first day, Rs 700 for the second day, Rs 1000 for the third day and so on. How much money the contractor has to pay as penalty, if he has delayed the work by 3 weeks? *Ans: Rs 71400*

5. (a) A person pays a loan of Rs 975 in monthly installments, each installment being less than a former by Rs 5. The amount of first installment is Rs 100. In how many installments will the entire amount be paid? Given reason. *Ans:25*

(b) A person pays a loan of Rs 720 in monthly installments, each installment being less than a former by Rs 40. The amount of first installment is Rs 240. In how many installments will the entire amount be paid? Given reason *Ans:4*

6. (a) A club consists of members whose ages are in AP. The common difference of their ages is 3 months. The youngest member of the club is 7 years old and the sum of their ages is 250 years. Find the number of members and the age of the eldest member. *Ans: 25, 13 years*

(b) A communityconsists of members whose ages are in AP. The common difference of their ages is 4 months. The youngest member of the club is 8 years old and the sum of their ages is 250 years. Find the number of members and the age of the eldest member. *Ans: 16, 13 years*

7. (a) A man borrows Rs 3465 without interest and repays the loan in 6 monthly installments, each installment being double the preceeding one. Find the first and last installments. *Ans: Rs 55, Rs 1760*

(b) A man borrows Rs 24200 without interest and repays the loan in 5 installments, each installment being treble the preceeding one. Find the first and last installments. *Ans: Rs 200, Rs 16200*

8. (a) If p^{th} , q^{th} and r^{th} terems of an arithmetic sequence are a, b and c respectively, then prove that: $p(b-c) + q(c-a) + r(a-b) = 0$

(b) If a, b , c are in an A.P. and x, y, z are in G.P., show that: $x^{b-c} . y^{c-a} . z^{a-b} = 1$

(c) The sum of first four terms of a GP is 30 and the sum of the last four terms is 960. If the first and the last terms are 2 and 512 respectively, find the common ratio. *Ans:2*

1.4 QUADRATIC EQUATION AND GRAPH

Knowledge (K) Based Problems

1. (a) Define linear programming.

(b) What do you mean by objective function?

2. (a) What is the coordinates of vertex of parabola $y= ax^2 + bx + c, a \neq 0$?

(b) What is the vertex of parabola whose equation is $y = a(x-h)^2 + k, a \neq 0$?

3. (a) For what value of 'a', the mouth of the graph of the quadratic equation $y = ax^2$, $a \neq 0$ faces upwards?

(b) In which side, the mouth of the graph of the quadratic equation $y = ax^2$, $a \neq 0$ faces when $a < 0$?

Understanding (U) Based Problems

1. (a) Show the inequality $x + y \leq 2$ in a graph.

(b) Show the inequality $2x + y \geq 3$ in a graph.

2. (a) What will be the points of intersection of the curve $f(x) = x^2 - 1$ and $f(x) = 3$? **[SEE MODEL-2076]** *Ans: (-2, 3), (2, 3)*

(b) What will be the points of intersection of the curve $f(x) = x^2 - 4$ and $f(x) = 5$? *Ans: (-3, 5) and (3, 5)*

3. (a) Find the vertex of the parabola $y = x^2 - 2x - 3$. *Ans: (1, -4)*

(b) Find the vertex of the parabola $y = x^2 - 4x - 5$. *Ans: (2, -9)*

4. (a) Find the y-intercept of the parabola $y = x^2 + x - 2$. *Ans: -2*

(b) Find the y-intercept of the parabola $y = x^2 - 3x - 4$. *Ans: -4*

5. (a) Find the x-intercept of the curve $y = x^2 - 3x + 2$ *Ans: 1, 2*

(b) Find the x-intercept of the curve $y = x^2 - 2x - 3$ *Ans: -1, 3*

Application (A) & Higher Ability (HA) Based Problems

1. Maximize the given objective functions subject to the following constraints:

SN	Constraints	Objective function	Answer
a.	$x + y \leq 6$, $x - y \leq 4$, $x \geq 0$, $y \geq 0$	$P = 3x + 5y$	$P_{Max} = 30$ at (0, 6)
b.	$x + y \leq 7$, $x - y \geq -1$, $x \geq 0$, $y \geq 0$	$F = 3x + 4y$	$F_{Max} = 25$ at (3, 4)
c.	$2y \geq x - 1$, $x + y \leq 4$, $x \geq 0$, $y \geq 0$	$P = 3x + y$	$P_{Max} = 10$ at (3, 1)
d.	$x + 2y \leq 6$, $x - 2y \leq 2$, $x \geq -2$	$P = 5x + 3y$	$P_{Max} = 23$ at (4, 1)

2. Minimize the given objective functions subject to the following constraints:

SN	Constraints	Objective function	Answer
a.	$2x + y \leq 20$, $2x + 3y \geq 24$, $x \geq 0$, $y \geq 0$	$Z = 5x + 3y$	$Z_{Min} = 0$ at (0, 0)
b.	$x + y \geq 0$, $x - y \leq 0$, $x \geq -1$, $y \leq 2$	$Z = 3x + 2y$	$Z_{Min} = -8$ at (-1, 2)
c.	$4x + 6y \geq 16$, $5x + 3y \leq 11$, $x \geq 0$, $y \geq 0$	$Z = 40x + 30y$	$Z_{Min} = 100$ at (1, 2)

3. Optimize the given objective functions subject to the following constraints:
OR, find the extreme values of the given objective functions under the following constraints:
OR, find the maximum and minimum values of the given objective functions under the following constraints:

SN	Constraints	Objective function	Answer
a.	$x - 2y \leq 1$, $x + y \leq 4$, $x \geq 0$, $y \geq 0$ **[SEE MODEL – 2076]**	$P = 5x + 4y$	$P_{Max} = 19$ at (3, 1) $P_{Min} = 0$ at (0, 0)
b.	$x - 2y \leq 2$, $x + y \leq 5$, $x \geq 0$, $y \geq 0$	$F = 2x + 3y$	$F_{Max} = 15$ at (0, 5) $F_{Min} = 0$ at (0, 0)

4. (a) Solve graphically: $x^2 + 2x - 3 = 0$ *Ans: x = -3, 1*

(b) Solve graphically: $x^2 - 2x - 3 = 0$ *Ans: x = -1, 3*

(c) Solve graphically: $x^2 - 3x - 4 = 0$ *Ans: x = -1, 4*

(d) Solve graphically: $x^2 - 3x = 10$ *Ans: x = -2, 5*

(e) Solve graphically: $x^2 - 2x = 15$ *Ans: x = -3, 5*

(f) Solve graphically: $y = x^2$ and $y = 2 - x$ *Ans: x = (1, 1), (-2, 4)*

(g) Solve graphically: $y = x^2$ and $y = 2x + 8$ *Ans: x = (4, 16), (-2, 4)*

UNIT-2 CONTINUITY

Knowledge (K) Based Problems

1. (a) Define continuity of a function.
(b) What does the discontinuity of a curve represent?

2. (a) Express "$-1 \le x \le 2$ in interval form.
(b) Express "$2 \le x \le 5$ in interval form.

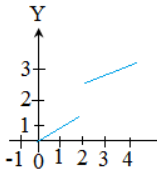
3. (a) Under what condition the limit of a function f (x) exists at x = a?
(b) Write the necessary condition for the continuity of a function f (x) at x = a.

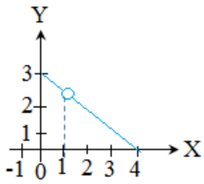
4. (a) Write the left hand limit of f (x) at x = 3 in notation.
(b) Write the right hand limit of f (x) at x = 2 in notation.

5. (a) Express $\lim_{x\to a^-} f(x)$ in sentence.
(b) Write $\lim_{x\to 2^-} f(x)$ in sentence.

6. (a) Write the set of numbers which is continuous in number line. **[SEE MODEL-2076]**
(b) Write a set of numbers which is discontinuous in number line.

7. (a) Is the function continuous at x = 3? Give reason.
(b) Discuss the continuity or discontinuity of the function at x = 2.
(c) Is the function continuous at x = 1? Give reason.





Application (A) Based Problems

1. (a) For a real valued function f (x) = x +3.
(i) Find the values of f (x) at x = 1.9, 1.99, 1.999, 1.9999, 2, 2.1, 2.01, 2.001, 2.0001
(ii) Find $\lim_{x\to2-} f(x)$, $\lim_{x\to2+}(x)$ and f (2).
(iii) Is this function continuous at x = 2?
(b) For a real valued function f (x) = 2x +1.
(c) Find the values of f (x) at x = 3.9, 3.99, 3.999, 3.9999, 4, 4.1, 4.01, 4.001, 4.0001
(ii) Find $\lim_{x\to3-} f(x)$, $\lim_{x\to3-} f(x)$ and f (4).
(iii) Is this function continuous at x = 4?

2. (a) Given that f (x) = x^2 is a real valued function.
(i) Find the left hand limit, right hand limit at x = 4
(ii) Find the value of f (4).
(iii) Is this function continuous at x = 4?
(b) Given that f (x) = x^3 is a real valued function.
(i) Find the left hand limit and right hand limit at x = 2
(ii) Find the value of f (2).
(iii) Is this function continuous at x = 2?

3. (a) For a real valued function f(x) = 2x + 3
(i) Find the values of f(2.95), f(2.99), f(3.01), f(3.05) and f(3).
(ii) Is this function continuous at x = 3? **[SEE MODEL-2076]**
(b) For a real valued function f(x) = 6x + 1
(i) Find the values of f(1.9), f(1.99), f(2.01), f(2.05) and f(2).
(ii) Is this function continuous at x = 2?

4. (a) Prove that the function $f(x) = \begin{cases} x+2 & \text{for } x \le 4 \\ 2x-1 & \text{for } x > 4 \end{cases}$ is continuous at x = 2.

(b) Prove that the function $f(x) = \begin{cases} 3x+1 & \text{for } x<1 \\ 4 & \text{for } x=1 \\ 5x-1 & \text{for } x>1 \end{cases}$ is continuous at x = 1.

UNIT-3 MATRIX

Knowledge (K) Based Problems

1. (a) Define singular matrix.
(b) What do you mean non-singular matrix?
(c) State inverse of a matrix.

2. (a) Under what condition the matrix becomes singular?
(b) Write the necessary condition for the possibility of inverse of a matrix.

3. (a) If A = [5], what is the value of $|A|$?
(b) If A = [-3], what is the value of $|A|$?

4. (a) If $A = \begin{pmatrix} p & q \\ r & s \end{pmatrix}$, what is the value of $|A|$? **[SEE MODEL-2076]**

(b) If $M = \begin{pmatrix} b & p \\ p & d \end{pmatrix}$, what is the value of $|M|$?

5. (a) Write the adjoint matrix of $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$.

(b) Write the adjoint matrix of $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$.

6. (a) Name the method for solving a pair of simultaneous linear equations by using determinant.
(b) If D, D_1 and D_2 are given then what is formula to find the value of x by using Cramer's rule?

7. (a) According to Cramer's rule for the system $a_1x + b_1y = c_1$ and $a_2x + b_2y = c_2$, what is D?
(b) According to Cramer's rule for the system $a_1x + b_1y = c_1$ and $a_2x + b_2y = c_2$, what is D_1?

Understanding (U) Based Problems

1. (a) If $A = \begin{pmatrix} 2 & 3 \\ 10 & 15 \end{pmatrix}$, find $|A|$. What type of matrix is it with respect to its determinant?

Ans:0, singular matrix

(b) If $P = \begin{pmatrix} 3 & -4 \\ -6 & 8 \end{pmatrix}$, find $|P|$. What type of matrix is it with respect to its determinant?

Ans:0, singular matrix

2. (a) Is $\begin{pmatrix} 4 & 8 \\ 3 & 6 \end{pmatrix}$ a singular matrix? Write with your reason. *Ans:Yes*

(b) Examine whether the matrix $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ is a singular or not. *Ans: No*

3. (a) If the matrix $\begin{pmatrix} 2 & 3 \\ 4 & x+5 \end{pmatrix}$ is singular, find the value of x. *Ans: 1*

(b) If the matrix $\begin{pmatrix} 2m & 2 \\ 10 & 5 \end{pmatrix}$ is singular, find the value of m. *Ans: 2*

4. (a) If $\begin{vmatrix} x & 3 \\ 4 & 2 \end{vmatrix} = 0$, find the values of x. *Ans: 6*

(b) If $\begin{vmatrix} x & 6 \\ 7 & x \end{vmatrix} = 7$, find the values of x. *Ans: ±7*

5. (a) If $A = \begin{pmatrix} 3 & 1 \\ 4 & 2 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & 2 \\ 3 & 5 \end{pmatrix}$, find $|2A + B|$ *Ans: 19*

(b) If $M = \begin{pmatrix} 2 & 5 \\ 1 & 3 \end{pmatrix}$ and $N = \begin{pmatrix} -4 & -6 \\ 3 & 2 \end{pmatrix}$, find $|MN|$. *Ans:10*

(c) If $A = \begin{pmatrix} -1 & 0 \\ 2 & 4 \end{pmatrix}$, find the determinant of AA^T. *Ans:24*

6. (a) If the determinant of the matrix $\begin{pmatrix} 2 & 4 \\ 6 & x \end{pmatrix}$ is 40, find the value of x. *Ans:32*

(b) If the determinant of the matrix $\begin{pmatrix} 3-p & -2 \\ p & 5 \end{pmatrix}$ is 6, find the value of p. Ans: 3

7. (a) If $A = \begin{pmatrix} 3 & 1 \\ 0 & 2 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & 2 \\ 0 & 4 \end{pmatrix}$, verify that: $|AB| = |A|\ |B|$

(b) If $A = \begin{pmatrix} 4 & 3 \\ -7 & 2 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & -5 \\ 3 & 4 \end{pmatrix}$, verify that: $|AB| = |A|\ |B|$

8. (a) Show that the matrices $\begin{pmatrix} 3 & 1 \\ 5 & 2 \end{pmatrix}$ and $\begin{pmatrix} 2 & -1 \\ -5 & 3 \end{pmatrix}$ are inverse to each other.

(b) Show that the matrices $\begin{pmatrix} -5 & 2 \\ -7 & 3 \end{pmatrix}$ and $\begin{pmatrix} -3 & 2 \\ -7 & 5 \end{pmatrix}$ are inverse to each other.

9. (a) If $A = \begin{pmatrix} 2 & -1 \\ 3 & 1 \end{pmatrix}$, find $|A|$ and write A^{-1} is defined or not. **[SEE MODEL-2076]**

(b) If $A = \begin{pmatrix} 4 & 2 \\ -1 & 3 \end{pmatrix}$, find $|A|$ and write A^{-1} is defined or not.

10. Find the inverse of the matrix:

(a) $A = \begin{pmatrix} 4 & 7 \\ 6 & 8 \end{pmatrix}$ Ans: $-\frac{1}{10}\begin{pmatrix} 8 & -7 \\ -6 & 4 \end{pmatrix}$ (b) $A = \begin{pmatrix} 2 & -3 \\ 3 & -4 \end{pmatrix}$ Ans: $\begin{pmatrix} -4 & 3 \\ -3 & 2 \end{pmatrix}$

11. (a) If the inverse of matrix $\begin{pmatrix} 1 & 2 \\ x & 3 \end{pmatrix}$ is the matrix $\begin{pmatrix} 3 & -2 \\ -1 & y \end{pmatrix}$, find the values of x and y. *Ans:x = 1, y = 1*

(b) If the matrices $\begin{pmatrix} 2x & 7 \\ 5 & 9 \end{pmatrix}$ and $\begin{pmatrix} 9 & y \\ -5 & 4 \end{pmatrix}$ are inverse to each other, find the values of x and y. *Ans: 2 -7*

12. (a) If the matrix $\begin{pmatrix} k & 3 \\ 8 & 4 \end{pmatrix}$ has no inverse, find the value of k. *Ans: 6*

(b) For what value of p, the inverse of matrix $\begin{pmatrix} 12 & p \\ 32 & 24 \end{pmatrix}$ is not defined? *Ans: 9*

13. (a) According to Cramer's rule, find the value of D_1 and D_2 for ax + by = c and px + qy = r. **[SEE MODEL-2076]** Ans: $D_1 = \begin{vmatrix} c & b \\ r & q \end{vmatrix} = cq - br, D_2 = \begin{vmatrix} a & c \\ p & r \end{vmatrix} = ar - cq$

(b) According to Cramer's rule, find the value of D and D_1 for $a_1x + b_1y = c_1$ and $a_2x + b_2y = c_2$ Ans: $D = \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix} = a_1b_2 - a_2b_1, D_1 = \begin{vmatrix} c_1 & b_1 \\ c_2 & b_2 \end{vmatrix} = c_1b_2 - c_2b_1$

14. (a) Using Cramer's rule, find the value of D_1 and D_2 for 3x + 2y = 8 and 4x – y = 7. *Ans: -22, -11*

(b) Using Cramer's rule, find the value of D and D_1 for 2x – 3y = 5 and 3x + y = 2 *Ans: 11, -11*

Application (A) Based Problems

1. Solve the given system of equations by matrix method:
(a) $3x + 5y = 11, 2x - 3y = 1$ **[SEE MODEL-2076]** *Ans:* $x = 2, y = 1$
(b) $3x + 2y = 10, 2x + 5y = 14$ *Ans:* $x = 2, y = 2$

2. Solve the given system of equations by matrix method:
(a) $2x - 3y - 7 = 0, 4y - 3x + 10 = 0$ *Ans:* $x = 2, y = -1$ (b) $2x + 3y - 18 = 0, 3x - 2y - 1 = 0$ *Ans:* $x = 3, y = 4$

3. Solve the given system of equations by matrix method:
(a) $x = \frac{2}{3}y, 4x - 3y = 1$ *Ans:* $x = -2, y = -3$ (b) $\frac{3}{2}x + 2y = 1, \frac{x}{3} - \frac{y}{3} = 1$ *Ans:* $x = 2, y = -1$

4. Solve the given system of equations by matrix method:
(a) $\frac{3x+5y}{8} = \frac{5x-2y}{3} = 3$ *Ans:* $x = y = 3$ (b) $\frac{3x+5y}{4} = \frac{7x+3y}{5} = 4$ *Ans:* $x = y = 2$

5. Solve the given system of equations by matrix method:
(a) $3x + \frac{5}{y} = 7, 7x - \frac{10}{y} = 12$ *Ans:* $x = 2, y = 5$ (b) $3x + \frac{4}{y} = 10, x + \frac{1}{y} = 3$ *Ans:* $x = 2, y = 1$
(c) $\frac{4}{x} + 3y = 5, \frac{6}{x} - y = 2$ *Ans:* $x = 2, y = 1$ (d) $\frac{3}{x} - 2y = 1, \frac{2}{x} - y = 1$ *Ans:* $x = 1, y = 1$

6. Solve the given system of equations by matrix method:
(a) $\frac{6}{x} - \frac{2}{y} = 1, \frac{3}{y} = \frac{2}{x} + 2$ *Ans:* $x = 2, y = 1$ (b) $\frac{2}{x} + \frac{6}{y} = 3, \frac{10}{x} - \frac{9}{y} = 2$ *Ans:* $x = 2, y = 3$

7. (a) If $A = \begin{pmatrix} 1 & 2 \\ 3 & 5 \end{pmatrix}$ and $B = \begin{pmatrix} 4 & 5 \\ 3 & 4 \end{pmatrix}$ verify that $(AB)^{-1} = B^{-1}A^{-1}$

(b) If $P = \begin{pmatrix} 0 & -1 \\ 2 & -3 \end{pmatrix}$ and $Q = \begin{pmatrix} -2 & 1 \\ -4 & 5 \end{pmatrix}$ verify that $(PQ)^{-1} = Q^{-1}P^{-1}$

8. Solve the following system of equations by Cramer's rule.
(a) $9x - 8y = 12, 2x + 3y = 17$ *Ans:* $x = 4, y = 3$ (b) $2x + 3y = 3, x + 2y = 1$ *Ans:* $x = 3, y = -1$

9. Solve the following system of equations by Cramer's rule.
(a) $2x + 3y - 18 = 0, 3x - 2y - 1 = 0$ *Ans:* $x = 3, y = 4$ (b) $3x = 5y + 3, 4x - 4 = -3y$ *Ans:* $x = 1, y = 0$

10. Solve the following system of equations by Cramer's rule.
(a) $x = \frac{2}{3}y, 4x - 3y = 1$ *Ans:* $x = -2, y = -3$ (b) $\frac{3}{2}x + 2y = 1, \frac{x}{3} - \frac{y}{3} = 1$ *Ans:* $x = 2, y = -1$

11. Solve the following system of equations by Cramer's rule.
(a) $\frac{3}{x} - 2y = 1, \frac{2}{x} - y = 1$ *Ans:* $x = 1, y = 1$ (b) $\frac{6}{x} - \frac{2}{y} = 1, \frac{3}{y} = \frac{2}{x} + 2$ *Ans:* $x = 2, y = 1$

UNIT-4 COORDINATE GEOMETRY

4.1 ANGLE BETWEEN TWO STRIAGHT LINES

Knowledge (K) Based Problems

1. (a) Write the formula to find the angle between the t lines $y = m_1x + c_1$ and $y = m_2x + c_2$.
(b) Write the formula to find the angle between the lines $y = m_1x$ and $y = m_2x$.
(c) If the slopes of two straight lines are m_1 and m_2 respectively and θ be the angle between them, write the formula for $\tan\theta$. **[SEE MODEL-2076]**

2. (a) If two straight lines $y = m_1x + c_1$ and $y = m_2x + c_2$ are parallel to each other, write the relation between m_1 and m_2.
(b) If two straight lines $y = m_1x + c_1$ and $y = m_2x + c_2$ are perpendicular (orthogonal) to each other, write the relation between m_1 and m_2.

Understanding (U) Based Problems

1. (a) Write the condition for the straight lines $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ to be perpendicular and parallel to each other. Ans: $a_1a_2 + b_1b_2 = 0$, $a_1b_2 = b_1a_2$

(b) Write down the conditions of being perpendicularity and parallelism of the lines $p_1x + q_1y+r_1 = 0$& $p_1x + q_1y+r_1= 0$. Ans: $p_1q_2 – p_2q_1 = 0$, $p_1p_2 + q_1q_2 = 0$.

2. (a) Show that the lines $2x + 3y + 4 = 0$ and $3x – 2y + 1 = 0$ are perpendicular to each other.

(b) Show that the lines $6x - 8y = 7$ and $4x + 3y + 8 = 0$ are perpendicular to each other.

(c) Show that the line $3x - 2y = 5$ and the line joining the points (2, 6) and (8, 2) are perpendicular to each other.

3. (a) If two straight lines $px + qy + r = 0$ and $lx + my + n = 0$ are perpendicular to each other, show that: $pl+ qm = 0$.

(b) If two lines $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ are orthogonal, show that: $a_1a_2 + b_1b_2 = 0$.

4. (a) Find the slopes of two straight lines $2x + 3y + 4 = 0$ and $3x – 2y + 1 = 0$ and write the relation between them. *Ans: perpendicular*

(b) Find the slopes of two straight line $x + 3y + 6 = 0$ and the line passing through the points (-2, -3) and (-6, 9) and write the relation between them. *Ans: perpendicular*

5. (a) Find the value of k if the straight lines $3x – 4y + 1 = 0$ and $8x +ky = 7$ are perpendicular. *Ans:6*

(b) For what value of p, the lines $px + 7y = 1$ and $7x – 5y = 6$ are perpendicular to each other? *Ans:5*

(c) For what value of p, line $px – 2y + 1 = 0$ is perpendicular to the line joining the points (1, 3) and (p, 2)? *Ans:2*

6. (a) Find the slope of the line perpendicular to the line $3x – 8y = 10$. *Ans:8/3*

(b) Find the slope of the line perpendicular to the line $5x + 3y + 1 = 0$. *Ans:-3/5*

8. (a) Find the slopes of two straight lines $3x + 4y + 5 = 0$ and $6x + 8y + 7 = 0$ and write the relation between them. **[SEE MODEL-2076]** *Ans: parallel*

(b) Find the slopes of two straight lines $5x + 3y + 1 = 0$ and $25x + 15y + 10 = 0$ and write the relation between them. *Ans: parallel*

(c) Find the slopes of two straight line $x + 3y + 8 = 0$ and the line passing through the points (2, 1) and (-7, 4) and write the relation between them. *Ans: parallel*

9. (a)Find the value of k if the pair of straight lines $2x + ky = 3$ and $x + 3y – 2 = 0$ are parallel to each other. *Ans: 6*

(b) Find the value of p if the pair of straight lines $px - 6y = 5$ and $4x + 3y – 2 = 0$ are parallel to each other. *Ans: -8*

(c) If a line passing through the points (4, – p) and (– 2, 6) is parallel to the line $2y + 3x = 4$, find the value of p. *Ans: 3*

3. (a) Find the acute angle between the lines with slopes $\sqrt{3}$ and $-\sqrt{3}$. *Ans:60^0*

(b) Find the acute angle between the lines $4x – y + 7 = 0$ and $3x – 5y =$. *Ans: 45^0*

(c) Find the acute angle between the lines $y = 2x + 3$ and $x = 3y – 5$ *Ans: 45^0*

4. (a) Find the obtuse angle between the lines $y =2$ and $x – \sqrt{3}y + 6 = 0$ *Ans: 150^0*

(b) Find the obyuse angle between the lines $x + 2y = 5$ and $2x – y = 1$ *Ans: 150^0*

5. (a) Find the angles between the lines $2y = 3$ and $x – \sqrt{3}y + 6 = 0$ *Ans: 30^0, 150^0*

(b) Find $\angle BAC$ if A (1, 2), B (2, 1) and C (4, 5) are any three points. *Ans: 45^0, 135^0*

Application (A) Based Problems

1. (a) Find the equation of a straight line passing through the point (4, 1) and parallel to the line $2x + 5y = 3$. *Ans: $2x + 5y - 13 = 0$*

(b) Find the equation of a straight line passing through the point (2, 3) and parallel to the line $7x – 3y + 1 = 0$. *Ans: $7x – 3y + 5 = 0$*

2. (a) Find the equation of a straight line which is parallel to the line $2x + y – 4 = 0$ and making an intercept of length 2 units along y –axis. *Ans: $2x + y = 2$*

(b) Find the equation of a straight line which is parallel to the line $5x + 3y = 9$ and making an intercept – 5 on the x-axis. *Ans: $5x + 3y = 25$*

3. (a) A (0, 3), B (1, -1) and C (5, -5) are the vertices of Δ ABC; find the equation of line passing through the centroid of Δ ABC and parallel to the side BC. *Ans: $x + y = 3$*

(b) Find the equation of a straight line passing through the centroid of ΔPQR with vertices P (3, 3), Q (-2, -6) and R (5, -3) amd parallel to side QR. *Ans: $3x - 7y = 20$*

4. (a) Find the equation of a line parallel to the line $7x - 2y + 1 = 0$ and passing through the mid-point of the line joining the points (1, 4) and (3, 0). *Ans: $7x - 2y = 10$*

(b) Find the equation of a line parallel to the line $x - 8y + 7 = 0$ and passing through the mid-point of the line joining the points (2, 1) and (-4, 1). *Ans: $x - 8y + 10 = 0$*

5. (a) Find the eqwustioj of a straight line passinfg through (3, 2) and perpendicular to the line $4x - 3y - 10 = 0$. *Ans: $3x + 4y = 17$*

(b) Find the eqwustioj of a straight line passinfg through (4, 6) and perpendicular to the line $x - 2y = 2$. *Ans: $2x + y = 14$*

6. (a) Find the equation of a straight line which is perpendicular to the line $x - 3y + 4 = 0$ and making an intercept of length 5 units along x –axis. Ans: $3x + y = 15$

(b) Find the equation of a straight line which is perpendicular to the line $8x + 3y = 11$ and making an intercept 4 on the x-axis. Ans: $3x - 8y = 12$

7. (a) Find the equation of line passing through the centroid of triangle ABCwith vertices A (0, 3), B (5, 1) and C (1, 2) and perpendicular to the side BC. *Ans: $4x + y = 6$*

(b) Find the equation of line passing through the centroid of triangle XYZ with vertices X (-5, -1), Y (-1, 2) and Z (0, 5) and perpendicular to the side XZ. *Ans: $5x + 6y = 2$*

8. (a) Find the equation of a straight line perpendicular to the line $2x + y = 1$ and passing through the mid-point of the line joining the points (0, 7) and (6, 1). *Ans: $x-2y+5 = 0$*

(b) Find the equation of a straight line perpendicular to the line $x + 3y = 4$ and passing through the mid-point of the line joining the points (3, 1) and (1, 3). *Ans: $3x - y = 4$*

9. (a) Find the equation of the altitude of triangle PQR with vertices P (2, 3), Q (-4, 1) and R (2, 0) drawn from the vertex P. *Ans: $6x - y = 9$*

(b) The points M (3, 4), N (-1, 1) and P (5, -1) are the vertices of a Δ MNP. Find the equation of altitude of the Δ MNP drawn from the point N (-1, 1) *Ans: $5y - 2x + 7 = 0$*

10. (a) M (4, 7) and N (5, -2) are two given points. Find the equation of the line PQ which is perpendicular to MN and passes through the point (2, 3) *Ans: $x - 9y + 25 = 0$*

(b) A (-3, 2) and B (5, -3) are two given points. Find the equation of the line XY which is perpendicular to AB and passes through the point (-6, 2) *Ans: $8x - 5y + 58 = 0$*

11. (a) Find the equation of the perpendicular bisector of line segment joining the points

(i) (3, 5) and (9, 3) Ans: $3x - y = 14$ (ii) (2, 4) and (8, 10) *Ans: $x + y = 12$*

12. (a) A (3, 5) and C (7, 9) are the opposite vertices of a rhombus ABCD, find the equation of the diagonal BD. *Ans: $x + y = 12$*

(b) If P (5, 7) and R (7, -2) are the opposite vertices of a square PQRS, find the equation of the diagonal QS. *Ans: $4x - 18y + 21 = 0$*

Higher Ability (HA) Based Problems

1. (a) Find the equation of the straight line passing through the point of intersection of the lines $3x + y = 7$ and $3y = 4x - 5$ and parallel to the line $2x - y = 3$. *Ans: $2x - y = 3$*

(b) Find the equation of the straight line passing through the point of intersection of the lines $2x + y = 3$ and $7x = 5y + 2$ and parallel to the line $4x + 5y = 9$ *Ans: $4x + 5y = -9$*

2. (a) Find the equation of the line passing through the point of intersection of the lines $3x + 4y = 7$ and $5x - 2y = 3$ and perpendicular to the line $2x + 3y = 5$. *Ans: $3x - 2y = -1$*

(b) Find the equation of the straight line passing through the point of intersection of the lines $x + y = 3$ and $x - y = 1$ and perpendicular to the line $2x - y = 1$. *Ans: $x + 2y = 3$*

3. (a) Find the equation the lines passing through the point (2, 3) and making an angle of 45^0 with the line $x - 3y = 5$. *Ans: $2x - y = 1$, $x + 2y = 8$*

(b) Find the equation the lines passing through the point (4, -1) and making an angle of 45^0 with the $2x - 3y = 5$. *Ans: $5x - y = 21$, $x + 5y + 1 = 0$*

4. (a) If B (1, 5) and D (7, 0) are the ends of diagonal BD of a square ABCD, find the equations of sides BC and CD. *Ans: $x - 11y + 54 = 0$, $11x + y = 16$*

(b) ABCD is a square having a vertex A (1, 2). If the equation of the diagonal AC is $x - 2y = 3$, find the equations of sides AB and AD. *Ans: $3x + y = 5$, $x - 3y + 5 = 0$*

5. (a) Find the equation of the sides of an equilatereal triangle whose vertex is (-1, 2) and base is $y = 0$.
Ans: $\sqrt{3}x - y + (\sqrt{3} + 2) = 0$, $\sqrt{3}x + y + (\sqrt{3} - 2) = 0$

(b) Find the equation of the remaing sides of a right angled isosceles triangle whose vertex is (-2, -3) and base is $x = 0$. *Ans: $x - y = 1$, $x + y + 5 = 0$*

6. (a) If the line $\frac{x}{a} + \frac{y}{b} = 1$ passes through the point of intersection of the lines $x + y = 3$ and $2x - 3y = 1$ and is parallel to the line $y = x - 6$, find the values of aand b. *Ans: $a = 1$, $b = -1$*

(b) If the line $\frac{x}{a} + \frac{y}{b} = 1$ passes through the point of intersection of the lines $x- y = 0$ and $4x - 3y = 1$ and is parallel to the line $2x - 2y = 5$, find the values of aand b. *Ans: $a = 2$, $b = 2$*

7. (a) Find the equation of line which is parallel to the line $4x + 5y = 6$ and makes the interepts on the axes whose sum is 9. *Ans: $4x+5y = 20$.*

(b) Find the equation of line which is parallel to the line $2x - 3y = 7$ and makes the interepts on the axes whose sum is 5. *Ans: $2x - 3y = 30$*

8. (a) Find the angle between the straight lines $y = m_1x + c_1$ and $y = m_2x + c_2$. Also, derive the conditions of the lines of perpendicularity and parallelism.

(b) Show that the equation of line passing through the point $(a\cos^3\theta, a\sin^3\theta)$ and perpendicular to the line $x\sec\theta + y\operatorname{cosec}\theta = a$ is $x\cos\theta - y\sin\theta = a\cos2\theta$.

4.2 PAIR OF STRAIGHT LINES

Knowledge (K) Based Problems

1. (a) Write the formula to find the angle between the pair of lines represented by $ax^2 + 2hxy + by^2 = 0$.

(b) If θ be the angles the pair of lines represented by $ax^2 + 2hxy + by^2 = 0$; a, b ≠ 0, what is the value of tanθ?

2. (a) If the pair of lines represented by $ax^2 + 2hxy + by^2 = 0$, $a \neq 0$, $b \neq 0$are coincident (parallel) to each other, write the relation between a, b and h.

(b) Write the condition under which the pair of straight lines represented by $ax^2 + 2hxy + by^2 = 0$ are perpendicular to each other.

Understanding (U) Based Problems

1. (a) Find the single equation for the pair of straight lines represented by $3x + 2y = 0$ and $2x - 3y = 0$.
[SEE MODEL-2076] *Ans: $6x^2 - 5xy - 6y^2 = 0$*

(b) Find the single equation for the pair of straight lines represented by $x + y = 0$ and $x - 2y = 0$.
Ans: $x^2 - xy - 2y^2 = 0$

2. (a) Find the separate equation of lines represented by $x^2 + 7xy + 6y^2 = 0$ *Ans: $x + 6y = 0$, $x + y = 0$*

(b) Find the separate equation of lines represented by $2x^2 + xy - y^2 = 0$ *Ans: $x + 2y = 0$, $x - y = 0$*

3. (a) Find the separate equation of lines represented by $x(x-1)-y(y-1)=0$ *Ans: $x - y = 0$, $x + y - 1 = 0$*

(b) Find the separate equation of lines represented by $ab(x^2-y^2)-(a^2 - b^2)xy=0$ *Ans: $px+qy=0$, $qx-py = 0$*

4. (a) Find the separate equation of lines represented by $x^2 + 2xy\sec\theta + y^2 = 0$ *Ans: $x+(\sec\theta \pm \tan\theta) y = 0$*

(b) Find the separate equation of lines represented by $x^2+2xy\operatorname{cosec}\theta+y^2 =0$ *Ans: $x+(\operatorname{cosec}\theta \pm \cot\theta)y=0$*

5. (a) Find the acute angle between the lines represented by the equation $3x^2 + 7xy + 2y^2 = 0$ *Ans: 45^0*

(b) Find the acute angle between the lines represented by the equation $x^2 + 4xy + y^2 = 0$ *Ans: 60^0*

6. (a) Find the obtuse angle between the lines represented by the equation $12x^2 - 23xy + 5y^2=0$ *Ans:135^0*

(b) Find the obtuse angle between the lines represented by the equation $\sqrt{3}\,x^2+4xy+\sqrt{3}\,y^2=0$. *Ans: 150^0*

7. (a) Show that the lines represented by $3x^2 + 4xy - 3y^2 = 0$ are perpendicular to each other.

(b) Prove that the pair of lines represented by $x(x + 2) + y(3 - y) = 0$ are perpendicular to each other.

8. (a) If the pair of straight lines represented by $(k + 1)x^2 - 3xy - 5y^2 = 0$ are perpendicular to each other, find the value of k? *Ans: 4*

(b) For what value of m, the two lines represented by $(3 - 2m)x^2 - 5xy + my^2 = 0$ are perpendicular to each other? Find it. *Ans: 3*

9. (a) Show that the pair of straight lines represented by $4x^2 + 12xy + 9y^2 = 0$ are coincident to each other.

(b) Show that the pair of straight lines represented by $x^2 - 14xy + 49y^2 = 0$ are coincident to each other.

10. (a) A pair of lines represented by $px^2 - 12xy + 9y^2 = 0$ are coincident, find the value of p. *Ans: 4*

(b) A pair of lines represented by $x^2 - 4xy + (k - 1)y^2 = 0$ are coincident, find the value of k. *Ans: 5*

11. (a) If the angle between the pair of straight lines represented by $kx^2 + 7xy + 2y^2 = 0$ is 45^0; find the positive value of k. *Ans: 3*

(b) If the angle between the pair of straight lines represented by $x^2 - 4xy + py^2 = 0$ is 60^0; find the positive value of p. *Ans: 1*

Application (A) Based Problems

1. (a) Find the separate equations of lines represented by $6x^2 - xy - y^2 = 0$. Also, find the angle between them. *Ans: $3x - y = 0,\ 2x + y = 0,\ 45^0,\ 135^0$*

(b) Find the separate equations of lines represented by $2x^2 + 7xy + 3y^2 = 0$. Also, find the angle between them. *Ans: $2x + y = 0,\ x + 3y = 0,\ 45^0,\ 135^0$*

2. (a) Find the separate equations of lines represented by $x^2 + 2xy\sec\theta + y^2 = 0$. Also, find the angle between them. *Ans: $x + (\sec\theta \pm \tan\theta)y = 0,\ \theta,\ 180^o - \theta$*

(b) Find the separate equations of lines represented by $x^2 - 2xy\,\mathrm{cosec}\theta + y^2 = 0$. Also, find the angle between them. *Ans: $x - (\mathrm{cosec}\theta \pm \cot\theta)y = 0,\ (90^0 + \theta),\ 90^0 - \theta)$*

3. (a) If α be the acute angle between the pair of lines represented by the equation $x^2 + 2xy\sec\theta + y^2 = 0$, prove that: $\alpha = \theta$

(b) If the acute angle between the pair of lines represented by $x^2 - 2xy\,\mathrm{cosec}\theta + y^2 = 0$ is α, prove that: $\alpha = 90^0 - \theta$.

(c) If the acute angle between the pair of lines represented by $x^2 + 2xy\sec\theta + y^2 = 0$ is α, prove that: $\alpha = 90^0 + \theta$.

4. (a) Find the single equation of the pair of straight lines passing through the origin and perpendicular to the lines represented by $2x^2 - 5xy + 2y^2 = 0$. **[SEE MODEL-2076]** *Ans: $2x^2 + 5xy + 2y^2 = 0$*

(b) Find the single equation of the pair of straight lines passing through the origin and perpendicular to the lines represented by $x^2 + xy - 2y^2 = 0$. *Ans: $2x^2 + xy - y^2 = 0$*

5. a) Find the single equation of the pair of straight lines passing through the point (1, 2) and parallel to the lines represented by $x^2 + 3xy + 2y^2 = 0$. *Ans: $x^2 + 3xy + 2y^2 - 8x - 11y + 15 = 0$*

(b) Find the equations of the pair of straight lines passing through the point (2, 1) and parallel to the lines represented by $20x^2 - 11xy - 3y^2 = 0$. *Ans: $4x - 3y - 5 = 0,\ 5x + y - 11 = 0$*

6. (a) Find the single equation of pair of straight lines passing through the point (2, 1) and perpendicular to the pair of straight lines represented by $x^2 + 3xy - 4y^2 = 0$ *Ans: $4x^2 + 3xy - y^2 - 19x - 4y + 21 = 0$*

(b) Find the single equation of pair of straight lines passing through the point (1, 2) and perpendicular to the pair of straight lines represented by $x^2 - xy - 2y^2 = 0$ *Ans: $2x^2 - xy - y^2 - 2x + 5y - 4 = 0$*

7. (a) If two straight lines represented by an equation $3x^2 + 8xy + my^2 = 0$ are perpendicular to each other, find the separate equation of two lines. *Ans: $x + 3y = 0,\ 3x - y = 0$*

(b) If an angle between the lines represented by $2x^2 + kxy + 3y^2 = 0$ is 45^0, find the positive value of k and then separate equation of lines. *Ans: 7, $2x + y = 0,\ x + 3y = 0$*

Higher Ability (HA) Based Problems

1. (a) Find the separate equations of pair of straight lines represented by the equation
$x^2 + 2xy + y^2 - 2x - 2y - 15 = 0$ *Ans:* $x + y + 3 = 0, x + y = 5$
(b) Find the equation of a pair of straight lines represented by $2x^2 - 5xy - 3y^2 + 3x + 19y - 20 = 0$.
Ans: $x - 3y + 4 = 0, 2x + y = 5$

4.3 CONIC SECTIONS

Knowledge (K) Based Problems

1. (a) Which geometrical figure will form if a plane intersects a cone parallel to its base?**[SEE MODEL-2076]**
(b) Which geometrical figure will form if a plane intersects a cone perpendicular to its axis?
(c) Which geometrical figure will form if a plane intersects a cone parallel to its generator?
(d) If the plane cuts the cone being parallel to its axis, what conic section will form?
(e) Which geometrical figure will form if a plane intersects a cone parallel to its generator?
(f) Name the conic section so formed when an intersecting plane is neither parallel nor perpendicular to the base.

2. (a) What type of conic section is formed if the angle between intersecting plane and axis of cone is a right angle (90^0)?
(b) What type of conic section is formed if the angle between intersecting plane and axis of cone is 0^0?

Understanding (U) Based Problems

1. (a) Define circle with figure. (b) Define parabola with figure.
(c) Define ellipse with figure. (d) Define hyperbola with figure.

4.4 CIRCLE

Knowledge (K) Based Problems

1. (a) What is the equation of the circle with centre (0, 0) and radius r units?
(b) What is the equation of the circle having centre (a, b) and radius c units?
(c) Write the equation of the circle having ends of a diameter are (x_1, x_2) and (y_1, y_2).

2. (a) What is the centre and radius of a circle with equation $(x - p)^2 + (y - q)^2 = k^2$?
(b) What is the centre and radius of a circle $x^2 + y^2 + 2gx + 2fy + c = 0$?

3. (a) What will be the length of radius of a circle having centre (h, k) and touches the x-axis?
(b) What will be the length of radius of a circle having centre (h, k) and y-axis is tangent to the circle?
(c) What will be the length of radius of a circle having centre (h, k) and touches both the axes?

Understanding (U) Based Problems

1. (a) Find the coordinates of centre and length of radius of circle having equation $(x-2)^2+(y-3)^2=16$
Ans: (2, 3); 4 units
(b) Find the coordinates of centre and length of radius of circle $(x + 1)^2 + (y - 2)^2 = 9$. *Ans: (-1, 2); 3*

2. (a) Find the coordinates of centre and length of radius of circle having equation $x^2+y^2+4x-6y+4=0$.
Ans: (- 2, 3); 3 units
(b) Find the centre and length of radius of circle $x^2 + y^2 - 2x - 4y + 1 = 0$. *Ans: (1, 2); 2 units*

3. (a) Find the circumference of the circle $x^2 + y^2 - 2y - 48 = 0$. *Ans: 44 units*
(b) If the equation of circle is $x^2 + y^2 + 10x - 171 = 0$, find its centre and circumference.
Ans: (- 5, 0); 88 units

4. (a) If the centre of a circle $x^2 + y^2 - ax + by + 1 = 0$ is (2, 3), find the values of 'a' and 'b'.
(b) If the centre of a circle $x^2 + y^2 + mx - ny = 7$ is (- 4, 1), find the values of 'm' and 'n'.
Ans: (a) $a = 4, b = -6$ (b) $m = 8, n = 2$

5. (a) If the radius of a circle with the equation $x^2 + y^2 - ay = 24$ is 5 units, find the value of 'a'.
(b) If the radius of a circle with the equation $x^2 + y^2 - 4x - 6y - k = 0$ is 4 units, find the value of 'k'.
Ans: (a) $a = 2$ (b) $k = 3$

6. (a) Find the equation of circle with centre (1, 0) and radius 3 units. *Ans:* $x^2 + y^2 - 2x = 8$

(b) Find the equation of circle with centre (0, 3) and radius 4 units. *Ans:* $x^2 + y^2 - 4y = 7$

7. (a) Find the equation of a circle with centre (2, 3) and x-axis is tangent to it. *Ans:* $x^2+y^2-4x-6y+4=0$

(b) Find the equation of a circle with centre (–6, 5) and tangent to x-axis. *Ans:* $x^2+y^2+12x-10y+36=0$

8. (a) Find the equation of a circle with radius 3 units, touching both the positive axes. *Ans:* $x^2 + y^2 - 6x - 6y + 9 = 0$

(b) Find the equation of a circle with radius 5 units, touching both the positive axes. *Ans:* $x^2 + y^2 - 10x - 10y + 25 = 0$

9. (a) Find the equation of a circle whose ends of a diameter are (2, 3) and (–1, 4). *Ans:* $x^2 + y^2 - x - 7y + 10 = 0$

(b) Find the equation of a circle whose end points of a diameter are (3, 4) and (0, -2). *Ans:* $x^2 + y^2 - 3x - 2y - 8 = 0$

10. (a) Find the equation of a circle having centre (0, 0) and passes through the point (2, -3). *Ans:* $x^2 + y^2 = 13$

(b) Find the equation of a circle with centre (2, 3) and passes through the point (-2, 0). *Ans:* $x^2 + y^2 - 4x - 6y = 12$

11. (a) Find the coordinates of centre of a circle having equations of two diameters $x + y = 5$ and $2x - y = 1$ *Ans: (2, 3)*

(b) Find the coordinates of centre of a circle having equations of two diameters $x - y = 3$ and $x + 2y = 15$ *Ans: (7, 4)*

12. (a) Find the coordinates of centre of the circle which is concentric with the circle having equation $x^2 + y^2 - 8x - 4y = 7$. *Ans: (4, 2)*

(b) Find the coordinates of centre of the circle which is concentric with the circle having equation $x^2 + y^2 - 2x + 6y = 1$. *Ans: (1, -3)*

13. (a) Find the equation of circle with centre (3, 1) and that touches the line having equation $5x + 12y - 22 = 0$ *Ans:* $x^2 + y^2 - 6x + 2y + 9 = 0$

(b) Find the equation of circle with centre (1, 2) and that touches the line having equation $3x - 4y + 10 = 0$. *Ans:* $x^2 + y^2 - 2x - 4y + 4 = 0$

Application (A) Based Problems

1. (a) Find the centre and the radius of the circle $9x^2+9y^2-36x+6y=107$. *Ans: (2, -1/3), 4units*

(b) Find the coordinates of centre and the radius of the circle $4x^2+4y^2-24x-20y=3$ *Ans: (3, 5/2), 8units*

2. (a) If (3, 4) is one end of a diameter of a circle $x^2 + y^2 - 4x - 6y + 11 = 0$, find the other end. *Ans: (1, 2)*

(b) If (2, 2) is one end of a diameter of a circle $x^2 + y^2 + 2x - 8y + 5 = 0$, find the other end. *Ans: (-4, 6)*

(c) If the coordinates of one end of a diameter of the circle $x^2 + y^2 + 2x + 4x - 6y + 8 = 0$ is (0, 2) then find the coordinates of other end of the diameter. *Ans: (-4, 4)*

3. (a) Find the equation of the circle which passes through the point (1, 4) and equations of two diameters are $2x + y = 5$ and $x - y = 1$. *Ans:* $x^2 + y^2 - 4x - 2y = 5$

(b) Find the equation of the circle which passes through the point (0, 2) and equations of two diameters are $4x - 2y = 5$ and $8x + 6y = -5$. *Ans:* $x^2 + y^2 - x + 3y = 10$

4. (a) Find the equation of the circle with centre (3, 4) and passing through the point of intersection of the lines given by $3x - y = 14$ and $2x - y = 7$ *Ans:* $x^2 + y^2 - 6x - 8y = 0$

(b) Find the equation of the circle with centre (1, 2) and passing through the point of intersection of the lines given by $x + 2y = 3$ and $3x + y = 4$ *Ans:* $x^2 + y^2 - 2x - 4y = 13$

5. (a) Find the equation of the circle with centre (3, 2) and passing through the centre of circle $x^2 + y^2 - 2x + 4y + 5 = 0$. *Ans:* $x^2 + y^2 - 6x - 4y + 5 = 0$

(b) Find the equation of a circle with centre (4, 3) and passes through the centre of the circle $x^2 + y^2 - 6x + 2y - 7 = 0$. *Ans:* $x^2 + y^2 - 8x - 6y + 8 = 0$

(c) Find the equation of a circle with centre (-1, 2) and passes through the centre of the circle $x^2 + y^2 - 6x - 10y - 2 = 0$. *Ans:* $x^2 + y^2 + 2x - 4y - 20 = 0$

6. (a) Find the equation of circle concentric with the circle $x^2 + y^2 - 6x + y = 1$ and passing through the point (4, -2). *Ans:* $x^2 + y^2 - 6x + y + 6 = 0$

(b) A circle P passes through the (2, 1) and concentric with the circle Q having equation $x^2 + y^2 - x - 2y + 1 = 0$, find the equation of circle P. *Ans:* $x^2 + y^2 - x - 2y - 1 = 0$

7. (a) The centres of two equal circles A and B are X and Y respectively. If the coordinates of X are (2, 3) and the equation of circle B is $x^2 + y^2 - 2x + 6y + 1 = 0$ then find the equation of circle A.
Ans: $x^2 + y^2 - 4x - 6y + 4 = 0$

(b) Find the equation of circle having centre (– 4, 1) and has the same radius as the circle $x^2 + y^2 - 2x + 2y = 7$. *Ans:* $x^2 + y2 + 8x - 2y = 8$

(c) A circle is concentric with the circle $25x^2 + 25y^2 - 200x - 250y + 989 = 0$ and has radius half of the radius of given circle. Find the equation of the circle. *Ans:* $25x^2 + 25y^2 - 200x - 250y + 1016 = 0$

8. (a) Find the equation of the circle having centre at (b, 4) and passing through the points (-2, 0) and (4, 0) of x-axis. *Ans:* $x^2 + y^2 - 2x - 8y = 8$

(b) Find the equation of the circle which touches the x-axis at (3, 0) and passing through the point (1, 2).
Ans: $x^2 + y^2 - 6x - 4y + 9 = 0$

9. (a) Find the equation of the circle which passes through the points (2, 3) and (-1, 2) and its centre lies on the straight line $2x - 3y + 1 = 0$. *Ans:* $x^2 + y^2 - 2x - 2y - 3 = 0$

(b) Find the equation of the circle which passes through the points (2, 3) and (5, 4) and its centre lies on the straight line $2x + 3y - 7 = 0$. *Ans:* $x^2 + y^2 - 10x + 2y + 1 = 0$

(c) Find the equation of the circle which passes through the points (4, 1)) and (6, 5) and its centre lies on the straight line $2x + y = 16$ *Ans:* $x^2 + y^2 - 6x - 8y + 15 = 0$

(d) Find the equation of the circle which passes through the origin and (4, 2) and its centre lies on the line $x + y = 1$. *Ans:* $x^2 + y^2 - 8x + 6y = 0$

Higher Ability (HA) Based Problems

10. (a) On a wheel there are three points (5, 7), (-1, 7) and (5, -1) located such that the distance from a fixed point to these points is always equal. Find the coordinates of the fixed point and then derive the equation of representing the locus that contains all three points. **[SEE MODEL-2076]**
Ans: (2, 3), $x^2 + y^2 - 4x - 6y - 12 = 0$

(b) A cow is tight to a stake by a rope. If the cow moves through the points (2, -2), (5, 7) and (6, 6) in a circular path always keeping the rope tight then find the coordinates of the point at which the stake is fixed and the equation repseenting the locus of cow. *Ans:* (2, 3), $x^2 + y^2 - 4x - 6y - 12 = 0$

11. (a) Find the equation of circle which passes through the points (2, 0), (0, 2) and (-2, 0). *Ans:* $x^2 + y^2 = 4$

(b) Find the equation of circle passing through the points (2, -2), (5, 7) and (6, 6)
Ans: $x^2+y^2-4x-6y-12= 0$

12. (a) Find the equation of the circle which passes through the origin and making intercepts of lengths 6 and 8 units on the positive x- axis and y- axis. *Ans:* $x^2 + y^2 - 6x - 8y = 0$

(b) A circle passing through the origin and makes intercepts of lengths 4 and 2 units on the positive x-axis and y- axis. Find the centre and the equation of the circle. *Ans:* (2, 1) units, $x^2 + y^2 - 4x - 2y=0$

13. (a) If the line $x + y = 1$ cuts the circle $x^2 + y^2 = 1$ at two points, find the distance between the points.
Ans: $\sqrt{2}$ units

(b) If the line $x + y = 5$ cuts the circle $x^2 + y^2 = 25$ at two points A and B, find the length of AB.
Ans: $5\sqrt{2}$ units

14. (a) Find the equation of circle which touches x- axis at the point (4, 0) and cuts off an intercept of 6 units from the positive y-axis. *Ans:* $x^2 + y^2 - 8x - 10y + 16 = 0$

(b) Find the equation of circle which lies in the fist quadrant, touches x- axis at the point (3, 0) and cuts off an intercept of 8 units from y-axis. *Ans:* $x^2 + y^2 - 6x - 10y + 9 = 0$

15. (a) Find the equation of circle whose diameter is the intercept between the coordiantre axes of the line $3x + 2y = 6$. *Ans:* $x^2 + y^2 - 2x - 3y = 0$

(b) Find the equation of circle whose diameter is the intercept between the coordiantre axes of the line $4x - 6y = 12$. *Ans:* $x^2 + y^2 - 3x + 2y = 0$

UNIT-5 TRIGONOMETRY

5.1 MULTIPLE AND SUB-MULTIPLE ANGLES

Knowledge (K) Based Problems

1. (a) Express sin2A in terms of sinA and cos A.
(b) Express sin2A in terms of tanA. **[SEE MODEL-2076]**
(c) Write sin3A in terms of sinA.

2. (a) Express sinθ in terms of $\sin\frac{\theta}{2}$ and $\cos\frac{\theta}{2}$
(b) Write sinθ in terms of $\tan\frac{\theta}{2}$

3. (a) Express cos2A in terms of cosA and sin A.
(b) Express cos2A in terms of cosA.
(c) Write cos2A in terms of sinA.
(d) Write cos2A in terms of tanA.
(e) Express cos3A in terms of cosA.

4. (a) Write cosα in terms of $\sin\frac{\alpha}{2}$ and $\cos\frac{\alpha}{2}$
(b) Express cosα in terms of $\cos\frac{\alpha}{2}$
(c) Express cosα in terms of $\sin\frac{\alpha}{2}$
(d) Express cosα in terms of $\tan\frac{\alpha}{2}$.
(e) Express cosα in terms of cosα /3

5. (a) Express tan2A in terms of tan A.
(b) Write tanA in terms of tan A/2.

Understanding (U) Based Problems

1. (a) If sinA = 3/4, find the value of cos2A. *Ans: 7/25*
(b) If cosψ = 3/4 find the value of cos2ψ. *Ans: 1/8*
(c) If tanθ = 3/4, find the value of cos2θ. *Ans:7/25*

2. (a) If sinA = 4/5, find the value of sin2A. *Ans: 24/25*
(b) If 5sinα = 3 find the value of sin2α. *Ans:24/25*
(c) If cosA = 12/13 find the value of sin2A. *Ans: 120/169*
(d) If tanθ = 1/3, find the value of sin2θ. *Ans: 3/5*

3. (a) If tanθ = 4/3, find the value of tan2θ. *Ans: -24/7*
(b) If 3tanθ = 2, find the value of tan2θ. *Ans: 12/5*

4. (a) If cosθ = 3/5, find the value of cos3θ. *Ans: -117/125*
(b) If 2cosθ = $\sqrt{3}$, find the value of cos3θ. *Ans: 0*

5. (a) If 2sinθ = 1, find the value of sin3θ. *Ans: 1*
(b) If 5sinA = 4, find the value of sin3A. *Ans: 44/125*

6. (a) If sinA/2 = 3/5, find the value of sinA. *Ans: 24/25*
(b) If sinA/2 = 1/2, find the value of sinA. *Ans: $\sqrt{3}/2$*
(c) If tanA/2 = 3/4, find the value of sinA. *Ans:24/25*

7. (a) If $\cos\frac{\alpha}{2} = \frac{4}{5}$, find the value of cosα. *Ans:7/25*
(b) If $\cos\frac{\theta}{2} = \frac{1}{\sqrt{2}}$, find the value of cosθ. *Ans: 0*
(c) If sinA/2 = 3/4, find the value of cosA. *Ans:-1/8*

8. (a) If $\sin\frac{\theta}{3} = \frac{1}{2}$, find the value of sinθ. *Ans: 1*
(b) If $\sin\frac{A}{3} = \frac{4}{5}$, find the value of sinθ. *Ans: 44/125*

9. (a) If $\cos\frac{\theta}{3} = \frac{3}{5}$, find the value of cosθ. *Ans: -117/125*

(b) If cosA/3 = 4/5, find the value of cosA. *Ans: 44/125*

10. (a) If $\cos A = \frac{1}{2}\left(a + \frac{1}{a}\right)$ then show that $\cos 2A = \frac{1}{2}\left(a^2 + \frac{1}{a^2}\right)$.

(b) If $\sin\frac{A}{2} = \frac{1}{2}\left(x + \frac{1}{x}\right)$ prove that $\cos A + \frac{1}{2}\left(x^2 + \frac{1}{x^2}\right) = 0$

(c) If $\cos\frac{\alpha}{3} = \frac{1}{2}\left(k + \frac{1}{k}\right)$, prove that $\cos\alpha = \frac{1}{2}\left(k^3 + \frac{1}{k^3}\right)$

(d) If $\sin\frac{\theta}{3} = \frac{1}{2}\left(m + \frac{1}{m}\right)$ prove that $\sin\theta + \frac{1}{2}\left(m^3 + \frac{1}{m^3}\right) = 0$

11. If $\cos 330^0 = \frac{\sqrt{3}}{2}$, prove that: (a) $\cos 15^o = \frac{1}{2}(\sqrt{2+\sqrt{3}})$ (b) $\sin 15^0 = \frac{1}{2}(\sqrt{2-\sqrt{3}})$

12. If $\cos 45^o = \frac{1}{\sqrt{2}}$, show that: (a) $\cos\left(22\frac{1}{2}\right)^o = \frac{1}{2}\sqrt{2+\sqrt{3}}$ (b) $\sin\left(22\frac{1}{2}\right)^o = \frac{1}{2}\sqrt{2-\sqrt{3}}$

13. (a) If $\tan^2\alpha = 1 + 2\tan^2\beta$, prove that $\cos 2\beta = 1 + 2\cos 2\alpha$

(b) If tanθ = b/a, prove that: $a\cos 2\theta + b\sin 2\theta = a$

14. (a) Prove that: $2\cos^2(45^0 - A) = 1 + \sin 2A$ (b) Prove that: $1 - 2\sin^2(45^0 - \theta) = \sin 2\theta$

15. (a) Prove that: $\cos 4A = 8\cos^4 A - 8\cos^2 A + 1$ (b) Prove that: $\cos 4\alpha = 8\sin^4\alpha - 8\sin^2\alpha + 1$

16. (a) Prove that: $\frac{1+\tan^2(45^0 - A)}{1-\tan^2(45^0 - A)} = \text{cosec}2A$ (b) Prove that: $\frac{1-\tan^2(45^0-\alpha)}{1+\tan^2(45^0-\alpha)} = \sin 2\alpha$

(c) Prove that: $1 - 2\sin^2\left(45^0 - \frac{A}{2}\right) = \sin\frac{A}{2}$ (d) Prove that: $\cos^2\left(\frac{\pi}{4} - \frac{A}{4}\right) - \sin^2\left(\frac{\pi}{4} - \frac{A}{4}\right) = \sin\frac{A}{2}$

17. (a) Express $\frac{\sin 2A}{1+\cos 2A}$ in terms of tangent. (b) Express $\frac{1\text{-co}2A}{\sin 2A}$ in terms of tangent.

18. (a) Express $\frac{\cos^3 x - \cos 3x}{\sin^3 x + \sin 3x}$ in terms of tangent. (b) Express $\frac{\sin^3 A + \sin 3A}{\cos^3 A - \cos 3A}$ in terms of cotangent.

19. (a) Express $\frac{\sin 2A}{\sin A} - \frac{\cos 2A}{\cos A}$ in terms of secant. (b) Express $\frac{\sin\alpha + \sin 2\alpha}{1 + \cos\alpha + \cos 2\alpha}$ in terms of tangent.

(c) Express $\frac{\sin 2A - \sin A}{\cos 2A - \cos A + 1}$ in terms of tangent. (d) Express $\frac{1 + \sin 2A - \cos 2A}{1 + \sin 2A + \cos 2A}$ in terms of tangent.

20. (a) Prove that: $\frac{\cos 2\theta}{1 - \sin 2\theta} = \frac{1 + \tan\theta}{1 - \tan\theta}$ (b) Prove that: $\frac{1 - \sin 2\theta}{\cos 2\theta} = \frac{1 - \tan\theta}{1 + \tan\theta}$

21. (a) Express $\frac{\sin\alpha - \sqrt{1+\sin 2\alpha}}{\cos\alpha - \sqrt{1+\sin 2\alpha}}$ in terms of co-tangent. (b) Express $\frac{\cos\alpha - \sqrt{1+\sin 2\alpha}}{\sin\alpha - \sqrt{1+\sin 2\alpha}}$ in terms of tangent.

22. (a) Express cosec2A – cot2A in terms of tangent. (b) Express cosec2A + cot2A in terms of cotangent.

23. (a) Prove that: $\frac{\cos^3\alpha + \sin^3\alpha}{\cos\alpha + \sin\alpha} = 1 - \frac{1}{2}\sin 2\alpha$ (b) Prove that: $\frac{\cos^3 A - \sin^3 A}{\cos A - \sin A} = 1 + \frac{1}{2}\sin 2A$

24. (a) Prove that: $\frac{1 - 2\sin A.\cos A}{2} = \sin^2\left(\frac{\pi^c}{4} - A\right)$ (b) Prove that: $\frac{1 + 2\sin\theta.\cos\theta}{2} = \sin^2\left(\frac{\pi^c}{4} + \theta\right)$

25. (a) Express $\frac{\sin A}{1 + \cos A}$ in terms of sub-multiple angle of tangent. **[SEE MODEL-2076]**

(b) Express $\frac{1 - \cos A}{\sin A}$ in terms of sub-multiple angle of tangent.

26. (a) Express $\dfrac{2\sin\beta-\sin 2\beta}{2\sin\beta + \sin 2\beta}$ in terms of sub-multiple angle of tangent.

(b) Express $\dfrac{2\sin A+\sin 2A}{2\sin A-\sin 2A}$ in terms of sub-multiple angle of co-tangent.

27. (a) Prove that: $\dfrac{\sin\alpha+\sin\frac{\alpha}{2}}{1+\cos\alpha+\cos\frac{\alpha}{2}}=\tan\dfrac{\alpha}{2}$ (b) Prove that: $\dfrac{1+\cos A+\cos\frac{A}{2}}{\sin A+\sin\frac{A}{2}}=\cot\dfrac{A}{2}$

28. (a) Prove that: $\dfrac{1+\tan\frac{\theta}{2}}{1-\tan\frac{\theta}{2}}=\dfrac{\cos\theta}{1-\sin\theta}$ (b) Prove that: $\dfrac{1-\sin\theta}{\cos\theta}=\dfrac{1-\tan\frac{\theta}{2}}{1+\tan\frac{\theta}{2}}$

29. (a) Prove that: $\tan\dfrac{\theta}{2}=\dfrac{1+\sin\theta-\cos\theta}{1+\sin\theta+\cos\theta}$ (b) Prove that: $\dfrac{1+\sin\theta+\cos\theta}{1+\sin\theta-\cos\theta}=\cot\dfrac{\theta}{2}$

30. (a) Prove that $\dfrac{\cos\frac{A}{2}-\sqrt{1+\sin A}}{\sin\frac{A}{2}-\sqrt{1+\sin A}}=\tan\dfrac{A}{2}$ (b) Prove that $\dfrac{\sin\frac{A}{2}-\sqrt{1+\sin A}}{\cos\frac{A}{2}-\sqrt{1+\sin A}}=\cot\dfrac{A}{2}$

23. (a) Find the value of $\cot 22\frac{1}{2}^0-\tan 22\frac{1}{2}^0$ (b) Find the value of $\tan 67\frac{1}{2}^0-\cot 67\frac{1}{2}^0$

Application (A) Based Problems

1. (a) Prove that: $\sqrt{3}$ cosec20^0 – sec20^0 = 4 (b) Find the value of cosec10^0 – $\sqrt{3}$ sec10^0

(c) Find the value of sec40^0 + $\sqrt{3}$ cosec40^0 (d) Find the value ofsec80^0 – $\sqrt{3}$ cosec80^0

2. (a) Prove that: $\cos^2A + \sin^2A.\cos2B = \cos^2B+ \sin^2B.\cos2A$

(b) Prove that: $\sin^2A - \cos^2A.\cos2B = \sin^2B - \cos^2B.\cos2A$

3. If $2\tan\alpha = 3\tan\beta$, prove that: (a) $\tan(\alpha+\beta)=\dfrac{5\sin 2\beta}{5\cos 2\beta-1}$ (b) $\tan(\alpha-\beta)=\dfrac{\sin 2\beta}{5-\cos 2\beta}$

4. (a) Prove that: $\cos^6\theta-\sin^6\theta=\cos2\theta\left(1-\frac{1}{4}\sin^2 2\theta\right)$ (b) Prove that: $\cos^6\alpha-\sin^6\alpha=\frac{1}{4}(3\cos2\alpha+\cos^3 2\alpha)$

(c) Prove that: $\cos^6\theta+\sin^6\theta=1-\frac{3}{4}\sin^2 2\theta$ (d) Prove that: $\cos^6\theta+\sin^6\theta=\frac{1}{8}(5+3\cos4\theta)$

5. (a) Prove that: $\cos^8\theta-\sin^8\theta=\cos2\theta\left(1-\frac{1}{2}\sin^2 2\theta\right)$ (b) Prove that: $\cos^8\theta + \sin^8\theta = 1 - \sin^2 2\theta + \frac{1}{8}\sin^4 2\theta$

6. (a) Prove that: $\sin^4x = \frac{1}{8}(3 - 4\cos2x + \cos4x)$ (b) Prove that: $\cos^4\theta = \frac{1}{8}(3 + 4\cos2\theta + \cos4\theta)$

(c) Prove that: $3 + 4\cos2\alpha + \cos4\alpha = 8\cos^4\alpha$ (d) Prove that: $3 - 4\cos2\alpha + \cos4\alpha = 8\sin^4\alpha$

7. (a) Prove that:coec2A + cot4A = cotA – cosec4A (b) Prove that: coec4A + cot8A = cot2A – cosec8A

(c) Prove that: cosec2A + cosec4A = cotA – cot 4A (d)Prove that: cosec4A + cosec8A = cot2A – cot8A

8. (a) Prove that: $\dfrac{\sec4A-1}{\sec2A-1}=\tan4A.\cot A$ (b) Prove that: $\dfrac{\sec4\theta-1}{\sec2\theta-1}=\dfrac{\tan4\theta}{\tan\theta}$

9. (a) Prove that: $\tan\theta+2\tan2\theta+4\cot4\theta=\cot\theta$ (b) Prove that: $\tan\theta+2\tan2\theta+4\tan4\theta+8\cot8\theta=\cot\theta$

10. (a) Prove that: (2cosA + 1) (2cosA – 1) (2cos2A – 1) = 2cos4A + 1

(b) Prove that: (2cosA + 1) (2cosA – 1) (2cos2A – 1) (2cos4A – 1) = 2cos8A + 1

11. (a) Prove that: $\sqrt{2+\sqrt{2+\sqrt{2+2\cos8\theta}}}=2\cos\theta$ (b) Prove that: $\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2+2\cos16\theta}}}}=2\cos\theta$

12. (a) Prove that: $\left(1+\sin\frac{\pi^c}{8}\right)\left(1+\sin\frac{3\pi^c}{8}\right)\left(1-\sin\frac{5\pi^c}{8}\right)\left(1-\sin\frac{7\pi^c}{8}\right)=\frac{1}{8}$

(b) Prove that: $\left(1+\cos\frac{\pi^c}{8}\right)\left(1+\cos\frac{3\pi^c}{8}\right)\left(1+\cos\frac{5\pi^c}{8}\right)\left(1+\cos\frac{7\pi^c}{8}\right)=\frac{1}{8}$

13. Prove that: (a) $\cot(\theta+45^0)-\tan(\theta-45^0)=\frac{2\cos 2\theta}{1+\sin 2\theta}$ (b) $\cot\left(\frac{A}{2}+\frac{\pi}{4}\right)-\tan\left(\frac{A}{2}-\frac{\pi}{4}\right)=\frac{2\cos A}{1+\sin A}$

14. Prove that: (a) $4(\cos^3 15^0+\sin^3 15^0)=3(\cos 15^0+\sin 15^0)$ (b) $4(\cos^3 20^0+\sin^3 10^0)=3(\cos 20^0+\sin 10^0)$

15. Prove that: (a) $\cos^3 A.\cos 3A+\sin^3 A.\sin 3A=\cos^3 2A$ (b) $\cos^3 A.\sin 3A+\sin^3 A.\cos 3A=\frac{3}{4}\sin 4A$

16. Prove that: (a) $\sin 5\theta=16\sin^5\theta-20\sin^3\theta+5\sin\theta$ (b) $\cos 5\theta=16\cos^5\theta-20\cos^3\theta+5\cos\theta$

17. Prove that: (a) $\sin^4\frac{\pi^c}{8}+\sin^4\frac{3\pi^c}{8}+\sin^4\frac{5\pi^c}{8}+\sin^4\frac{7\pi^c}{8}=\frac{3}{2}$ (b) $\cos^4\frac{\pi^c}{8}+\cos^4\frac{3\pi^c}{8}+\cos^4\frac{5\pi^c}{8}+\cos^4\frac{7\pi^c}{8}=\frac{3}{2}$

18. Prove that: (a) $\cos\frac{\pi^c}{7}.\cos\frac{2\pi^c}{7}.\cos\frac{3\pi^c}{7}=\frac{1}{8}$ (b) $\cos\frac{\pi^c}{7}.\cos\frac{2\pi^c}{7}.\cos\frac{4\pi^c}{7}=-\frac{1}{8}$

19. Prove that: (a) $\frac{2\sin x}{\cos 3x}+\frac{2\sin 3x}{\cos 9x}+\frac{2\sin 9x}{\cos 27x}=\tan 27x-\tan x$ (b) $\frac{2\cos x}{\sin 3x}+\frac{2\cos 3x}{\sin 9x}+\frac{2\cos 9x}{\sin 27x}=\cot x-\cot 27x$

5.2 TRANSFORMATION OF TRIGONOMETRIC FORMULAE

Knowledge (K) Based Problems

1. (a) Express 2sinA.cosB in terms of sine. (b) Express 2cosx.siny in terms of sine.
(c) Convert 2cosA.cosB in terms of cosine. (d) Express 2sinP.sinQ in terms of cosine.

2. (a) Convert sin C + sin D in terms of product of sine or cosine.
(b) Convert sin C - sin D in terms of product of sine or cosine.
(c) Convert cos α + cos β in terms of product of cosine.
(d) Convert cos X - cos Y in terms of product of sine.

Understanding (U) Based Problems

1. (a) Convert sin6A.cos4A into sum of difference of sine or cosine. **[SEE MODEL-2076]** $\rightarrow\frac{1}{2}$(sin10A+sin2A)

(b) Convert cos4A.sin2A into sum of difference of sine or cosine. Ans: $\frac{1}{2}$(sin6A-sin2A)

(c) Convert cos9θ.cos5θ into sum of difference of sine or cosine. Ans: $\frac{1}{2}(\cos 14\theta+\cos 4\theta)$

(d) Convert sin5α.sin20α into sum of difference of sine or cosine. Ans: $\frac{1}{2}(\cos 15\alpha-\cos 25\alpha)$

2. (a) Express sin8A + sin2A into product of sine or cosine. Ans: 2sin5A.cos3A
(b) Express sin25A – sin15A into product of sine or cosine. Ans: 2cos20A.sin5A
(c) Express cos5A + cos3A into product of cosine. Ans: 2cos4A.cosA
(d) Express cos40A – cos60A into product of sine. Ans: 2sin50A.sin10A

3. Find the value of (a) $\sin 75^0+\sin 15^0$ *Ans:* $\sqrt{\frac{3}{2}}$ (b) $\sin 75^0-\sin 105^0$ *Ans:* $\frac{1}{\sqrt{2}}$
(c) $\cos 15^0+\cos 75^0$ *Ans:* $\sqrt{\frac{3}{2}}$ (d) $\cos 15^0-\cos 75^0$ *Ans:* $\frac{1}{\sqrt{2}}$

4. Find the value of (a) $\sin 75^0.\sin 15^0$ *Ans: 1 / 4* (b) $4\cos 105^0.\cos 15^0$ *Ans: -1*

5. (a) Prove that: $2\cos 70^0.\cos 20^0=\cos 50^0$ (b) Prove that: $2\sin 50^0.\sin 40^0=\cos 10^0$
(c) Prove that: $2\sin 55^0.\cos 35^0=1+\sin 20^0$ (d) Prove that: $2\cos 80^0.\sin 10^0=1-\sin 70^0$
(e) Prove that: $2\cos(45^0+A).\cos(45^0-A)=\cos 2A$ (f) Prove that: $2\cos(45^0+\alpha).\sin(45^0+\alpha)=\cos 2\alpha$

6. (a) Prove that: $\cos 10^0+\cos 110^0+\cos 130^0=0$ (b) Prove that: $\cos 40^0+\cos 80^0+\cos 160^0=0$
(c) Prove that: $\cos 40^0+\sin 40^0=\sqrt{2}\cos 5^0$ (d) Prove that: $\cos 18^0-\sin 18^0=\sqrt{2}\sin 27^0$

7. (a) Prove that: $\frac{\sin A+\sin 5A}{\cos A+\cos 5A}=\tan 3A$ (b) Prove that: $\frac{\cos A-\cos 5A}{\sin 5A-\sin A}=\tan 3A$

Application (A) Based Problems

1. (a) Prove that: $(\cos A + \cos B)^2 + (\sin A + \sin B)^2 = 4\cos^2\left(\frac{A-B}{2}\right)$

(b) Prove that: $(\cos A - \cos B)^2 + (\sin A - \sin B)^2 = 4\sin^2\left(\frac{A-B}{2}\right)$

2. Prove that: (a) $\sin\theta.\sin(60^0-\theta).\sin(60^0+\theta) = \frac{1}{4}\sin3\theta$ (b) $\cos(60^0+\theta).\cos(60^0-\theta).\cos\theta = \frac{1}{4}\cos3\theta$

3. Prove that: (a) $\frac{\sin^2\alpha - \sin^2\beta}{\sin\alpha.\cos\alpha - \sin\beta.\cos\beta} = \tan(\alpha+\beta)$ (b) $\frac{\cos^2\alpha - \sin^2\beta}{\sin\alpha.\cos\alpha + \sin\beta.\cos\beta} = \cot(\alpha+\beta)$

4. Prove that: (a) $16\sin20^0.\sin40^0.\sin60^0.\sin80^0 = 3$ (b) $\sin10^0.\sin30^0.\sin50^0.\sin70^0 = \frac{1}{16}$

(c) $\sin10^0.\sin50^0.\sin70^0 = \frac{1}{8}$ (d) $16\sin6^0.\sin42^0.\sin66^0.\sin78^0 = 1$

5. Prove that: (a) $\cos10^0.\cos30^0.\cos50^0.\cos70^0 = \frac{3}{16}$ (b) $\cos40^0.\cos100^0.\cos160^0 = \frac{1}{8}$

(c) $8\cos10^0.\cos50^0.\cos70^0 = \sqrt{3}$ (d) $8\cos40^0.\cos100^0.\cos160^0 = 1$

6. (a) Find the value of $\sin20^0.\sin30^0.\sin40^0.\sin80^0$ [SEE MODEL-2076] *Ans:* $\frac{\sqrt{3}}{16}$

(b) Find the value of $\sin10^0.\sin30^0.\sin50^0.\sin70^0$ *Ans: 1/16*

(c) Find the value of $\cos10^0.\cos30^0.\cos50^0.\cos70^0$ *Ans: 3/16*

7. Prove that: (a) $\frac{\sin2A + \sin5A - \sin A}{\cos2A + \cos5A + \cos A} = \tan2A$ (b $\frac{\cos A - \cos2A + \cos3A}{\sin A - \sin2A + \sin3A} = \cot2A$

8. Prove that: (a) $\frac{\sin A + \sin2A + \sin4A + \sin5A}{\cos A + \cos2A + \cos4A + \cos5A} = \tan3A$ (b) $\frac{\sin2A + \sin5A + \sin7A + \sin9A}{\cos3A + \cos5A + \cos7A + \cos9A} = \tan6A$

9. Prove that: (a) $\frac{(\sin4A + \sin2A)(\cos4A - \cos8A)}{(\sin7A + \sin5A)(\cos A - \cos5A)} = 1$ (b) $\frac{(\cos A - \cos3A)(\sin8A + \sin2A)}{(\cos4A - \cos8A)(\sin5A - \sin A)} = 1$

10. Prove that: (a) $\frac{\sin8A.\cos A - \sin6A.\cos3A}{\cos2A.\cos A - \sin2A.\sin5A} = \tan2A$ (b) $\frac{\cos4A.\cos3A - \cos5A.\cos2A}{\cos4A.\sin3A - \cos5A.\sin2A} = \tan2A$

11. Prove that: (a) $\sec\left(\frac{\pi}{4}+\frac{\theta}{2}\right).\sec\left(\frac{\pi}{4}-\frac{\theta}{2}\right) = 2\sec\theta$ (b) $\text{cosec}\left(\frac{\pi}{4}+\frac{\theta}{2}\right).\text{cosec}\left(\frac{\pi}{4}-\frac{\theta}{2}\right) = 2\sec\theta$

12. Prove that: (a) $\cos^3x.\sin^2x = \frac{1}{16}(2\cos x - \cos3x - \cos5x)$ (b) $\cos^2x.\sin^3x = \frac{1}{16}(2\sin x + \sin3x - \sin5x)$

13. Prove that: (a) $\cos(36^o - A).\cos(36^o + A) + \cos(54^o + A)\cos(54^o - A) = \cos2A$

(b) $\cos(25^o - A).\cos(25^o + A) + \cos(65^o + A)\cos(65^o - A) = \cos2A$

14. Prove that: (a) $\sin^2A + \sin^2(A - 120^o) + \sin^2(A + 120^o) = \frac{3}{2}$

(b) $\cos^2A + \cos^2(A - 120^o) + \cos^2(A + 120^o) = \frac{3}{2}$

15. (a) If $\frac{1}{\sin A} + \frac{1}{\cos A} = \frac{1}{\sin B} + \frac{1}{\cos B}$, prove that: $\cot\left(\frac{A+B}{2}\right) = \tan A.\tan B$

(b) If sinA + sinB = A and cosA + cosB = b, prove that: $\tan\left(\frac{A+B}{2}\right) = \frac{a}{b}$

5.3 CONDITIONAL TRIGONOMETRIC IDENTITIES

Understanding (U) Based Problems

1. (a) If $A + B + C = 180^0$, convert sin (A + B) in terms of $\angle C$.

(b) If $A + B + C = 200^g$, express cos (A + B) in terms of $\angle C$.

(c) If $A + B + C = \pi^c$, find tan (A + B) in terms of angle C.

2. (a) If A + B + C = 90^0, find the value of $\sin\left(\frac{A}{2}+\frac{B}{2}\right)$

(b) If $\alpha + \beta + \gamma = 90^0$, find the value of $\cos\left(\frac{A}{2}+\frac{B}{2}\right)$

Application (A) Based Problems

1. If A, B and C are the intrereior angles of a triangle ABC, prove that:

a) $\cot\frac{A}{2}.\cot\frac{B}{2}.\cot\frac{C}{2} = \cot\frac{A}{2} + \cot\frac{B}{2} + \cot\frac{C}{2}$ b) $\tan\frac{A}{2}\tan\frac{B}{2} + \tan\frac{B}{2}.\tan\frac{C}{2} + \tan\frac{C}{2}.\tan\frac{A}{2} = 1$

2. If A + B + C = π^c, then prove that:

a) sin2A + sin2B + sin2C = 4sinA.sinB.sinC

b) sin2A – sin2B + sin2C = 4cosA.sinB.cosC

c) sin (B + C – A) + sin (C + A – B) + sin (A + B – C) = 4 sinA.sinB.sinC

d) $\frac{\cos A}{\sin B.\sin C} + \frac{\cos B}{\sin C.\sin A} + \frac{\cos C}{\sin A.\sin B} = 2$

3. If A + B + C = π^c, then prove that:

a) cos2A + cos2B + cos2C = - (1 + 4cosA.cosB.cosC)

b) cos2A – co2B – cos2C = 4cosA.sinB.sinC – 1

c) cos (B + C – A) + cos (C + A – B) + cos (A + B – C) = 4 cosA.cosB.cosC + 1

4. If A, B and C are the intrereior angles of a triangle ABC, prove that:

a) sinA.cosB.cosC + sinB.cosC.cosA + sinC.cosA.cosB = sinA.sinB.sinC

b) cosA.sinB.sinC + cosB.sinC.sinA + cosC.sinA.sinB = 1 + cosA.cosB.cosC

5. If A + B + C = π^c, then prove that:

a) $\sin^2A + \sin^2B + \sin^2C = 2 + 2\cos A.\cos B.\cos C$

b) $\sin^2A + \sin^2B - \sin^2C = 2\sin A.\sin B.\cos C$

c) $\sin^2A - \sin^2B + \sin^2C = 2\sin A.\cos B.\sin C$ [SEE MODEL-2076]

d) $\sin^2A - \sin^2B - \sin^2C = -2\cos A.\sin B.\sin C$

6. If A, B and C are the intrereior angles of a triangle ABC, prove that:

a) $\cos^2A + \cos^2B + \cos^2C = 1 - 2\cos A.\cos B.\cos C$ b) $\cos^2A - \cos^2B + \cos^2C + 2\sin A.\cos B.\sin C = 1$

c) $\cos^2A + \cos^2B - \cos^2C = 1 - 2\sin A.\sin B.\cos C$ d) $\cos^2A - \cos^2B - \cos^2C + 1 = 2\cos A.\sin B.\sin C$

7. If A + B + C = 180^0, then prove that:

a) $\sin A + \sin B + \sin C = 4\cos\frac{A}{2}.\cos\frac{B}{2}.\cos\frac{C}{2}$ b) $\sin A + \sin B - \sin C = 4\sin\frac{A}{2}.\sin\frac{B}{2}.\cos\frac{C}{2}$

8. If A, B and C are the intrereior angles of a triangle ABC, prove that:

a) $\cos A + \cos B + \cos C = 1 + 4\sin\frac{A}{2}.\sin\frac{B}{2}.\sin\frac{C}{2}$ b) $\cos A + \cos B + \cos C = 1 + 4\sin\frac{A}{2}.\sin\frac{B}{2}.\sin\frac{C}{2}$

9. If A + B + C = 180^0, then prove that:

a) $\sin^2\frac{A}{2} + \sin^2\frac{B}{2} + \sin^2\frac{C}{2} = 1 - 2\sin\frac{A}{2}.\sin\frac{B}{2}.\sin\frac{C}{2}$ b) $\cos^2\frac{A}{2} + \cos^2\frac{B}{2} + \cos^2\frac{C}{2} = 2\left(1 + \sin\frac{A}{2}.\sin\frac{B}{2}.\sin\frac{C}{2}\right)$

10. If A + B + C = 180^0, then prove that:

(a) $\sin\frac{A}{2} + \sin\frac{B}{2} + \sin\frac{C}{2} = 1 + 4\sin\left(\frac{\pi^c - A}{4}\right).\sin\left(\frac{\pi^c - B}{4}\right).\sin\left(\frac{\pi^c - C}{4}\right)$

b) $\sin\frac{A}{2} + \sin\frac{B}{2} + \sin\frac{C}{2} - 1 = 4\sin\left(\frac{A+B}{4}\right).\sin\left(\frac{B+C}{4}\right).\sin\left(\frac{C+A}{4}\right)$

c) $\sin(A+2B) + \sin(B+2C) + \sin(C+2A) = 2\sin\left(\frac{A-B}{2}\right).\sin\left(\frac{B-C}{2}\right).\sin\left(\frac{C-A}{2}\right)$

11. If A + B + C = 180^0, prove that:

a) $\cos\frac{A}{2} + \cos\frac{B}{2} + \cos\frac{C}{2} = 4\cos\left(\frac{\pi^c - A}{4}\right).\cos\left(\frac{\pi^c - B}{4}\right).\cos\left(\frac{\pi^c - C}{4}\right)$

b) $\cos\frac{A}{2} + \cos\frac{B}{2} - \cos\frac{C}{2} = 4\cos\left(\frac{\pi^c + A}{4}\right).\cos\left(\frac{\pi^c + B}{4}\right).\cos\left(\frac{\pi^c - C}{4}\right)$

c) $\cos\frac{A}{2} - \cos\frac{B}{2} + \cos\frac{C}{2} = 4\cos\left(\frac{\pi^c + A}{4}\right).\cos\left(\frac{\pi^c - B}{4}\right).\cos\left(\frac{\pi^c + C}{4}\right)$

5.4 TRIGONOMETRIC EQUATIONS

Knowledge (K) Based Problems

1. (a) Define trigonometric equation.
(b) What is trigonometric equation differ from trigonometric identities?

Understanding (U) Based Problems

Range: ($0^0 \le \theta \le 180^0$)

1. (a) If $2\sin 2\theta = \sqrt{3}$, find the value of θ. **[SEE MODEL-2076]** *Ans: 30^0, 60^0*
(b) If $2\cos 2\theta = \sqrt{3}$, find the value of θ. *Ans: 30^0, 165^0*
(c) If $\sqrt{3}\tan 3A - 3 = 0$, find the value of θ. *Ans: 20^0, 80^0*

2. (a) If $\sin\theta = \cos\theta$, find the value of θ. ($0^0 \le \theta \le 180^0$) *Ans: 45^0*
(b) If $\mathrm{cosec}\theta = \sec\theta$, find the value of θ ($0^0 \le \theta \le 180^0$) *Ans: 45^0*
(c) If $\tan\theta = \cot\theta$, find the value of θ.($0^0 \le \theta \le 180^0$) *Ans: 45^0*

3. (a) If $\tan^2\theta = 3$, find the value of θ.($0^0 \le \theta \le 180^0$) *Ans: 60^0,120^0*
(b) If $\cot^2\theta = 3$, find the value of θ.($0^0 \le \theta \le 180^0$) *Ans: 30^0,150^0*
(c) If $4\sin^2\theta = 1$,find the value of θ.($0^0 \le \theta \le 180^0$) *Ans: 30^0,150^0*
(d) If $4\sin\theta - 3\mathrm{cosec}\theta = 0$, find the value of θ. *Ans: 60^0,120^0*
(e) If $4\cos\theta - 3\sec\theta = 0$, find the value of θ. *Ans: 30^0,150^0*

4. (a) If $\tan x + \cot x = 2$, find the value of x. *Ans: 45^0*
(b) If $\sec^2\theta + \mathrm{cosec}^2\theta = 4$, find the value of θ. *Ans: 45^0*
(c) If $\sin 2\theta = \sin^2\theta + \cos^2\theta$, find the value of θ. *Ans: 45^0*

5. (a) If $\sin(x + 24^0) = \cos x$, find the value of x. *Ans: 33^0*
(b) If $\sec(x - 6^0) = \mathrm{cosec}3x$, find the value of x. *Ans: 24^0*

6. (a) If $\sin 2\theta = \sin^2\theta + \cos^2\theta$, find the value of θ. *Ans: 45^0*
(b) If $\cos^2 2A - 4\cos 2A + 1 = 0$, find A. *Ans: 30^0,150^0*
(c) If $\cos^2\frac{\theta}{2} - \cos\frac{\theta}{2} + \frac{1}{4} = 0$, find the value of θ. *Ans: 120^0*
(d) If $\sin^2\frac{\theta}{2} - \sin\frac{\theta}{2} + \frac{1}{4} = 0$, find the value of θ. *Ans: 60^0*

Application (A) Based Problems

1. Solve:

a) $\cos^2\theta - \sin\theta - \frac{1}{4} = 0$	($0^o \le \theta \le 360^o$)	*Answer: 30^0, 150^0*
b) $3\sin^2\theta + 4\cos\theta = 4$	($0^o \le \theta \le 360^o$)	*Answer: 0^o, 360^0, $\cos^{-1}\left(\frac{1}{3}\right)$*
c) $6\sin^2 A + \cos A = 5$	($0^o \le A \le 360^o$)	*Answer: 60^o, 300^o, $\cos^{-1}\left(\frac{1}{3}\right)$*

2. Solve:

(a) $4\cos^2\theta + 4\sin^2\theta - 5 = 0$	($0^o \le \theta \le 360^o$)	*Answer: 30^0, 150^0*
(b) $7\sin^2\theta + 3\cos^2\theta = 4$	($0^o \le \theta \le 360^o$)	*Answer: 30^0, 150^0, 210^0, 330^0*

3. Solve:

(a) $3\tan^2\theta - 4\sec\theta - 1 = 0$	($0^o \le \theta \le 360^o$)	*Answer: 60^o, 300^o*
(a) $4\sec^2\theta - 7\tan\theta - 3 = 0$	($0^o \le \theta \le 360^o$)	*Answer: 30^0, 150^0, 210^0, 330^0*

4. Solve:

(a) $2\sqrt{3}\sin^2\theta = \cos\theta$	($0^o \le \theta \le 360^o$)	*Answer: 30^o, 330^o*
(b) $2\sqrt{3}\cos^2\theta = \sin\theta$	($0^o \le \theta \le 360^o$)	*Answer: 60^o, 120^o*

5. Solve:

a) $\sec\theta.\tan\theta = \sqrt{2}$ $(0^\circ \leq \theta \leq 360^\circ)$ *Answer: 45°, 135°*

b) $\text{cosec}\theta.\cot\theta = \sqrt{2}$ $(0^\circ \leq \theta \leq 360^\circ)$ *Answer: 45°, 135°*

6. Solve:

a) $\cot^2\theta + \left(\sqrt{3} + \frac{1}{\sqrt{3}}\right)\cot\theta + 1 = 0$ $(0^\circ \leq \theta \leq 360^\circ)$ *Answer: 120°, 150°, 300°, 330°*

b) $\cot^2\theta + \left(\sqrt{3} - \frac{1}{\sqrt{3}}\right)\cot\theta = 1$ $(0^\circ \leq \theta \leq 360^\circ)$ *Answer: 60°, 150°, 240°, 330°*

7. Solve:

a) $\sqrt{3}\sin A + \cos A = \sqrt{2}$ $(0^\circ \leq A \leq 360^\circ)$ *Answer: 15°, 105°*

b) $\sin A = \sqrt{3}(1 - \cos A)$ $(0^\circ \leq A \leq 360^\circ)$ *Answer: 0°, 60°, 360°*

c) $\sqrt{3}\sin x - \cos x = \sqrt{2}$ $(0^\circ \leq x \leq 360^\circ)$ *Answer: 75°, 165°*

d) $\sin x - \sqrt{3}\cos x = \sqrt{2}$ $(0^\circ \leq x \leq 360^\circ)$ *Answer: 75°, 165°*

8. Solve:

a) $\cos A + \cos 2A + \cos 3A = 0$ $(0^\circ \leq A \leq 360^\circ)$ *Answer: 45°, 120°, 135°*

b) $\cos 3\theta + \cos\theta = \cos 2\theta$ $(0^\circ \leq \theta \leq 360^\circ)$ *Answer: 45°, 60°, 135°, 300°*

c) $\sin 3\theta + \sin\theta = \sin 2\theta$ $(0^\circ \leq \theta \leq 360^\circ)$ *Answer: 0°, 60°, 180°, 270°, 300°, 360°*

9. Solve:

a) $\tan\theta + \tan 2\theta + \sqrt{3}\tan\theta.\tan 2\theta = \sqrt{3}$ $(0^\circ \leq \theta \leq 360^\circ)$ *Answer: 20°, 80°, 140°*

b) $\cot\theta + \cot 2\theta - \sqrt{3}\cot\theta.\cot 2\theta + \sqrt{3} = 0$ $(0^\circ \leq \theta \leq 360^\circ)$ *Answer: 20°, 80°, 140°*

8. a) If $\tan x + \tan y = 2$ and $2\cos x.\cos y = 1$, find the values of x and y. Ans: $x = y = 45^0$

b) If $\cot A + \cot B = 2$ and $2\sin A.\sin B = 1$, find the values of A and B. Ans: $A = B = 45^0$

5.5 HEIGHT AND DISTANCE

Knowledge (K) Based Problems

1. (a) Define angle of elevation **[SEE MODEL-2076]** (b) Define angle of depression.

2. (a) What are the conditions under which the angle of elevation becomes smaller?
(b) What are the conditions under which the angle of depression becomes larger?

Application (A) Based Problems

1. a) The angle of elevation of the top of a tower was observed to be 60^0 from a point. On walking 200m away from the point it was found to be 30^0. Find the height of the tower. *Ans:173.2m*

b) The angle of elevation of the top of a tower from two places due east of its foot are 45^0 and 30^0. If two places are 180 m apart, find the height of the tower. *Ans: 245.9m*

c) The shadow of a tower on the ground is found to be 45m longer when sun's altitude is 45^0 than when it is 60^0, find the height of the tower. *Ans: 106.46m*

2. a) The angle of depression and elevation of the top of a building 40m high from the top and bottom of a tower are found to be 60^0 and 30^0 respectively, find the height of the tower. *Ans:160m*

b) The angle of depression and elevation of the top of a building 25m high from the top and bottom of a cliff are found to be 60^0 and 30^0 respectively, find the height of the cliff. *Ans:100m*

c) From the top of a tree 30m high, the angles of depression of the top and bottom of a pole are observed to be 45^0 and 60^0 respectively. Find the height of the pole the distance between the tree and the pole. *Ans:12.68m, 17.32m*

d) From the top of a cliff 100m high, the angles of depression of the top and bottom of a building are observed to be 30^0 and 45^0 respectively. Find the height of the building. Also, find the distance between the tower and the building. *Ans:42.27m*

3. a) A poster hanging on the wall has a vertical height 3.66m. From the point 5m away from the wall on the same plane, the angle of elevation of the bottom edge of the poster was found to be 45^0. What was the angle of elevation of the top edge of the poster, as observed from the same place? *Ans:60°*

b) There are two posts of height 90m and 30m respectively. From the foot of the second post, the angle of elevstion of the first post is found to be 60^0. Fi nd the angle of elevation of the top f the second post from the foot of the first post. *Ans: 30^0*

4. a) From a place at the ground level in front of a tower the angle of elevations of the top and bottom of flagstaff 6m high situated at the top of a tower are observed 60° and 45° respectively. Find the height of the tower and the distance between the base of the tower and point of observation.
[SEE MODEL-2076] *Ans: 8.19 m, 8.19 m*

b) A flagstaff of height 7m stands on the top of a tower. The angles subtended by the tower and the flagstaff at a point on the ground are 45^0 and 15^0 respectively, find the height of the tower. *Ans:9.56m*

c) A flagstaff stands on the top of a 10m high pillar. The angles elevation of the top of the flagstaff from a point on the ground is 60^0 and from the same point the angle of elevation of the top of the pillar is 45^0; find the height of the flagstaff. *Ans: 7.32m*

d) The angle of elevation of the top of an incomplete tower from a distance of 100m is 45^0. What height should it be raised so that the angle of elevation of the may change to 60^0? *Ans: 73.20m*

5. a) The angles of elevation of the top of the tower as observed from the distances of 20m and 45m from its foot are found to be complementary. Find the height of the tower. *Ans:30m*

b) The angles of elevation of the top of the tower as observed from the distances of 18m and 32m are found to be complementary. Find the height of the tower. *Ans: 24m*

c) The angles of elevation of the top of the tower as observed from the two places from its foot are found to be complementary. If the distance of a place from the tower is 36m and the other place is 13m away from the first place, find the height of the tower. *Ans: 42m*

6. a) Two posts are 180m apart and the height of one is double that of the other. From the mid point of the line joining their feet, an observer finds the angles of the elevation of their tops to be complementary, find the height of the longer post. *Ans: 127.28m*

e) Two poles are 40m apart and the height of one is double that of the other. The angles of elevation of the top of poles from the mid way between them are complementary; find the height of the longer post. *Ans: $10\sqrt{2}\,m$, $20\sqrt{2}\,m$*

7. a) A vertical tower is divided by any point in the ratio 9: 1 from the top. If both the segments subtend equal angles to each other at a distance of 20m away from the foot of the pole, find the height of the tower. *Ans:178.9m*

b) A vertical tower is divided by any point in the ratio 3: 1 from the top. If both the segments subtend equal angles to each other at a distance of 10m away from the foot of the pole, find the height of the tower. *Ans: $20\sqrt{2}\,m$*

8. a) A ladder 10m long reaches a point 10m below the top of a vertical flagstaff. From the foot of the ladder the angle of elevation of the flagstaff is 60^0, find the height of the flagstaff. *Ans:15m*

b) A ladder 6m long reaches a point 6m below the top of a vertical flagstaff. From the foot of the ladder the angle of elevation of the flagstaff is 60^0, find the height of the flagstaff. *Ans: 9m*

9. a) An aero-plane flying horizontally 750m above the ground is observed at an elevation of 60^0. After 5 seconds, the elevation of the plane changes to 30^0, find the speed of the aero-plane in km/hr.
Ans:623.54km/hr

b) At the foot of the mountain, the angle of elevation of its summit is 45^0, after ascending 1km forwards at an inclination of 30^0, the elevation changes to 60^0, find the height of the mountain. *Ans: 1.366km*

10. a) The angle of elevation of a bird from a point 100 feet above a lake is 30^0 and the angle of depression on its image in the lake is 60^0; find the height of the bird from the water surface. *Ans: 200 ft.*

b) The angle of elevation of a bird from a point 400 feet above a lake is 30^0 and the angle of depression on its image in the lake is 60^0; find the height of the bird from the water surface. *Ans: 800 ft.*

UNIT-6 VECTOR

Knowledge (K) Based Problems

1. (a) What is the scalar product of two vectors $\vec{a}$ and $\vec{b}$ if the angle between them is θ? **[SEE MODEL-2076]**
   (b) Define scalar product of two vectors.
2. (a) Write the condition of parallelism of vectors in term of their scalar product.
   (b) Under what the condition two vectors are perpendicular (orthogonal) to each other? Write in term of their scalar product.
3. (a) If $\vec{a}.\vec{b} = 0$ then what is the relation between $\vec{a}$ and $\vec{b}$?
   (b) If $\vec{a}.\vec{b} = 0$ then what is the angle between $\vec{a}$ and $\vec{b}$?
4. (a) If $\vec{i}$ the standard unit vector along x-axis, what is the value of $\vec{i}.\vec{i}$?
   (b) If $\vec{j}$ the standard unit vector along y-axis, what is the value of $\vec{i}^2$?
   (c) If $\vec{i}$ and $\vec{j}$ are the unit vectors along x-axis and y-axis respectively, what is the value of $\vec{i}.\vec{j}$?
5. (a) In a triangle ABC, if AM ⊥ BC, what is the value of $\overrightarrow{AM}.\overrightarrow{BC}$?
   (b) In a rectangle PQRS, what is the value of $\overrightarrow{PQ}.\overrightarrow{QR}$?
6. (a) If $|\vec{a}| = 5$, what is the value of $\vec{a}^2$? (b) If $|\vec{p}| = \sqrt{7}$, what is the value of $\vec{p}^2$?
7. (a) If M is the mid-point of line segment joining the points A and B and O is the reference origin, write down the relation of $\overrightarrow{OM}, \overrightarrow{OA}$ and $\overrightarrow{OB}$.
   (b) If the point P divides the line segment AB joining the points A and B externally in the ratio $m_1:m_2$, and O the reference origin, write down the relation of $\overrightarrow{OP}, \overrightarrow{OA}$ & $\overrightarrow{OB}$ along with m_1 and m_2.

Understanding (U) Based Problems

2.1 SCALAR PRODUCT OF VECTORS

1. (a) If $|\vec{a}| = 4\sqrt{2}, |\vec{b}| = 6$ and angle between $\vec{a}$ and $\vec{b}$ $(\theta) = 45^0$, find the value of $\vec{a}.\vec{b}$ *Ans: 24*
   (b) If $|\vec{a}| = 3\sqrt{3}, |\vec{b}| = 4$ and angle between $\vec{a}$ and $\vec{b}$ $(\theta) = 30^0$, find the value of $\vec{a}.\vec{b}$ *Ans: 18*
   (c) In an equilateral triangle ABC, AB = BC = CA = 6cm, find of $\overrightarrow{AB}.\overrightarrow{BC}$ *Ans: 18*
2. (a) If $|\vec{a}| = 6$, $\vec{a}\vec{b} = 12$ and $\theta = 60^0$, find the value of $|\vec{b}|$ *Ans:4*
   (b) If $|\vec{q}| = 6\sqrt{2}, \vec{p}.\vec{q} = 30$ and θ = 45^0, find the value of $|\vec{p}|$ *Ans:5*
3. (a) Find the angle between two vectors $\vec{a}$ and $\vec{b}$ if $|\vec{a}| = 2$, $|\vec{b}|$=12 and $\vec{a}.\vec{b}$=12 **[SEE MODEL-2076]** *Ans: 60^0*
   (b) If $|\vec{a}| = 4$, $|\vec{b}| = 5$ and $\vec{a}.\vec{b}$ =10 then find the angle between $\vec{a}$ and $\vec{b}$. Ans: 60^0
   (c) If $|\vec{a}| = 10$, $|\vec{b}| = 3$ and $\vec{a}.\vec{b} = 15\sqrt{2}$ then find the angle between $\vec{a}$ and $\vec{b}$. Ans: 45^0
   (d) If $|\vec{a}| = 5$, $|\vec{b}| = 3$ and $\vec{a}.\vec{b} = \frac{15}{2}$ then find the angle between $\vec{a}$ and $\vec{b}$. Ans: 60^0
4. (a) If $\vec{a} = 2\vec{i}+\vec{j}$ and $\vec{b} = \vec{i}+3\vec{j}$ then find the angle between $\vec{a}$ and $\vec{b}$. *Ans:: 45^0*
   (b) If $\vec{a} = -\vec{i} + 2\vec{j}$ and $\vec{b} = 4\vec{i} + 2\vec{j}$, find the angle between $\vec{a}$ and $\vec{b}$. *Ans: 90^0*
   (c) If $\vec{a} = 10\vec{i} + 7\vec{j}$ and $\vec{b} = 10\vec{i} - 7\vec{j}$, find the angle between $\vec{a}$ and $\vec{b}$. *Ans: 90^0*
   (d) Find the angle between $\vec{i}$ and $\vec{a} = \sqrt{3}\vec{i} + \vec{j}$ *Ans: 30^0*
5. (a) If $\vec{a}+\vec{b}+\vec{c} = 0$, $|\vec{a}| = 3$, $|\vec{b}| = 5$ and $|\vec{c}| = 7$, find the angle between $\vec{a}$ and $\vec{b}$. *Ans:60^0*
   (b) If $\vec{p} + \vec{q} + \vec{r} = 0$, $|\vec{p}| = 6$, $|\vec{q}|$=7 and $|\vec{r}| = \sqrt{127}$, find the angle between $\vec{p}$ & $\vec{q}$ *Ans:60^0*
6. (a) If $\vec{a} + 2\vec{b}$ and $5\vec{a} - 4\vec{b}$ are perpendicular to each other and $\vec{a}$ and $\vec{b}$ are unit vectors, find the angle between $\vec{a}$ and $\vec{b}$. *Ans: 60^0*

(b) If $4\vec{a}+5\vec{b}$ and $2\vec{a}-\vec{b}$ are perpendicular to each other and $\vec{a}$ and $\vec{b}$ are unit vectors, find the angle between $\vec{a}$ and $\vec{b}$. *Ans: 60^0*

7. (a) Prove that the vectors $\vec{a}=2\vec{i}-5\vec{j}$ and $\vec{b}=15\vec{i}+6\vec{j}$ are perpendicular to each other.

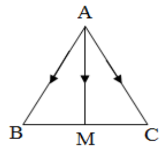
(b) If $|\vec{a}+\vec{b}|=|\vec{a}-\vec{b}|$, prove that $\vec{a}$ and $\vec{b}$ are perpendicular to each other.

(c) If $(\vec{x}+\vec{y})^2=(\vec{x}-\vec{y})^2$, prove that $\vec{x}$ and $\vec{y}$ are perpendicular to each other.

8. (a) For what value of k, are the vectors $\vec{a}=9\vec{i}-k\vec{j}$ and $\vec{b}=2\vec{i}+6\vec{j}$ are perpendicular to each other? *Ans:3*

(b) If $\vec{a}=\begin{pmatrix}-5\\3\end{pmatrix}$ and $\vec{b}=\begin{pmatrix}p\\p+2\end{pmatrix}$ are perpendicular, find the value of p. *Ans: 3*

6.2. VECTOR GEOMETRY



1. (a) If AM is the median of ΔABC. Prove that: $\overrightarrow{AM}=\frac{1}{2}(\overrightarrow{AB}+\overrightarrow{AC})$

(b) If the position vectors of A and B are $3\vec{i}+4\vec{j}$ and $7\vec{i}+8\vec{j}$ respectively, find the position vector of the mid-point of the line joining A and B. *Ans: $5\vec{i}+6\vec{j}$*

(c) If the position vectors of M and N are $7\vec{i}+2\vec{j}$ and $\vec{i}+4\vec{j}$. Find the position vector of a point P such that $\overrightarrow{MP}=\overrightarrow{PN}$ *Ans: $\overrightarrow{OP}=4\vec{i}+3\vec{j}$*

(d) If M is the mid-point of the join of A (2, 5) and B (-6, -1), find the position vector of M. $\rightarrow -2\vec{i}+2\vec{j}$

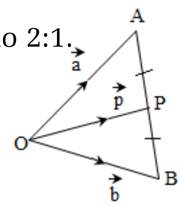
2. (a) The position vectors of A and B are $\vec{i}+2\vec{j}$ and $6\vec{i}+7\vec{j}$. Find the position vector of a point P which divides AB internally in the ratio 2:3. *Ans: $\overrightarrow{OP}=3\vec{i}+4\vec{j}$*

(b) Two points A and B have their position vectors are $\begin{pmatrix}5\\2\end{pmatrix}$ and $\begin{pmatrix}-3\\-6\end{pmatrix}$ respectively. Find the position vector of P which divides AB internally in the ratio 2:3. *Ans: $\overrightarrow{OP}=\frac{3}{5}(3\vec{i}-2\vec{j})$*

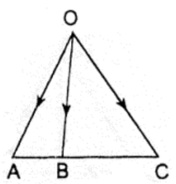
(c) If the point P divides the line segment joining the points A (3, 1) and B (6, 4) in the ratio 2:1, find the position vector of P. *Ans: $\overrightarrow{OP}=5\vec{i}+3\vec{j}$*

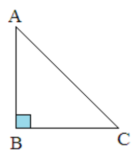
3. (a) The position vectors of A and B are $\vec{i}+\vec{j}$ and $2\vec{i}-\vec{j}$. Find the position vector of a point P which divides AB externally in the ratio 3:2. *Ans: $\overrightarrow{OP}=4\vec{i}-\vec{j}$*

(b) Two points A and B have their position vectors are $\begin{pmatrix}1\\2\end{pmatrix}$ and $\begin{pmatrix}3\\2\end{pmatrix}$ respectively. Find the position vector of P which divides AB externally in the ratio 2:1. *Ans: $\overrightarrow{OP}=5\vec{i}+2\vec{j}$*

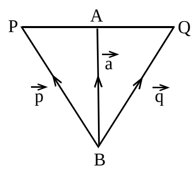


4. (a) In the given figure; find $\overrightarrow{AP}$ and express $\vec{p}$ in terms of $\vec{a}$ and $\vec{b}$ **[SEE MODEL-2076]**



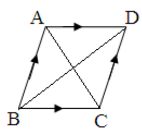


(b) In the given figure; $\overrightarrow{OA}=\vec{a}$ and $\overrightarrow{OB}=\vec{b}$.
If $\overrightarrow{AC}=3\overrightarrow{AB}$, find $\overrightarrow{OC}$ in terms of $\vec{a}$ and $\vec{b}$

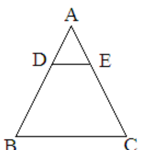


(c) In the given figure; $\angle ABC=90^0$, prove that: $\overrightarrow{AC}^2=\overrightarrow{AB}^2+\overrightarrow{BC}^2$

(d) In the given figure, $\overrightarrow{PA}=\frac{1}{4}\overrightarrow{PQ}$, prove that $\vec{a}=\frac{1}{4}(3\vec{p}+\vec{q})$.



(e) In the given figure, ABCD is a parallelogram. If AC and BD are diagonals, prove that: $\overrightarrow{AC}+\overrightarrow{BD}=2\overrightarrow{BC}$



(f) In ΔABC; D and E divide AB and AC in the ratio 1:2 respectively.

Prove that: $\overrightarrow{DE} = \frac{1}{3}\overrightarrow{BC}$

Higher Ability (HA) Based Problems

1. Prove by vector method that the median of an isosceles triangle is perpendicular to the base.
2. Prove by vector method that the mid-point of hypotenuse of a right angled triangle is equidistance from its every vertex.
3. Prove by vector method that the angle in a semi-circle is right angle.
4. Prove by vector method that the diagonals of a rectangle are equal to each other.
5. By using vector method, prove that the quadrilateral formed by joining the midpoints of adjacent sides of a quadrilateral is a parallelogram. **[SEE MODEL-2076]**
6. Prove by vector method that the diagonals of a parallelogram bisect to each other.
7. Prove by vector method that the line joining the mid-points of any two sides of a triangle is parallel and half of the third side.
8. Prove by vector method that the diagonals of a rhombus bisect to each other perpendicularly.
9. Prove by vector mthod that the position vector of centroid o triangle is one third of sum of the position vectors of its vertices.

UNIT-7 TRANSFORMATION

7.1 COMBINED TRANSFORMATION

Knowledge (N) Based Problems

1. (a) What do you mean by combined transformation?
(b) What will be the combined transformation if a reflection is followed by another reflection and the axes of reflections do intersect at a point?
(c) What will be the combined transformation if a reflection is followed by another reflection and the axes of reflections being parallel?

2. (a) What is the single transformation when the reflection about x-axis is followed by the reflection about y-axis?
(b) If $r_1$represents the reflection in y = x and r_2 represents the reflection in y = -x, what does $r_2 o r_1$ represent?

3. (a) What is the single transformation when the reflection in x-axis is followed by the reflection in y = x?
(b) If R_1denotes the reflection on y-axis and R_2 denotes the reflection in y = -x, what does $R_1 o R_2$ denote?
(c) If $r_1$and r_2 represent the reflections about the lines x =1and y = 2 respectively, what does $r_2 o r_1$ represent?
(d) If $r_1$and r_2 represent the reflections about the line x = -2 and y = 1 respectively, what does $r_1 o r_2$ represent?

4. (a) What will be single transformation when a rotation through θ_1 about origin is followed by another rotation through θ_2 about origin?
(b) What will be single transformation when a rotation through 60^0 about origin is followed by another rotation through 30^0 about origin?

5. (a) What will be single transformation when an enlargement with scale factor k_1 about origin is followed by another enlargement with scale factor k_1 about origin?
(b) If an enlargement E_1 [(a, b), h] is followed by another enlargement E_2 [(a, b), k], what is the single enlargement $E_2 o E_1$?

6. (a) What will be the image of a point P (x, y) if it is first reflected in x-axis and then rotated through $+90^0$ about origin?

(b) What will be the image of a point P (x, y) if it is first rotated through $+90^0$ about origin and then reflected about y-axis?

7. (a) What will be the image of a point (x, y) if it is first reflected in y = x and then enlarged by E [O, k]?
(b) What will be the image of a point (x, y) if it is first reflected in y = -x and then enlarged by E [O, k]?

Application (A) Based Problems

1. (a) A (-1, 1), B (4, 2) and C (5, 6) are the vertices of a ΔABC. Find the coordinates of the vertices of image of ΔABC under the reflection on x-axis followed by the reflection on y-axis and draw the triangles on the same graph paper. *Ans: A' (-2, -3), B' (-4, -5), C' (-1, -4*

(b) B (2, 1), U (4, 5) and Y (-1, 4) are the vertices of ΔBUY. Δ Find the coordinates of the vertices of image of ΔABC under the reflection on about y = x followed by the reflection on x = 2 and draw the triangles on the same graph paper. *Ans: A' (3, 1), B' (-1, 4), C' (0, -1)*

2. a) The vertices of ΔABC are A (2, 0), B (3, 1) and C (1, 1). ΔABC is translated by $\begin{pmatrix} 2 \\ -3 \end{pmatrix}$ the vector. The image so obtained is reflected on the line x + y = 0, find the coordinates of the images under combined transformation and draw all the triangles on the same graph paper.
Ans: A' (4, -3), B' (5, -2), C'(3, -2);A''(3, -4), B''(2, -5), C''(2, -3)

b) Translate the parallelogram OABC having vertices O (0, 0), A (2, 0), B (3, 1) and C (1, 1) by translation vector $\begin{pmatrix} 0 \\ 2 \end{pmatrix}$. Reflect the image so formed on the line x = 3. Represent the object and image on the same graph. *Ans: O' (0, 2), A' (2, 2), B' (3, 4), C' (1, 4); O'' (6, 2), A'' (4, 2), B'' (3, 4), C'' (5, 4)*

3. a) A (2, 5), B (-1, 3) and C (4, 1) are the vertices of a ΔABC. Find the coordinates of the vertices of image of ΔABC under the rotation of positive 90^0 about origin followed by enlargement E [(0, 0); 2]. Represent the object and the images on the same graph paper.
Ans: A' (-5, 2), B'(-3, -1), C'(-1, 4);A''(-10, 4), B''(-6, -2), C''(-2, 8)

b) I (4, -2), L (2, 1) and U (5, 2) are the vertices of ΔILU. ΔILU is transformed by a single transformation obtained by a rotation [(0, 0); 180^0] and on the same direction a rotation [(0, 0); 90^0]. Find the coordinates of the images of ΔILU and draw both the triangles on the same graph paper.
Ans: I' (-4, 2), L' (-2, -1), U' (-5, -2); I'' (-2, -4), L'' (1, -2), U'' (2, -5)

4. a) Δ PQR having vertices P (2, 1), Q (4, 3) and R (0, 4) is reflected on the y-axis. The image so formed is enlarged by E [(0, 0); 2]. Write the vertices of the images thus obtained and present the ΔPQR and its images on the same graph paper. *Ans: P' (-2, 1), Q'(-4, 3), R'(0, 4); P''(-4, 2), Q''(-8, 6), R''(0, 8)*

b) The vertices of ΔABC are A (2, 3), B (-2, 1) and C (0, 2). If E_1 [(0, 0); 2] and E_2 [(0, 2); 2], then find the coordinates of the images of ΔABC under enlargement $E_2 0 E_1$. Draw both triangles on the same graph. *Ans: A' (4, 6), B' (-4, 2), C' (0, 4); A'' (8, 10), B'' (-8, 2), C'' (0, 6)*

Higher Ability (Ha) Based Problems

1. a) The coordinates of vertices of a quadrilateral ABCD are A (1, 1), B (2,3), C (4, 2) and D(3,−2). Rotate this quadrilateral about origin through180°. Reflect this image of quadrilateral about y = −x. Write the name of transformation which denotes the combined transformation of above two transformations. **[SEE Model-2076]** *Ans: y = x, A' (1, 1), B' (3, 2), C' (2, 4) and D' (-2, 3)*

b) State the single transformation equivalent to combination of reflections on the x-axis and y-axis respectively. Using this single transformation, find the coordinates of the vertices of the image of ΔPQR with vertices P (4, 3), Q (1, 1) and R (5, -1). Also draw the object and the image on the same graph paper. *Ans: R [180^0, (0, 0)]; P' (-4, -3), Q' (-1, -1), R' (-5,-1)*

c) State the single transformation equivalent to combination of reflections on the y-axis and y = x respectively. Using this single transformation, find the coordinates of the vertices of the image of ΔABC with vertices A (2, 3), B (3, -4) and C (1, -2). Also draw the object and the image on the same graph paper. *Ans: R [-90^0, (0, 0)]; A' (3, -2), B' (-4, -3), C' (-2,-1)*

d) ΔABC with vertices A (1, 2), B (4, -1) and C (2, 5) is reflected successively in the line y = x and x-axis. Find by stating coordinates and graphically represent images under these transformations. State also the single transformation given by the combination of these transformations.
Ans: A' (2, 1), B' (-1, 4), C' (5, 2); A'' (2, -1), B'' (-1, -4), C'' (5, -2); R [-90^0, (0, 0)]

2. a) A triangle with vertices A (1, 2), B (4, -1) and C (2, 5) is reflected successively in the lines x = 5 and y = -2. Find by stating coordinates and graphically represent images under these transformations. State also the single transformation given by the combination of these transformations.
Ans: A' (9, 2), B' (6, -1), C' (8, 5); A'' (9, -6), B'' (6, -3), C'' (8, -9); R [-180^0, (5,-2)]

b) A triangle with vertices A (1, 2), B (4, -1) and C (2, 5) is reflected successively in the lines x = -1 and y = 2. Find by stating coordinates and graphically represent images under these transformations. State also the single transformation given by the combination of these transformations. *Ans: A' (-3, 2), B' (-6, -1), C' (-4, 5); A'' (-3, 2), B'' (-6, 5), C'' (-4, -1); R [-180^0, (-1, 2)]*

7.2 INVERSION TRANSFORMATION AND INVERSION CIRCLE

Knowledge (N) Based Problems

1. (a) What do you mean by inversion transformation?
(b) Define inversion circle.
(c) Write down a property of inversion transformation.
(d) Write the definition of inversion centre and radius.

2. (a) In an inversion transformation, if P' is the image of the P and r is the radius of inversion circle with centre O, write the relation of OP, OP' and r. **[SEE MODEL-2076]**
(b) Under what condition, the point on the inversion transformation is invariant?
(c) If a point P is inside the inversion circle, where does its inverse lie?
(d) If a point P is outside the inversion circle, where does its inverse lie?

3. (a) If P' (x', y') is the inversion point of P (x, y) in the circle with centre origin and radius r, write down the formula of finding x'.
(b) If P' (x', y') is the inversion point of P (x, y) in the circle with centre origin and radius r, write down the formula of y'.

4. (a) If P' (x', y') is the inversion point of P (x, y) in the circle with centre (h, k) and radius r, write down the formula of finding x'.
(b) If P' (x', y') is the inversion point of P (x, y) in the circle with centre (h, k) and radius r, write down the formula of Finding y'.

5. (a) In a circle with centre O, P' is the inversion point of P. If OP × OP' = 36 inch^2, find the radius of the inversion circle.
(b) In a circle with centre O, P' is the inversion point of P. If OP × OP' = 100 cm^2, find the radius of the inversion circle.

6. (a) In a circle with centre O, P' is the inversion point of P. If the radius (r) = 2 cm, what is the value of OP × OP'.
(b) In a circle with centre O, P' is the inversion point of P. If the radius (r) = 3 cm, what is the value of OP × OP'.

7. (a) In a circle with centre O, P' is the inversion point of P. If the radius (r) = 4 cm and OP = 2cm, what is the length of OP'?
(b) In a circle with centre O, P' is the inversion point of P. If the radius (r) = 6 cm and OP' = 9cm, what is the length of OP?

Application (A) Or Higher Ability (Ha) Based Problems

1. (a) Find the inverse of the point (2, 0) with respect to the circle $x^2 + y^2 = 9$. *Ans: (18, 0)*

(b) Find the inverse of the point (4, 4) with respect to the circle $x^2 + y^2 = 64$. *Ans: (8, 8)*

(b) Find the inverse of the point (3, 4) with respect to the circle $x^2 + y^2 = 100$. *Ans: (12, 16)*

2. (a) Find the inverse of the point (6, 7) with respect to the circle $x^2 + y^2 - 4x - 6y = 51$. *Ans: (10, 11)*

(b) Find the inverse of the point (3, 4) with respect to the circle $x^2 + y^2 - 4x - 6y = 23$. *Ans: (20, 21)*

(c) Find the inverse of the point (4, 5) with respect to the circle $x^2 + y^2 - 4x - 6y = 3$. *Ans: (6, 7)*

7.3 MATRIX TRANSFORMATION

Knowledge (N) Based Problems

1. (a) What does the matrix $\begin{pmatrix} k & 0 \\ 0 & k \end{pmatrix}$ represent?

(b) To what transformation is the matrix $\begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$ associated?

2. (a) To what transformation is the matrix $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ associated?

(b) Which type of transformation does $\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$ represent?

(c) To what transformation is the matrix $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$ associated?

(d) What transformation does the matrix $\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$ denote?

(e) Which type of transformation does $\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$ represent?

3. (a) Which transformation matrix is associated to reflection on x-axis?

(b) Which transformation matrix is associated to reflection on y-axis?

(c) Which transformation matrix is associated to reflection in $y = x$?

(d) Write the matrix that represents the rotation through $+90^0$ about origin.

(e) Write the matrix that represents the rotation through 180^0 about origin.

(f) Write the matrix is representing to the enlargement [k ; O] .

(g) Write the matrix is representing to the enlargement with scale factor 4 and centre at origin.

Application (A) OR HIGHER ABILITY (HA) BASED PROBLEMS

1. a) Find the 2×2 matrix which transforms the unit square into the parallelogram $\begin{pmatrix} 0 & 3 & 4 & 1 \\ 0 & 0 & 1 & 1 \end{pmatrix}$.

[SEE Model-2076] Ans: $\begin{pmatrix} 3 & 1 \\ 0 & 1 \end{pmatrix}$

b) Find the 2×2 matrix which transforms the unit square into the parallelogram $\begin{pmatrix} 0 & 3 & 5 & 2 \\ 0 & 1 & 2 & 1 \end{pmatrix}$. $\rightarrow \begin{pmatrix} 3 & 2 \\ 1 & 1 \end{pmatrix}$

(c) Find the2×2 matrix which transforms the unit square into a parallelogram $\begin{pmatrix} 0 & 2 & 5 & 3 \\ 0 & 1 & 0 & -1 \end{pmatrix}$. $\rightarrow \begin{pmatrix} 2 & 3 \\ 1 & -1 \end{pmatrix}$

(c) Find the2×2 matrix that transforms $\begin{pmatrix} 0 & -2 & 1 & 3 \\ 0 & 4 & 3 & -1 \end{pmatrix}$ into $\begin{pmatrix} 0 & 2 & -1 & -3 \\ 0 & -4 & -3 & 1 \end{pmatrix}$. *Ans:* $\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$

2. a) A square $\begin{pmatrix} 0 & a & 3 & 2 \\ b & 1 & c & d \end{pmatrix}$ is transformed by the matrix $\begin{pmatrix} 1 & 2 \\ 1 & -2 \end{pmatrix}$ to get a parallelogram $\begin{pmatrix} 6 & 3 & 7 & 10 \\ -6 & -1 & -1 & -6 \end{pmatrix}$.

Find the values of a, b, c and d. *Ans: $a = 1, b = 2, c = 3 , d = 4$*

b) If the matrix $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ transforms a rectangle $\begin{pmatrix} 0 & 2 & 2 & 0 \\ 0 & 0 & 1 & 1 \end{pmatrix}$ into a rectangle $\begin{pmatrix} 0 & 2 & 2 & 0 \\ 0 & 0 & -1 & -1 \end{pmatrix}$, find the values of a, b, c and d. *Ans: a = 1, b = 0, c = 0 , d = -1*

3. a) A square ABCD with vertices A (0, 3), B (1, 1), C (3, 2) and D (2, 4) is mapped on to □ A′B′C′D′ by a 2× 2 transformation matrix so that the vertices of □ A′B′C′D′ are A′ (6, -6), B′ (3, -1), C′ (7, –1) and D′ (10, –6). Find the2× 2 matrix. *Ans:* $\begin{pmatrix} 1 & 2 \\ 1 & -2 \end{pmatrix}$

b) Δ ABC with vertices A (3, 6), B (4, 2) and C (2, 2) is mapped on to aΔA′B′C’ by a 2×2 transformation matrix so that the vertices of ΔA′B′C’ are A′ (-6, 3), B′ (-2, 4) and C′ (-2, 2). Find the 2×2 matrix . *Ans:* $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$

4. a) Find the coordinates of vertices of image ΔA’B’C’ which is transformed by the transformation of ΔABC having vertices A (3, 1), B (-2, -1) and C (4, 2) under the matrix $\begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$

Ans: A’ (6, 2), B’ (-4, -2) and C’ (8, 4)

b) A square WXYZ has the vertices W (0, 3), X (1, 1), Y (3, 2) and Z (2, 4). Transform the given square WXYZ under the matrix $\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$ and find the coordinates of vertices of its image.

Ans: W’ (-3, 0), X’ (-1, 1) ,Y’ (-2, 3) and Z’ (-4, 2)

5. a) A (4, 1), B (-2, 3) and C (-3, 5) are the vertices of ΔABC. Find the vertices of image triangle under rotation through $+90^0$ about origin by using matrix method. Ans: A’ (-1, 4), B’ (-3, -2) and C’ (-5, -3)

b) X (-2, 3), Y (1, 4) and Z (0, -2) are the vertices of ΔXYZ. Find the vertices of image triangle under reflection on x-axis by using matrix method. *Ans: X’ (-2, -3), Y’ (1, -4) and Z’ (0, 2)*

UNIT-8 STATISTICS

8.1 QUARTILE DEVIATION

Understanding (U) Based Problems

1. (a) In a continuous series, if the first quartile (Q_1) is 10 and the third quartile (Q_3) is 30, find the quartile deviation and its coefficient. *Ans:10, 0.25*

(b) In a continuous series, the first quartile (Q_1) is 25 and the third quartile (Q_3) is 100, find the quartile deviation and its coefficient. *Ans: 37.5, 0.5*

2. (a) In a continuous series, the third quartile is two times the first quartile. If the first quartile is 24, find the third quartile and quartile deviation. *Ans:12, 0.33*

(b) In a continuous series, the third quartile is three times the first quartile. If the third quartile is 60, find the first quartile and quartile deviation. *Ans:20, 0.5*

3. (a) In a grouped data, if the first quartile and the quartile deviation are 15 and 10 respectively, find the coefficient quartile deviation.

(b) In a grouped data, if the first quartile and the quartile deviation are 24.5 and 65 respectively, find the coefficient quartile deviation.

4. (a) In a grouped data, the quartile deviation and its coefficient are 5 and 0.2 respectively then find the lower quartile.

(b) In a continuous series, the quartile deviation and its coefficient are 30 and 0.6 respectively then find the lower quartile.

Application (A) Based Problems

5. Calculat e the quartile deviantion and its coeffient of the data given below.
OR, find the inter-quartile range and its coeffient of the data given below.

(a) *Ans: 6.44, 0.11*

Marks obtained	0-20	20-40	40-60	60-80	80-100
No. of students	4	10	14	8	6

(b) *Ans: 6.68 years, 0.18*

Age in years	10-20	20-30	30-40	40-50	50-60
No. of people	2	6	22	13	7

(c) *Ans: 3.14 kg, 0.20*

Weight in kg	8-10	10-12	12-14	14-16	16-18	18-20	20-22
No. of girls	5	11	9	10	15	8	12

(d) *Ans: Rs 27.74 , 0.24*

Wage/Hr (Rs)	40-60	60-80	80-100	100-120	120-140	140-160
No. of workers	6	9	13	12	15	20

8.2 MEAN DEVIATION

Understanding (U) Based Problems

1. (a) In a grouped data $\sum fm = 200$, $\sum f|m - \bar{x}| = 480$ and $N = 40$ then calculate the mean deviation and its coefficient. *Ans: 12, 3*

(b) In a grouped data $\sum fm = 480$, $\sum f|m - \bar{x}| = 200$ and $N = 40$ then calculate the mean deviation and its coefficient. *Ans: 5, 0.4167*

2. (a) In a continuous data if $\sum fm = 600$, $\sum f|m - \bar{x}| = 70$ and MD from mean = 14 find the mean. *Ans: 120*

(b) In a continuous data if $\sum fm = 2000$, $\sum f|m - \bar{x}| = 600$ and MD from mean = 6 find the mean. *Ans: 20*

3. (a) In a continuous data if $\sum f|m - M_{dn}| = 544$, $N = 40$ and median $(M_{dn}) = 37$ calculate the mean deviation and its coefficient from median. *Ans: 13.6, 0.37*

(b) In grouped data, if $\sum f|m - M_{dn}| = 560$, $N = 50$ and median $(M_{dn}) = 27$ calculate the mean deviation and its coefficient from median. *Ans: 11.2, 0.41*

Application (A) Based Problems

1. Find the mean deviation from mean and its coefficient of the given data.

(a) **[SEE MODEL-2076]** *Ans: 10.3, 0.3678*

Marks obtained	0-10	10-20	20-30	30-40	40-50
No. of students	2	3	6	5	4

(b) *Ans: 11.2, 0.41*

Marks obtained	0-10	10-20	20-30	30-40	40-50
No. of students	5	8	15	16	6

2. Find the mean deviation and its coefficient from median of the data given alongside.

(a) *Ans: 6.2, 0.32*

Marks obtained	0-10	10-20	20-30	30-40	40-50	50-60
No. of students	2	4	6	2	4	2

(b) *Ans: 11.33, 0.32*

Weight (in kg)	0-10	10-20	20-30	30-40	40-50	50-60	60-70
No. of people	4	6	10	20	10	6	4

15.1 8.3 STANDARD DEVIATION

Understanding (U) Based Problems

1. (a) If the standard deviation of set of data is 0.25, find its variance. **[SEE MODEL-2076]** *Ans:0.0625*

(b) If the standard deviation of set of data is 0.5, find its variance. *Ans:0.25*

2. (a) If the variance of set of data is 25, find its standard deviation. *Ans:5*

(b) If the variance of set of data is 12.25, find its standard deviation. *Ans: 3.5*

3. (a) In a continuous series, if N = 50, mean ($\bar{x}$) = 20 and $\sum f(m - \bar{x})^2 = 1250$, find the standard deviation and its coefficient. *Ans: 5, 0.25*

(b) In a grouped data, if N = 7, mean ($\bar{x}$) = 44 and $\sum f(m - \bar{x})^2 = 432$, find the standard deviation and its coefficient. *Ans: 7.8, 0.18*

4. (a) In a grouped data, if N = 20, $\sum fm = 480$ and $\sum fm^2 = 12400$, find the standard deviation and its coefficient. *Ans: 6.633, 0.276*

(b) In a grouped data, N = 100, $\sum fm = 3100$ and $\sum fm^2 = 112800$, find the standard deviation and its coefficient. *Ans: 12.92, 0.416*

5. (a) In a grouped data, if N = 30, $\sum fd = 90$ and $\sum fd^2 = 5700$ where d = A –m , find the standard deviation . *Ans: 13.453*

(b) In a grouped data, if N = 50, $\sum fd = 200$ and $\sum fd^2 = 1250$ where d = A –m , find the standard deviation . *Ans: 3*

Application (A) Based Problems

1. Find the standard deviation, coefficient of standard deviation and coeficent of variation of the given data.

Marks obtained	0-10	10-20	20-30	30-40	40-50	50-60
No of students	8	12	20	40	12	8

[SEE MODEL-2076] *Ans: 6.03, 0.5042, 50.42%*

(b)

Age in years	0-4	4-8	8-12	12-16	16-20	20-24
No of students	7	7	10	15	7	6

Ans: 12.96, 0.418, 41.8%

Happy Learning….