Complete Solution Set – 2080

First Term Exam – 2080

Class:- X F.M:100
Subject:- Optional I (Mathematics) Time:- 3 hrs

Attempt all the questions

Group 'A' (10 × 1 = 10)

1. (a) Write down the condition for the existence of inverse function.

Solution:

The inverse of a function exists if it is a one to one and onto function.

(b) What is the nature of graph of constant function?

Solution:

The graph of constant function is always parallel to x-axis.

2. (a) If a polynomial $p(x)$ is divided by a linear polynomial $(x + a)$, what will be its remainder?

Solution:

The remainder (R) = $p(-a)$

(b) If $f(x)$ is a dividend, $g(x)$ is a divisor, $Q(x)$ is quotient and R is a remainder, then write down the relation among them.

Solution:

By division algorithm, polynomial = Divisor × Quotient + Remainder.

Hence, $f(x) = g(x) \times Q(x) + R$

3. (a) If A = [–7] is a square matrix of order 1 × 1, then what will be the determinant of A?

Solution:

Here, A = [–7] is a square matrix of order 1 × 1. $\therefore |A| = -7$

(b) What is the inverse of identity matrix of order 2 × 2? Write it.

Solution:

The inverse of identity matrix of order 2 × 2 is identity matrix itself.

4. (a) Express $\cos\theta$ in terms of $\tan\frac{\theta}{2}$.

Solution:

$$\text{We know, } \cos\theta = \frac{1 - \tan^2\frac{\theta}{2}}{1 + \tan^2\frac{\theta}{2}}$$

(b) What is the expanded form of sin3A in terms of sinA?

Solution:

We know, $\sin 3A = 3\sin A - 4\sin^3 A$.

5. (a) If θ be the angle between the two straight lines $y = m_1x + c_1$ and $y = m_2x + c_2$ having the slopes m_1 and m_2 respectively, then write the formula to calculate angle between them.

Solution:

We know, $\tan\theta = \pm \dfrac{m_1 - m_2}{1 + m_1m_2}$

(b) Write the condition of coincident of two lines when the slopes m_1 and m_2 are given.

Solution:

The condition of coincident of two lines is $m_1 = m_2$.

Group 'B' (13 × 2 = 26)

6. (a) If $(x - 3)$ is a factor of the polynomial $f(x) = x^3 + 4x^2 + kx - 30$, find the value of k.

Solution:

The given polynomial is $f(x) = x^3 + 4x^2 + kx - 30$ and $g(x) = x - 3$

Comparing $x - 3$ with $x - a$, we get

$a = 3$

Since, $(x - 3)$ is a factor of $f(x)$.

So, remainder (R) $= f(a) = 0$

or, $f(3) = 0$

or, $(3)^3 + 4(3)^2 + k(3) - 30 = 0$

or, $27 + 36 + 3k - 30 = 0$

or, $3k = -33$

or, $k = \dfrac{-33}{3} = -11$

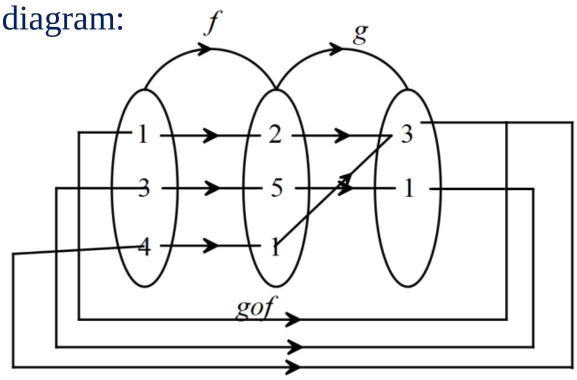
Hence, the required value of k is -11.

(b) If $f = \{(1, 2), (3, 5), (4, 1)\}$ and $g = \{(2, 3), (5, 1), (1, 3)\}$ are two given functions. Find gof by representing them into arrow diagrams.

Solution:

The given functions are $f = \{(1, 2), (3, 5), (4, 1)\}$ and $g = \{(2, 3), (5, 1), (1, 3)\}$

Representing the function gof in arrow diagram:



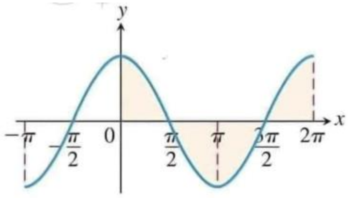
From above arrow-diagram, we get

$gof = \{(1, 3), (3, 1), (4, 3)\}$

(c) From the adjoining graph of trigonometric function, answer the following questions:

(i) Which type of function is shown by given graph? Write it.

(ii) Write down its range.



Solution:

(i) The function $y = \cos x$ is shown in the graph.

(ii) The range of the function $y = \cos x$ is the set of all real numbers from -1 to $+1$ inclusive. i.e., $[-1, +1]$

7. (a) Solve for t: $\begin{vmatrix} t-1 & t \\ t^2+1 & t^2+t+1 \end{vmatrix} = 0.$

Solution:

Here, $\begin{vmatrix} t-1 & t \\ t^2+1 & t^2+t+1 \end{vmatrix} = 0$

or, $(t-1)(t^2+t+1) - t(t^2+1) = 0$

or, $t^3 - 1^3 - t^3 - t = 0$

or, $-t = 1$

or, $t = -1$

Hence, the required value of t is -1.

(b) Find the inverse of the matrix $A = \begin{pmatrix} -1 & 3 \\ 2 & -8 \end{pmatrix}$ if exists.

Solution:

Here,

The given matrix is $A = \begin{pmatrix} -1 & 3 \\ 2 & -8 \end{pmatrix}$

Now, determinant of $A = \begin{vmatrix} -1 & 3 \\ 2 & -8 \end{vmatrix}$

$= (-1)(-8) - 2 \times 3$

$= 8 - 6$

$= 2$

Since, $|A| \neq 0$ So, A^{-1} exists.

We have, $A^{-1} = \frac{1}{|A|}$ Adjoint of A

$= \frac{1}{2}\begin{pmatrix} -8 & -3 \\ -2 & -1 \end{pmatrix}$

Hence, $A^{-1} = -\frac{1}{2}\begin{pmatrix} 8 & 3 \\ 2 & 1 \end{pmatrix}$

8. (a) If the line $2x - 3y = 6$ is perpendicular to the line $\frac{x}{p} + \frac{y}{9} = 3$, calculate the value of p.

Solution:

Here,

The slope of line $2x - 3y = 6$ is $m_1 = -\frac{\text{coefficient of } x}{\text{coefficeint of } y}$

$= -\frac{2}{-3}$

$= \frac{2}{3}$

The slope of line $\frac{x}{p} + \frac{y}{9} = 3$is $m_2 = -\frac{\text{coefficient of } x}{\text{coefficeint of } y}$

$= -\frac{\frac{1}{p}}{\frac{1}{9}}$

$= -\frac{9}{p}$

Since, the lines are perpendicular to each other.

So, $m_1 \times m_2 = -1$

or, $\frac{2}{3} \times \left(-\frac{9}{p}\right) = -1$

or, $-18 = -3p$

or, $p = 6$

Hence, the required value of p is 6.

(b) What is the obtuse angle between two straight lines whose slopes are $\frac{7}{4}$ and $\frac{3}{11}$? Find it.

Solution:

Here, the slopes of two straight lines are $m_1 = \frac{7}{4}$ and $m_2 = \frac{3}{11}$

Let, θ be the angle between the lines.

We have, $\tan\theta = \pm \frac{m_1 - m_2}{1 + m_1 m_2}$

$= \pm \frac{\frac{7}{4} - \frac{3}{11}}{1 + \frac{7}{4} \times \frac{3}{11}}$

$= \pm \frac{\frac{7}{4} - \frac{3}{11}}{1 + \frac{21}{44}}$

$$= \pm \frac{\frac{77-12}{44}}{\frac{44+21}{44}}$$

$$= \pm \frac{65}{65}$$

$$= \pm 1$$

To find the obtuse angle, the value of tanθ should be negative.

Thus, taking (-) ve sign, we get

$\tan\theta = -1$

or, $\theta = \tan^{-1}(-1) = 135^o$

Hence, the obtuse angle between the lines is 135^o.

9. (a) If $\sin\theta = \frac{3}{5}$, find the value of cosθ and sin2θ.

Solution:

Here, $\sin\theta = \frac{3}{5}$

We have, $\cos\theta = \sqrt{1 - \sin^2\theta}$

$$= \sqrt{1 - \left(\frac{3}{5}\right)^2}$$

$$= \sqrt{1 - \frac{9}{25}}$$

$$= \sqrt{\frac{25-9}{25}}$$

$$= \sqrt{\frac{16}{25}}$$

$$= \frac{4}{5}$$

Again, $\sin 2\theta = 2\sin\theta.\ \text{Cos}\theta$

$$= 2 \times \frac{3}{5} \times \frac{4}{5}$$

$$= \frac{24}{25}$$

(b) Prove that: : $\frac{3\cos x + \cos 3x}{3\sin x - \sin 3x} = \cot^3 x$.

Solution:

Here, L.H.S. $= \frac{3\cos x + \cos 3x}{3\sin x - \sin 3x}$

$$= \frac{3\cos x + 4\cos^3 x - 3\cos x}{3\sin x - (3\sin x - 4\sin^3 x)}$$

$$= \frac{4\cos^3 x}{3\sin x - 3\sin x + 4\sin^3 x}$$

$$= \frac{4\cos^3 x}{4\sin^3 x}$$

$$= \cot^2 x$$

= R.H.S.

Hence, proved

(c) Simplify: $\frac{1}{2}\left(\cot\frac{A}{2} - \tan\frac{A}{2}\right)$

Solution:

Here, $\frac{1}{2}\left(\cot\frac{A}{2} - \tan\frac{A}{2}\right)$

$$= \frac{1}{2}\left(\frac{\cos\frac{A}{2}}{\sin\frac{A}{2}} - \frac{\sin\frac{A}{2}}{\cos\frac{A}{2}}\right)$$

$$= \frac{1}{2}\left(\frac{\cos^2\frac{A}{2} - \sin^2\frac{A}{2}}{\sin\frac{A}{2}.\cos\frac{A}{2}}\right)$$

$$= \frac{\cos^2\frac{A}{2} - \sin^2\frac{A}{2}}{2\sin\frac{A}{2}.\cos\frac{A}{2}}$$

$$= \frac{\cos A}{\sin A}$$

$$= \cot A$$

10. (a) If $\cos 30° = \frac{\sqrt{3}}{2}$, show that the value of $\sin 15° = \frac{\sqrt{3}-1}{2\sqrt{2}}$.

Solution:

Here,

Given: $\cos 30° = \frac{\sqrt{3}}{2}$

Need to show: $\sin 15° = \frac{\sqrt{3}-1}{2\sqrt{2}}$

We have,

$$\cos A = 1 - 2\sin^2\frac{A}{2}$$

or, $\cos 30° = 1 - 2\sin^2\frac{30^o}{2}$

or, $\frac{\sqrt{3}}{2} = 1 - 2\sin^2 15^\circ$

or, $2\sin^2 15^\circ = 1 - \frac{\sqrt{3}}{2}$

or, $2\sin^2 15^\circ = \frac{2-\sqrt{3}}{2}$

or, $\sin^2 15^\circ = \frac{2-\sqrt{3}}{4}$

or, $\sin 15^\circ = \sqrt{\frac{2-\sqrt{3}}{4}}$

or, $\sin 15^\circ = \sqrt{\frac{2-\sqrt{3}}{4} \times \frac{2}{2}}$

or, $\sin 15^\circ = \sqrt{\frac{4-2\sqrt{3}}{8}}$

or, $\sin 15^\circ = \sqrt{\frac{3-2\sqrt{3}+1}{8}}$

or, $\sin 15^\circ = \sqrt{\frac{(\sqrt{3})^2 - 2\sqrt{3}.1 + 1^2}{8}}$

or, $\sin 15^\circ = \sqrt{\frac{(\sqrt{3}-1)^2}{2^2 \times 2}}$

$\therefore \sin 15^\circ = \frac{\sqrt{3}-1}{2\sqrt{2}}$

Hence, proved

(b) For a grouped data, if the value of lower quartile (Q_1) is 31.42 and quartile deviation (Q.D) is 4.75. Find the value of upper quartile (Q_3) and coefficient of quartile deviation.

Solution:

Here,

Lower quartile (Q_1) = 31.42

Quartile deviation (Q.D) = 4.75

Upper quartile (Q_3) =?

Coefficient of quartile deviation =?

Now, Q.D. = $\frac{Q_3 - Q_1}{2}$

or, $4.75 = \frac{Q_3 - 31.42}{2}$

or, $9.5 = Q_3 - 31.42$

$\therefore Q_3 = 40.92$

Again, coefficient of quartile deviation $= \dfrac{Q_3 - Q_1}{Q_3 + Q_1}$

$$= \frac{40.92 - 30.42}{40.92 + 30.42}$$

$$= \frac{9.5}{71.34}$$

$$= 0.1331$$

(c) Find the standard deviation and variance of a continuous series having N = 10, $\Sigma fm = 72$ and $\Sigma fm^2 = 720$.

Solution:

Here, N = 10, $\Sigma fm = 72$ and $\Sigma fm^2 = 720$

Standard deviation =?

Variance =?

$$\text{Now, S. D. } (\sigma) = \sqrt{\frac{\Sigma fm^2}{N} - \left(\frac{\Sigma fm}{N}\right)^2}$$

$$= \sqrt{\frac{720}{10} - \left(\frac{72}{10}\right)^2}$$

$$= \sqrt{20.16}$$

$$= 4.49$$

Again, variance $= \sigma^2 = (4.49)^2 = 20.16$

Group 'C' (11 × 4 = 44)

11. If $f(x)$ and $g(x)$ be two functions which are defined by $f(x) = x + 2$ and $g(x) = \dfrac{3x - 2}{4}$ such that $f(x) = g^{-1}(x)$. Find the value of x.

Solution:

The given functions are $f(x) = x + 2$ and $g(x) = \dfrac{3x - 2}{4}$

Given relation: $f(x) = g^{-1}(x)$

Let, $g(x) = y$ then $y = \dfrac{3x - 2}{4}$

Now, interchanging the role of x and y, we get

$$x = \frac{3y - 2}{4}$$

or, $3y - 2 = 4x$

or, $3y = 4x + 2$

or, $y = \dfrac{4x + 2}{3}$

$\therefore g^{-1}(x) = \dfrac{4x + 2}{3}$

According to the question, $f(x) = g^{-1}(x)$

$$\text{or, } x + 2 = \frac{4x + 2}{3}$$

$$\text{or, } 3x + 6 = 4x + 2$$

$$\text{or, } 4 = x$$

Hence, the required value of x is 4.

12. It is given that $f(x) = 3x + 5$ and $g(x) = \frac{3x + 2}{4}$. What value of x makes $fog^{-1}(x)$ an identity function? Find it.

Solution:

Here,

The given functions are $f(x) = 3x + 5$ and $g(x) = \frac{3x + 2}{4}$

Given relation: $fog^{-1}(x)$ an identity function i.e., $fog^{-1}(x) = x$

Let, $g(x) = y$ then $y = \frac{3x + 2}{4}$

Now, interchanging the role of x and y, we get

$$x = \frac{3y + 2}{4}$$

$$\text{or, } 3y + 2 = 4x$$

$$\text{or, } 3y = 4x - 2$$

$$\text{or, } y = \frac{4x - 2}{3}$$

$$\text{or, } g^{-1}(x) = \frac{4x - 2}{3}$$

According to the question, $fog^{-1}(x) = x$

$$\text{or, } f\left(\frac{4x - 2}{3}\right) = x$$

$$\text{or, } 3\left(\frac{4x - 2}{3}\right) + 5 = x$$

$$\text{or, } 4x - 2 + 5 = x$$

$$\text{or, } 3x = -3$$

$$\text{or, } x = -1$$

Hence, the required value of x is -1.

13. The polynomial $f(x) = 3x^3 + 2x^2 - nx + m$ is exactly divisible by $(x - 1)$ but leaves a remainder 10 when divided by $(x + 4)$, then find the values of m and n.

Solution:

The given polynomial $f(x) = 3x^3 + 2x^2 - nx + m$

<u>Case-I:</u> $(x - 1)$ exactly divides $f(x)$.

So, remainder (R) $= 0$

or, $f(1) = 0$

or, $3(1)^3 + 2(1)^2 - n \times 1 + m = 0$

or, $5 - n + m = 0$

$\therefore \quad m = n - 5$ ... (i)

Case-II:

Divisor = $(x + 4)$ and remainder = 10

So, remainder (R) = 10

or, $f(-4) = 10$

or, $3(-4)^3 + 2(-4)^2 - n \times (-4) + m = 10$

or, $-192 + 32 + 4n + m = 10$

or, $4n + m = 170$ ... (ii)

Putting the value of 'm' in equation (ii) from equation (i), we get

$4n + m = 170$

or, $4n + n - 5 = 170$

or, $5n = 175$

$\therefore \quad n = 35$

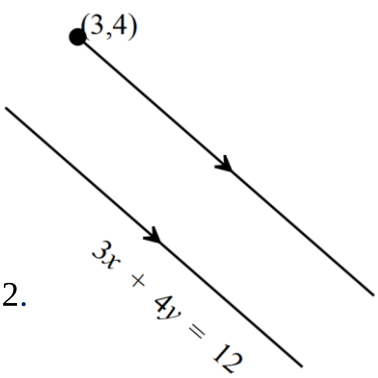
Again, putting the value of 'n' in equation (i), we get

$m = 35 - 5 = 30$

Hence, the required value of 'm' is 30 and the value of 'n' is 35.

14. Find the equation of a straight line which passes through a point (3, 4) and parallel to the line $3x + 4y = 12$.

Solution:



Here,

The slope of line $3x + 4y = 12$ is $m_1 = -\dfrac{\text{coefficient of } x}{\text{coefficeint of } y}$

$= -\dfrac{3}{4}$

Let, m_2 be the slope of the line parallel to the given line $3x + 4y = 12$.

Then, $m_1 = m_2$

or, $-\dfrac{3}{4} = m_2$

$\therefore m_2 = -\dfrac{3}{4}$

Also, passing point $(x_1, y_1) = (3, 4)$ and slope $(m_2) = -\dfrac{3}{4}$

Again,

Equation of required line is given by $y - y_1 = m_2(x - x_1)$

or, $y - 4 = -\dfrac{3}{4}(x - 3)$

or, $4y - 16 = -3x + 9$

or, $3x + 4y = 25$

Hence, the required equation is $3x + 4y = 25$.

15. Solve by matrix method: $x + y = 20$ and $x - y = 4$.

Solution:

Here,

The given equations are:

$x + y = 20$ … (i)

$x - y = 4$ … (ii)

Expressing equations (i) and (ii) in matrix form. We get

$$\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 20 \\ 4 \end{pmatrix}$$

or, AX = B where $A = \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$, $B = \begin{pmatrix} 20 \\ 4 \end{pmatrix}$ and $X = \begin{pmatrix} x \\ y \end{pmatrix}$

$\therefore X = A^{-1} B$ … (iii)

Also, determinant of $A = \begin{vmatrix} 1 & 1 \\ 1 & -1 \end{vmatrix} = -1 - 1 = -2$

Since, $|A| \neq 0$ So, A^{-1} exists and the given system has a unique solution.

Again, $A^{-1} = \frac{1}{|A|}$ Ad joint of A

$= \frac{1}{-2}\begin{pmatrix} -1 & -1 \\ -1 & 1 \end{pmatrix}$

Putting the value of A^{-1} in equation (iii), we get

$$\begin{pmatrix} x \\ y \end{pmatrix} = \frac{1}{-2}\begin{pmatrix} -1 & -1 \\ -1 & 1 \end{pmatrix}\begin{pmatrix} 20 \\ 4 \end{pmatrix}$$

or, $\begin{pmatrix} x \\ y \end{pmatrix} = \frac{1}{-2}\begin{pmatrix} -20 - 4 \\ -20 + 4 \end{pmatrix}$

or, $\begin{pmatrix} x \\ y \end{pmatrix} = \frac{1}{-2}\begin{pmatrix} -24 \\ -16 \end{pmatrix}$

or, $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 12 \\ 8 \end{pmatrix}$

Equating the corresponding elements, we get

$x = 12$ and $y = 8$

Hence, $x = 12$ and $y = 8$

16. Solve the equations $2(x - 1) = y$ and $3(x - 1) = 4y$ by Cramer's rule.

Solution:

Here,

The given equations are;

$2(x - 1) = y$ or, $2x - 2 = y$ $\therefore 2x - y = 2$ … (i) and $3(x-1) = 4y$ or, $3x - 3 = 4y$ $\therefore 3x - 4y = 3$ … (ii)

Coefficient of x	Coefficient of y	Constant
2	– 1	2
3	– 4	3

Now,

$$D = \begin{vmatrix} 2 & -1 \\ 3 & -4 \end{vmatrix} = -8 + 3 = -5$$

$$D_x = \begin{vmatrix} 2 & -1 \\ 3 & -4 \end{vmatrix} = -8 + 3 = -5$$

$$D_y = \begin{vmatrix} 2 & 2 \\ 3 & 3 \end{vmatrix} = 6 - 6 = 0$$

Again, by using Cramer's rule

$$x = \frac{D_x}{D} = \frac{-5}{-5} = 1$$

$$\text{and } y = \frac{D_y}{D} = \frac{0}{-5} = 0$$

Hence, the value of x is 1 and that of y is 0.

17. Without using calculator or table, find the value of following trigonometric expression: $\sin^4\frac{\pi}{8} + \sin^4\frac{3\pi}{8} + \sin^4\frac{5\pi}{8} + \sin^4\frac{7\pi}{8}$

Solution:

Here,

$$\sin^4\frac{\pi}{8} + \sin^4\frac{3\pi}{8} + \sin^4\frac{5\pi}{8} + \sin^4\frac{7\pi}{8}$$

$$= \sin^4\frac{\pi}{8} + \sin^4\frac{3\pi}{8} + \sin^4\left(\pi - \frac{3\pi}{8}\right) + \sin^4\left(\pi - \frac{\pi}{8}\right)$$

$$= \sin^4\frac{\pi}{8} + \sin^4\frac{3\pi}{8} + \sin^4\frac{3\pi}{8} + \sin^4\frac{\pi}{8}$$

$$= 2\sin^4\frac{\pi}{8} + 2\sin^4\frac{3\pi}{8}$$

$$= \frac{1}{2} \times 2\left(2\sin^4\frac{\pi}{8} + 2\sin^4\frac{3\pi}{8}\right)$$

$$= \frac{1}{2}\left(4\sin^4\frac{\pi}{8} + 4\sin^4\frac{3\pi}{8}\right)$$

$$= \frac{1}{2}\left[\left(2\sin^2\frac{\pi}{8}\right)^2 + \left(2\sin^2\frac{3\pi}{8}\right)^2\right]$$

$$= \frac{1}{2}\left[\left(1 - \cos2 \times \frac{\pi}{8}\right)^2 + \left(1 - \cos2 \times \frac{3\pi}{8}\right)^2\right]$$

$$= \frac{1}{2}\left[(1 - \cos45^\circ)^2 + (1 - \cos135^\circ)^2\right]$$

$= \frac{1}{2}\left[\left(1 - \frac{1}{\sqrt{2}}\right)^2 + \left(1 + \frac{1}{\sqrt{2}}\right)^2\right]$

$= \frac{1}{2}\left(1 - 2\times\frac{1}{\sqrt{2}} + \frac{1}{2} + 1 + 2\times\frac{1}{\sqrt{2}} + \frac{1}{2}\right)$

$= \frac{1}{2}\left(1 + \frac{1}{2} + 1 + \frac{1}{2}\right)$

$= \frac{1}{2}\left(\frac{2 + 1 + 2 + 1}{2}\right)$

$= \frac{1}{2}\left(\frac{6}{2}\right)$

$= \frac{3}{2}$

18. Prove that: $8\sin^4\alpha = \cos4\alpha - 4\cos2\alpha + 3$.

Solution:

Here,

L.H.S. $= 8\sin^4\alpha$

$= 2 \times 4\sin^4\alpha$

$= 2\,(2\sin^2\alpha)^2$

$= 2\,(1 - \cos2\alpha)^2$

$= 2\,(1 - 2\cos2\alpha + \cos^2 2\alpha)$

$= 2 - 4\cos2\alpha + 2\cos^2 2\alpha$

$= 2 - 4\cos2\alpha + 1 + \cos2(2\alpha)$

$= 3 - 4\cos2\alpha + \cos4\alpha$

$= \cos4\alpha - 4\cos2\alpha + 3$

= R.H.S.

Hence, proved

19. Reduce $\cos^6\frac{A}{2} + \sin^6\frac{A}{2}$ in terms of sinA.

Solution:

Here, $\cos^6\frac{A}{2} + \sin^6\frac{A}{2} = \left(\cos^2\frac{A}{2}\right)^3 + \left(\sin^2\frac{A}{2}\right)^3$

$= \left(\cos^2\frac{A}{2} + \sin^2\frac{A}{2}\right)^3 - 3\cos^2\frac{A}{2}.\sin^2\frac{A}{2}\left(\cos^2\frac{A}{2} + \sin^2\frac{A}{2}\right)$

$= 1 - 3\cos^2\frac{A}{2}.\sin^2\frac{A}{2} \times 1$

$= 1 - \frac{1}{4} \times 4 \times 3\cos^2\frac{A}{2}.\sin^2\frac{A}{2}$

$= 1 - \frac{3}{4}\left(2\sin\frac{A}{2}.\cos\frac{A}{2}\right)^2$

$$= 1 - \frac{3}{4}\left(\sin 2 \times \frac{A}{2}\right)$$

$$= 1 - \frac{3}{4}\sin A$$

20. Find the mean deviation from mean and its coefficient from the following frequency distribution:

Marks Obtained	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50
No. of Students	3	5	7	3	4

Solution:

Here,

Computation of the mean deviation from the mean:

Marks	No. of students (*f*)	*m*	*fm*	$\|m - \overline{X}\|$	$f\|m - \overline{X}\|$
0-10	3	5	15	20	60
10-20	5	15	75	10	50
20-30	7	25	175	0	0
30-40	3	35	105	10	30
40-50	4	45	180	20	80
	N = 22		Σfm =550		$\Sigma f\|m - Md\|$ = 220

Now, mean ($\overline{X}$) = $\frac{\Sigma fm}{N} = \frac{550}{22} = 25$

Also, M.D. from mean = $\frac{\Sigma f|m - Md|}{N} = \frac{220}{22} = 10$

Again, coefficient of M.D. = $\frac{\text{M.D. from mean}}{\text{Mean}} = \frac{10}{25} = 0.4$

Hence, the mean deviation is 10 and its coefficient is 0.4

21. The following table gives the weight (in kg) of 20 workers in a certain company.

Weight in kg	30 – 40	40 – 50	50 – 60	60 – 70	70 – 80
No. of workers	2	3	6	5	4

Then, calculate the arithmetic mean, standard deviation and coefficient of variation.

Solution:

Computation of the standard deviation:

Ages group	No. of people (*f*)	*m*	*fm*	fm^2
30-40	2	35	70	2450
40-50	3	45	135	6075
50-60	6	55	330	18150
60-70	5	65	325	21125
70-80	4	75	300	22500
	N = 20		Σfm = 1160	Σfm^2 = 70300

Now, mean ($\overline{X}$) $= \dfrac{\Sigma fm}{N} = \dfrac{1160}{20} = 58$

Also, S.D. (σ) $= \sqrt{\dfrac{fm^2}{N} - \left(\dfrac{fm}{N}\right)^2}$

$= \sqrt{\dfrac{70300}{20} - \left(\dfrac{1160}{20}\right)^2}$

$= \sqrt{3515 - 3364}$

$= \sqrt{151}$

$= 12.29$

Again, coefficient of variation $= \dfrac{\sigma}{\overline{X}} \times 100\%$

$= \dfrac{12.29}{58} \times 100\%$

$= 21.19\%$

Group 'D' (4 × 5 = 20)

22. Given that $f(x) = 2x^3 + 3x^2 - 11x - 6 = 0$ is a polynomial equation in x, find the difference between the greatest and the smallest roots of $f(x)$.

Solution:

Here, $f(x) = 2x^3 + 3x^2 - 11x - 6$

The possible factors of 6 are ± 1, ± 2, ± 3 and ± 6.

Comparing $(x - 2)$ with $x - a$, we get

$a = 2$

Now, reminder (R) $= f(a)$

$= f(2)$

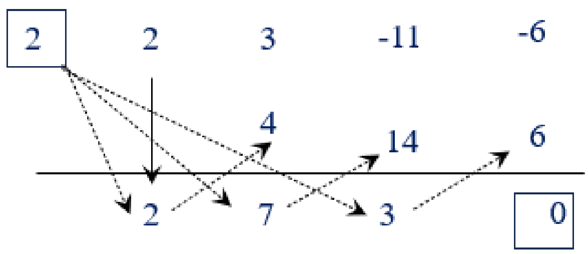
$= 2(2)^3 + 3(2)^2 - 11(2) - 6$

$= 16 + 12 - 22 - 6$

$= 0$

Since, $f(2) = 0$. Thus, $(x - 2)$ is a factor of $f(x)$.

By using synthetic division method, we get



Thus, quotient, Q $(x) = 2x^2 + 7x + 3$ and remainder (R) = 0

Also, $f(x) = 2x^3 + 3x^2 - 11x - 6 = (x - 2) \times Q(x) + R$

or, $0 = (x - 2)(2x^2 + 7x + 3) + 0$

or, 0 $= (x - 2)\{2x^2 + (6 + 1)x + 3\}$

or, 0 $= (x - 2)(2x^2 + 6x + x + 3)$

or, 0 $= (x - 2)\{2x(x + 3) + 1(x + 3)\}$

or, 0 $= (x - 2)(x + 3)(2x + 1)$

Either, $x - 2 = 0$ $\therefore x = 2$

OR, $x + 3 = 0$ $\therefore x = -3$

OR, $2x + 1 = 0$ $\therefore x = -\frac{1}{2}$

Hence, the roots of the polynomials are 2, – 3 and $-\frac{1}{2}$

Again, the greatest root is 2 and the smallest root is – 3.

Thus, the difference = 2 – (– 3) = 5

23. Let f: R→R and g: R→R are two functions defined by $f(x) = \frac{x}{2} - 5$ and $g(x) = 2x + 10$, then find $fog(x)$ and $gof(x)$. Are the functions $f(x)$ and $g(x)$ inverse to each other or not, Why?

Solution:

Here,

The given functions are $f(x) = \frac{x}{2} - 5$ and $g(x) = 2x + 10$

Now, $fog(x) = f(g(x))$

$= f(2x + 10)$

$= \frac{2x + 10}{2} - 5$

$= x + 5 - 5$

$= x$

Again, $gof(x) = g(f(x))$

$= g\left(\frac{x}{2} - 5\right)$

$= 2\left(\frac{x}{2} - 5\right) + 10$

$= x - 10 + 10$

$= x$

Since, $fog(x) = gof(x) = x$

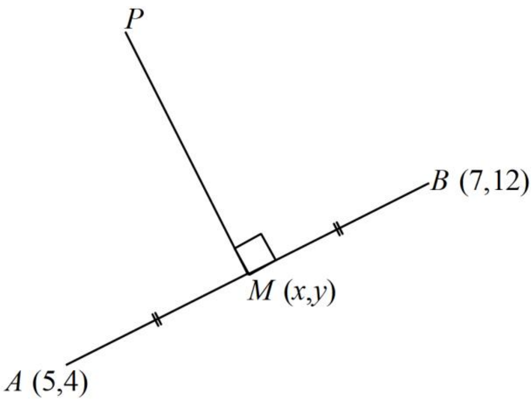
i.e., both the functions $gof(x)$ and $fog(x)$ are identity functions and equal too.

Thus, the functions $f(x)$ and $g(x)$ inverse to each other

24. Find the equation of the perpendicular bisector of the line joining the points (5, 4) and (7, 12).

Solution:

Let, PM be the perpendicular bisector of the line joining the points A (5, 4) and B (7, 12) where M is the mid-point of AB.



Now,

By using mid-point theorem, $M(x, y) = M\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

$= M\left(\frac{5 + 7}{2}, \frac{4 + 12}{2}\right)$

$= M(6, 8)$

Also, slope of AB (m_1) $= \frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{12 - 4}{7 - 5}$

$= \frac{8}{2}$

$= 4$

Let, m_2 be the slope of PM which is perpendicular to AB.

Then, $m_1 \times m_2 = -1$

or, $4 \times m_2 = -1$

$\therefore m_2 = -\frac{1}{4}$

Again, equation of required line is given by $y - y_1 = m_2 (x - x_1)$

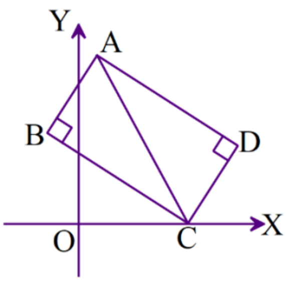
or, $y - 8 = -\frac{1}{4}(x - 6)$

or, $4y - 32 = -x + 6$

or, $x + 4y = 38$

Hence, the required equation of perpendicular bisector is $x + 4y = 38$.

25. The diagram shows a rectangle ABCD. The coordinate of corner points of a rectangle ABCD are A(2, 14), B(–2, 8) and corner C lies on the x – axis. Find the equation of the side BC and AC.



Solution:

Here, the coordinate of corner points of a rectangle ABCD are A(2, 14), B(–2, 8) and corner C lies on the x – axis.

Now, slope of AB (m_1) $= \frac{y_2 - y_1}{x_2 - x_1} = \frac{8 - 14}{-2 - 2} = \frac{-6}{-4} = \frac{3}{2}$

Let, m_2 be the slope of BC which is perpendicular to AB.

Then, $m_1 \times m_2 = -1$

or, $\frac{3}{2} \times m_2 = -1$

or, $3m_2 = -2$

$\therefore m_2 = -\frac{2}{3}$

Also, for side BC; passing point (x_1, y_1) = B (–2, 8) and slope $(m_2) = -\frac{2}{3}$

Equation of required side BC is given by $y - y_1 = m_2 (x - x_1)$

or, $y - 8 = -\frac{2}{3}(x + 2)$

or, $3y - 24 = -2x - 4$

or, $2x + 3y = 20$

Hence, the required equation of side BC is $2x + 3y = 20$.

Again,

Let C $(x, 0)$ be the coordinates of vertex C.

Then, B (–2, 8) $\rightarrow (x_1, y_1)$ and C $(x, 0) \rightarrow (x_2, y_2)$

We have, slope of BC $= \frac{y_2 - y_1}{x_2 - x_1}$

or, $-\frac{2}{3} = \frac{0 - 8}{x + 2}$

or, $-\frac{2}{3} = \frac{-8}{x + 2}$

or, $-2(x + 2) = -24$

or, $x + 2 = 12$

or, $x = 10$

Thus, coordinates of C $(x, 0)$ = C (10, 0)

To find the equation of AC; A (2, 14) $\rightarrow (x_1, y_1)$ and C (10, 0) $\rightarrow (x_2, y_2)$

Equation of side AC is given by $y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$

or, $y - 14 = \frac{0 - 14}{10 - 2}(x - 2)$

or, $y - 14 = \frac{-14}{8}(x - 2)$

or, $y - 14 = \frac{-7}{4}(x - 2)$

or, $4y - 56 = -7x + 14$

or, $7x + 4y = 70$

Hence, the required equation of side AC is $7x + 4y = 70$.