

Basic Set-I

SEE MODEL SET- 1

Optional- I (Mathematics)

Time: 3 hours

F.M: 75

Answer all the questions.

Group-A

10 × 1 = 10

1. Write the period of the function $f(x) = \sin x$.

Ans: The period of the function $f(x) = \sin x$ is 360° or 2π .

2. What is the sum of first 'n' terms of a geometric series having first term 'a' and common ratio 'r'?

Ans: The sum of first 'n' terms is given by $S_n = \frac{a(r^n - 1)}{r - 1}$.

3. Write condition for continuity of a function $f(x)$ at $x = a$.

Ans: The function $f(x)$ is continuous at $x = a$ if $\lim_{x \rightarrow a} f(x) = f(a)$

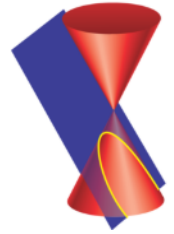
4. Find the determinant of the matrix $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$.

Ans: The determinant of the matrix $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ is $|A| = ad - bc$

5. If θ is the angle between the pair of straight lines represented by the equation $ax^2 + 2hxy + by^2 = 0$, find the value of $\tan\theta$.

Ans: The value of $\tan\theta$ is given by $\tan\theta = \pm \frac{2\sqrt{h^2 - ab}}{a + b}$

6. In the given figure, the plane intersects a double cone in such a way that it is parallel to the generator of the cone. Which conic section is formed in the given figure?



Ans: The parabola is formed.

7. Write the formula of $\cos A$ in terms of $\sin \frac{A}{2}$.

Ans: $\cos A = 1 - 2\sin^2 \frac{A}{2}$

8. Define angle of depression.

Ans: The angle made by upward line of sight with horizontal line or ground is called angle of depression

9. Write the formula to find the angle between vectors \vec{a} and \vec{b} .

Ans: The angle between vectors \vec{a} and \vec{b} is $\cos\theta = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}| |\vec{b}|}$

10. If O is the centre of circle, radius = r and P' is inversion of point P, write down the relation of OP, OP' and r .

Ans: The relation of OP, OP' and r is $OP \times OP' = r^2$

Group-B

8 × 2 = 16

11. Let $f: \mathbb{R} \rightarrow \mathbb{R}$ be defined by $f(x) = 2x + 3$ then find $f^{-1}(11)$. Ans: 4
12. Find the vertex of the parabola $y = x^2 - 4x$. Ans: $(h, k) = (2, -4)$

13. If $A = \begin{pmatrix} x-1 & 3 \\ x-2 & 4 \end{pmatrix}$ is a singular matrix, find the value of x . Ans: -2

14. If the equation $(k^2 + 2)x^2 + 5xy - 3ky^2 = 0$ represents two perpendicular lines, find the value of k . Ans: $k = 1$ or 2

15. Prove that: $\frac{\cos 2\theta}{1 + \sin 2\theta} = \frac{1 - \tan \theta}{1 + \tan \theta}$

16. If $\sec A = \operatorname{cosec} 2A$, find the value of A . ($0^\circ \leq A \leq 180^\circ$) Ans: 30°

17. If $|\vec{AB}| = 12$, $|\vec{AC}| = 15$ and $\angle BAC = 60^\circ$, find the value of $\vec{AB} \cdot \vec{AC}$. Ans: 90

18. In a continuous data, the first quartile and third quartile are 20 and 80 respectively. Find the quartile deviation and its coefficient. Ans: 30, 0.4

Group-C **11 × 3 = 33**

19. Solve the cubic polynomial equation $6x^3 - 13x^2 + x + 2 = 0$ by using factor theorem. Ans: $2, \frac{1}{2}, -\frac{1}{3}$

20. A sum of Rs. 7,750 is to be used to give cash prizes to 10 students of a school for their overall academic performance. If each prize is 50 rupees less than its preceding prize, find the amount of the first prize and the last prize. Ans: Rs 1,000 and Rs 550

21. A real valued function $f: R \rightarrow R$ is defined by $f(x) = 2x + 5$.
 (i) Find the values of $f(7.999)$, $f(8.001)$ and $f(8)$.
 (ii) Is f continuous at $x = 8$? Ans: Yes

22. Using Cramer's rule, solve the equations $\frac{3x+5y}{8} = \frac{5x-2y}{3} = 4$. Ans: $x = 4, y = 4$

23. Find the angles between the lines $2x - y + 3 = 0$ and $x - 3y + 2 = 0$. Ans: 45° and 135°

24. Prove that: $\frac{1}{2}\sqrt{2 + \sqrt{2 + \sqrt{2 + 2\cos 8A}}} = \cos A$

25. If A, B and C are the interior angles of a triangle ABC , prove that, $\cos(B+C-A) + \cos(C+A-B) + \cos(A+B-C) = 4\cos A \cos B \cos C + 1$

26. From the top of a house 200 m high, the angles of depression of two rested cars are observed as 60° and 45° respectively. Find the distance between the two cars if they are on the same side of house. Ans: 84.52 m

27. Find the 2×2 matrix which transforms a unit square into the rectangle $\begin{pmatrix} 0 & 2 & 2 & 0 \\ 0 & 0 & 3 & 3 \end{pmatrix}$. Ans: $\begin{pmatrix} 2 & 0 \\ 0 & 3 \end{pmatrix}$

28. Compute the mean deviation from the mean and its coefficient.

Weight (in kg)	0-10	30-40	40-50	10-20	20-30
No. of students	3	5	7	3	4

Ans: 10, 0.4

29. The table represents the marks obtained by 20 students in an exam. Find S.D. and coefficient of variation. Ans: 12.29, 21%

Marks obtained	30-40	40-50	50-60	60-70	70-80
No. of students	2	3	6	5	4

Group-D

 $4 \times 4 = 16$

30. Optimize the objective function $Z(x, y) = 2x + y$ under the constraints $x + y \leq 6$, $x - y \leq 4$, $x \geq 0$, $y \geq 0$.

Ans: $Z_{\text{Max}} = 11$ at $(5, 1)$ and $Z_{\text{min}} = 0$ at $(0, 0)$

31. Find the equation of the circle which passes through the points $(1, 1)$, $(4, 4)$ and $(5, 1)$.

Ans: $x^2 + y^2 - 6x - 4y + 8 = 0$

32. WXYZ is a rectangle. Prove by using vector method that $WY = XZ$.

33. $P(2, 2)$, $Q(1, -1)$ and $R(3, 0)$ are vertices of ΔPQR . If r_1 is reflection about x-axis and r_2 is rotation about origin through $+90^\circ$, find the rule which is equivalent to $r_2 \circ r_1$. Then transform the ΔPQR by $r_2 \circ r_1$. Also, represent the object and image in same graph.

Ans: Reflection about $y = x$

Congratulations for securing A⁺ in advance...

SEE MODEL SET- 2

Basic Set-II

Optional- I (Mathematics)

Time: 3 hours

F.M: 75

Answer all the questions.

Group-A

 $10 \times 1 = 10$

1. Write the maximum value of the function $f(x) = \cos x$.

Ans: The maximum value of the function $f(x) = \cos x$ is 1.

2. State remainder theorem.

Ans: When a polynomial $f(x)$ is divided by $x - a$, then remainder is $f(a)$

3. Write $\{x: 2 < x \leq 5\}$ in interval notation.

Ans: The interval notation for $\{x: 2 < x \leq 5\}$ is $(2, 5]$

4. Define singular matrix.

Ans: A square matrix having determinant zero is called singular matrix.

5. For what condition, the straight lines $y = m_1x + c_1$ and $y = m_2x + c_2$ will be perpendicular?

Ans: Condition for perpendicularity: $m_1 \times m_2 = -1$

6. Name the conic section which is formed when a plane intersects a cone parallel to the base of the cone?

Ans: The name of conic section is circle.

7. Write the formula of $\sin 3A$ in terms of $\sin A$.

Ans: $\sin 3A = 3\sin A - 4\sin^3 A$

8. Write $\cos C + \cos D$ into product form.

Ans: $2\cos\left(\frac{C+D}{2}\right) \cdot \cos\left(\frac{C-D}{2}\right)$

9. State the condition for orthogonality of vectors \vec{a} and \vec{b} .

Ans: The condition for orthogonality of vectors \vec{a} and \vec{b} is $\vec{a} \cdot \vec{b} = 0$.

10. Which transformation is associated to the matrix $\begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$?

Ans: The matrix is associated to the enlargement $E[2; (0, 0)]$

Group-B

8 × 2 = 16

11. Test whether $(x - 3)$ is a factor of $x^3 - 6x^2 + 11x - 6$ or not. *Ans: Yes*

12. Show the inequality $2x + 3y \leq 6$ in a graph. *Ans: Do yourself*

13. If I is the identity matrix of order 2×2 and $A = \begin{pmatrix} 2 & 4 \\ 3 & 4 \end{pmatrix}$, find the determinant of $4A + 3I$. *Ans: 17*

14. If the lines $\frac{x}{p} + \frac{y}{3} = 10$ and $6x - 10y + 1 = 0$ are parallel to each-other, find the value of p . *Ans: $p = -5$*

15. If $\cos\theta = \frac{12}{13}$, find the value of $\sin 2\theta$. *Ans: $\frac{120}{169}$*

16. If $\sec A = \operatorname{cosec} A$, find the value of A . ($0^\circ \leq A \leq 90^\circ$) *Ans: 45°*

17. If $\vec{a} = 5\vec{i} + 2\vec{j}$ and $\vec{b} = (k - 1)\vec{i} - 15\vec{j}$ are perpendicular vectors, Find the value of k . *Ans: 7*

18. In a continuous data, $Q_3 - Q_1 = 30$ and $Q_1 = 10$. Find the quartile deviation and its coefficient. *Ans: 25, 0.6*

Group-C

11 × 3 = 33

19. If $f(x) = \frac{3x + 10}{2}$ and $g(x) = 3x - 5$ such that $gg(x) = f^{-1}(x)$, find the value of x . *Ans: 2*

20. Solve graphically: $x^2 + 2x - 3 = 0$ *Ans: $x = -3$ or $x = 1$*

21. Test whether a function $f(x)$ defined by $f(x) = \begin{cases} x + 5 & \text{for } 1 \leq x < 2 \\ 4x - 1 & \text{for } x \geq 2 \end{cases}$ is continuous or not at $x = 2$. *Ans: Yes*

22. Solve by inverse matrix method: $7x - 2y = 1$ and $y - 2x = 4$ *Ans: $x = 3, y = 10$*

23. Find the single equation of two lines passing through the origin and perpendicular to the lines represented by $2x^2 + 3xy - 2y^2 = 0$ *Ans: $2x^2 - 3xy - 2y^2 = 0$*

24. Prove that: $\sin 20^\circ \cdot \sin 30^\circ \cdot \sin 40^\circ \cdot \sin 80^\circ = \frac{\sqrt{3}}{8}$

25. If $A + B + C = 180^\circ$, prove that:
 $\cos^2 A + \cos^2 B + \cos^2 C = 1 - 2\cos A \cdot \cos B \cdot \cos C$

26. The angles of elevation of the top of a tower observed from the distances of 44 feet and 176 feet from the foot of the tower are found to be complementary. Find the height of tower. *Ans: 88 feet*

27. $A(0, 1)$, $B(3, 2)$ and $C(-1, 3)$ are vertices of triangle ABC . If $E_1 [(0, 0), 3]$ and $E_2 [(0, 0), \frac{2}{3}]$ are two enlargements. Transform ΔABC by $E_1 \circ E_2$. Represent the object and image triangles in same graph.

Ans: $A'(0, 2)$, $B'(6, 4)$ and $C'(-2, 6)$

28. Calculate the mean deviation from median and its coefficient from the following data:

Age (in years)	10-20	20-30	30-40	40-50	50-60	60-70
No. of people	6	8	11	14	8	3

Ans: 11.8, 0.295

29. Compute the standard deviation from the data given below. Ans: 10.2

Class	0-10	10-20	20-30	30-40	40-50	50-60	60-70
Frequency	1	4	17	45	26	5	2

Group-D $4 \times 4 = 16$

30. The sum of three numbers in A.P. is 15. If 1 is added to the second number which form the consecutive term of an G.P. Find the original numbers. Ans: 2, 5, 8

31. Find the equation of a circle having equations of its two diameters are $x + y = 5$ and $x - y = 1$ and it passes through point (-4,-3). Ans: $x^2 + y^2 - 6x - 4y - 61 = 0$

32. The line segment joining the vertex and the midpoint of the base of an isosceles triangle is perpendicular to the base.

33. Find the inverse point of A(3, 4) with respect to the circle having equation $x^2 + y^2 - 4x - 6y - 23 = 0$ Ans: A'(20, 21)

Congratulations for securing A⁺ in advance...

Moderate Set-I

SEE MODEL SET-3

Optional- I (Mathematics)

Time: 3 hours

F.M: 75

Answer all the questions.

Group-A

$10 \times 1 = 10$

1. Write the nature of graph of constant function.

Ans: It is always parallel to x-axis.

2. What is the formula to find the common ratio of a geometric sequence having 'n' means between two numbers 'a' and 'b'?

Ans: $r = \left(\frac{b}{a}\right)^{\frac{1}{n+1}}$

3. Write the right-hand limit of $f(x)$ at $x = 2$ in notation. Ans: $\lim_{x \rightarrow 2^+} f(x)$

4. For a matrix $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$, what does $\frac{1}{ad - bc} \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$ represent?

Ans: A^{-1}

5. Write the condition under which the pair of lines represented by the equation $ax^2 + 2hxy + by^2 = 0$ are perpendicular to each other.

Ans: $a + b = 0$

6. Define hyperbola on the basis of conic section.

Ans: When a plane cuts a double napped right circular cone in such a way that the semi-vertical angle (α) is greater than the angle between the plane and the axis of cone ($0^\circ \leq \beta$) then the conic section so obtained is called hyperbola.

OR

If the intersecting plane is parallel to the axis of the cone, then the curve so obtained is called hyperbola.

7. Express $\cos\theta$ in terms of $\cos 2\theta$.

$$\text{Ans: } \cos\theta = \sqrt{\frac{1 + \cos 2\theta}{2}}$$

8. Write $2\sin P \cdot \sin Q$ as the sum and difference of sine.

9. What should be the angle between \vec{a} and \vec{b} to get the maximum value of $\vec{a} \cdot \vec{b}$? Ans: 0°

10. If an enlargement $E_1[(0, 0); k_1]$ is followed by another enlargement $E_2[(0, 0); k_2]$, what will be the single transformation for the combined enlargements? Ans: Enlargement: $E_2 \circ E_1 = [(0, 0); k_1 \times k_2]$

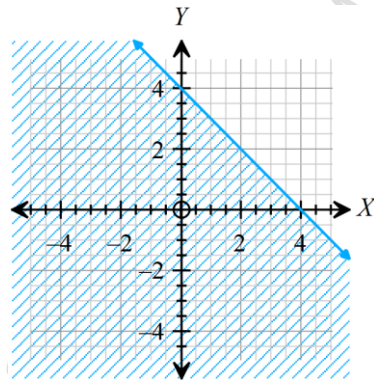
Group 'B'

[8 × 2 = 16]

11. If $f = \{(1, 2), (2, 5), (3, 10)\}$ and $g \circ f = \{(1, 1), (2, 8), (3, 27)\}$, by using arrow diagram, find the function g and write it in ordered pair form.

$$\text{Ans: } g = \{(2, 1), (5, 8), (10, 27)\}$$

12. Write the inequality represented by the shaded region from the adjoining figure.



$$\text{Ans: } x + y \leq 4$$

13. If the inverse of the matrix $\begin{pmatrix} 1 & 4 \\ -2 & 1 - a^2 \end{pmatrix}$ does not exist, find the value of a . Ans: ± 3

14. If the lines $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ are perpendicular to each other, then prove that $a_1a_2 + b_1b_2 = 0$

15. If $\cos 30^\circ = \frac{\sqrt{3}}{2}$ show that: $\sin 15^\circ = \frac{1}{2}(\sqrt{2} - \sqrt{3})$

16. Solve: $\sin^2 \frac{A}{4} - \sin \frac{A}{4} + \frac{1}{4} = 0$ ($0^\circ \leq A \leq 180^\circ$) Ans: 120°

17. If \vec{a} and \vec{b} are unit vectors such that $\vec{a} + 2\vec{b}$ and $5\vec{a} - 4\vec{b}$ are orthogonal to each other, find the angle between \vec{a} and \vec{b} . Ans: 60°

18. In a continuous data, if $N = 50$, $\bar{x} = 20$ and $\sum f(m - \bar{x})^2 = 1250$, find the standard deviation and its coefficient. Ans: $5, 0.25$

Group 'C'

11 × 3 = 33

19. Find the roots of the given cubic equation: $3x^3 - 13x^2 + 16 = 0$ Ans: $-1, 4, 4/3$

20. If the arithmetic mean and geometric mean of two numbers are 13 and 12 respectively, find the numbers. Ans: 8 and 18 or 18 and 8

21. Prove that the function $f(x) = \begin{cases} 3x - 1 & \text{for } x < 2 \\ 5 & \text{for } x = 2 \\ 2x + 1 & \text{for } x > 2 \end{cases}$ is continuous at a point $x = 2$.

22. Solve the following system of linear equations by a matrix method: $2xy - 5x + 3 = 0$ and $5xy - 7x = 6$ Ans: $x = 3, y = 2$

23. If an angle between the lines represented by $2x^2 + kxy + 3y^2 = 0$ is 45° , find the positive value of k and then separate equation of lines. Ans: $7, 2x + y = 0, x + 3y = 0$

24. Prove that: $\left(1 + \cos \frac{\pi^c}{8}\right) \left(1 + \cos \frac{3\pi^c}{8}\right) \left(1 + \cos \frac{5\pi^c}{8}\right) \left(1 + \cos \frac{7\pi^c}{8}\right) = \frac{1}{8}$

25. If $A + B + C = \pi^c$, prove that: $\frac{\cos A}{\sin B \cdot \sin C} + \frac{\cos B}{\sin C \cdot \sin A} + \frac{\cos C}{\sin A \cdot \sin B} = 2$

26. A flagstaff of height 7m stands on the top of a tower. The angles subtended by the tower and the flagstaff at a point on the ground are 45° and 15° respectively, find the height of the tower. **Ans: 9.56m**

27. If the matrix $\begin{pmatrix} a & 0 \\ c & -1 \end{pmatrix}$ transforms a rectangle $\begin{pmatrix} 0 & 2 & 2 & 0 \\ 0 & b & 1 & 1 \end{pmatrix}$ into a rectangle $\begin{pmatrix} 0 & 2 & 2 & 0 \\ 0 & 0 & -1 & d \end{pmatrix}$, find the values of a, b, c and d.

Ans: $a = 1, b = 2, c = 1, d = -1$

28. Find the quartile deviation from the following data. **Ans: 14**

Marks	20-30	30-40	40-50	50-60	60-70	70-80
No. of Students	4	5	2	4	3	2

29. Calculate the standard deviation and coefficient of variation from the following distribution table. **Ans: 4.7231, 49.72%**

Age (in years)	0-4	0-8	0-12	0-16	0-20
No. of boys	6	15	27	37	40

Group 'D'

$4 \times 4 = 16$

30. The minimum value of objective function $F = 6x + 10y + k$ subject to constraints $5x + 2y \leq 10$, $3x + 5y \leq 15$, $x \geq 0$, $y \geq 0$ is 20, find the value of k. **Ans: $k = 20$**

31. Find the equation of a circle which is obtained by reflecting the circle $25x^2 + 25y^2 - 200x - 250y + 989 = 0$ on x-axis.

Ans: $25x^2 + 25y^2 + 200x - 250y + 989 = 0$

32. Prove by vector method that the diagonals of parallelogram bisect to each other.

33. A triangle with vertices A (1, 2), B (4, -1) and C (2, 5) is reflected successively in the lines $x = -1$ and $y = 2$. Find by stating coordinates and graphically represent images under these transformations. State also the single transformation given by the combination of these transformations.

Ans: $A' (-3, 2), B' (-6, -1), C' (-4, 5); A'' (-3, 2), B'' (-6, 5), C'' (-4, -1); R [180^\circ, (-1, 2)]$

Congratulations for securing A⁺ in advance...

Moderate Set-I

SEE MODEL SET-4

Optional- I (Mathematics)

Time: 3 hours

F.M: 75

Answer all the questions.

Group-A

10 × 1 = 10

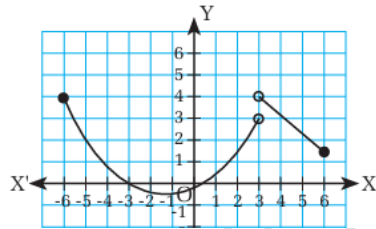
1. Under what condition, the inverse of a function is possible?

Ans: When the function is one to one and onto both

2. If $f(x)$, $q(x)$, $d(x)$ and $r(x)$ represent a polynomial, quotient, divisor and remainder respectively, write the relation among them.

Ans: $f(x) = d(x) \times q(x) + r(x)$

3. In the given figure, at which point the function is discontinuous? Give reason.



Ans: The function is discontinuous at $x = 3$ because there is a jump.

4. Define inverse matrix.

Ans: Let, A and B be any two square matrices of same order such that $AB = BA = I$, where I is an identity matrix of order same as A and B, then matrices A and B are called inverse of each other.

5. If the pair of lines represented by $px^2 + 2qxy + ry^2 = 0$ are coincident to each other, write the relation between p , q and r .

Ans: $q^2 - pr = 0$

6. What will be the length of radius of a circle having centre (h, k) and touches the x -axis?

Ans: $r = |k|$

7. Express $\sin A$ in terms of $\tan \frac{A}{2}$.

Ans: $\sin A = \frac{2 \tan \frac{A}{2}}{1 + \tan^2 \frac{A}{2}}$

8. If $A + B + C = \pi^c$, what is the value $\sin\left(\frac{3A}{2} + \frac{3B}{2}\right)$?

Ans: $-\cos \frac{C}{2}$

9. If the point P divides the line segment AB joining the points A and B externally in the ratio $m_1:m_2$, and O the reference origin, write down the relation of \vec{OP} , \vec{OA} and \vec{OB} along with m_1 and m_2 .

Ans: $\vec{OP} = \frac{m_1 \vec{OB} - m_2 \vec{OA}}{m_1 - m_2}$

10. Write the 2×2 matrix which represents the reflection about the line

$y = -x$

Ans: $\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$

Group 'B'

[8 × 2 = 16]

11. If $f(x + 2) = 3x + 4$ and $f \circ g(x) = 6x - 2$, find $g(x)$.

Ans: $2x$

12. Draw a graph of $y = x^2$.

13. If the matrices $\begin{pmatrix} 2x & 7 \\ 5 & 9 \end{pmatrix}$ and $\begin{pmatrix} 9 & y \\ -5 & 4 \end{pmatrix}$ are inverse to each other, find the values of x and y .

Ans: $x = 2, y = -7$

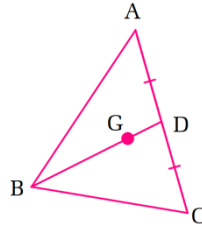
14. Find the equation of a circle whose ends of a diameter are $(2, 3)$ and $(-1, 4)$.

Ans: $x^2 + y^2 - x - 7y + 10 = 0$

15. Prove that: $\sin 55^\circ + \cos 55^\circ = \sqrt{2} \cos 10^\circ$

16. Prove that: $\frac{2 \sin \beta - \sin 2\beta}{2 \sin \beta + \sin 2\beta} = \tan^2 \frac{\beta}{2}$

17. In the given figure, $AD = DC$ and the point G is the centroid of the triangle ABC . If the position vectors of the points B and D are $3\vec{i} + 7\vec{j}$ and $3\vec{i} - 2\vec{j}$ respectively, find the position vector of G .



Ans: $3\vec{i} + \vec{j}$

18. In a continuous data, if $N = 25$, $\sum fm^2 = 12500$ and $\sigma = 10$, find the mean (\bar{x}).

Ans: 20

Group 'C'

$11 \times 3 = 33$

19. Solve: $y = x^3 - 4x^2 + x + 8$ and $y = 2$ Ans: -1, 2, 3
20. Minimize the objective function $Z = 3x + 2y$ subject to the constraints $x + y \geq 0$, $x - y \leq 0$, $x \geq -1$, $y \leq 2$ Ans: $Z_{Min} = -8$ at $(-1, 2)$
21. By finding the right hand limit, left hand limit and functional value at $x = 3$, test whether the function $f(x) = \begin{cases} \frac{2x^2 - 8}{x - 2} & \text{for } x \neq 3 \\ 3x + 1 & \text{for } x = 3 \end{cases}$ is continuous or not. Ans: Yes
22. Two buses were coming from two villages situated just in the opposite direction. The uniform speed of one bus is 10 km/hr more than that of another one and they had started their travelling in the same time. If the distance between the villages is 450 km and they meet after 5 hours, make the pair of linear equations and find their speeds by using matrix method. Ans: 50 km/hr and 40 km/hr

23. The vertices of a triangle PQR are $P(1, 3)$, $Q(5, -4)$ and $R(-3, 2)$. Find the equation of the altitude of the triangle drawn from vertex P .

Ans: $4x - 3y + 5 = 0$

24. Prove that: $\frac{(\sin A + \sin B)(\sin A - \sin B)}{\sin A \cdot \cos A - \sin B \cdot \cos B} = \tan(A + B)$

25. Solve: $\cot^2\theta + \left(\sqrt{3} + \frac{1}{\sqrt{3}}\right)\cot\theta + 1 = 0$ ($0^\circ \leq \theta \leq 360^\circ$)

Ans: $120^\circ, 150^\circ, 300^\circ, 330^\circ$

26. A ladder 10 m long reaches a point 10 m below the top of a vertical flagstaff. From the foot of the ladder, the elevation of the flagstaff is 60° . Find the height of the flagstaff. Ans: 15 m

27. Find the inverse of the point $(4, 5)$ with respect to the circle having equation $x^2 + y^2 - 4x - 6y = 3$. Ans: (6, 7)

28. The heights of 100 students of class X are given below.

Height (in cm)	Number of students
95-105	9
105-115	13
115-125	25
125-135	30
135-145	13
145-155	10

Calculate the mean deviation from the median. Also, find its coefficient. Ans: 11.34, 0.009

29. Calculate the standard deviation and coefficient of standard deviation from the following distribution table. Ans: 12.96, 41

Marks obtained	0-60	10-60	20-60	30-60	40-60	50-60
No. of students	50	46	40	30	10	4

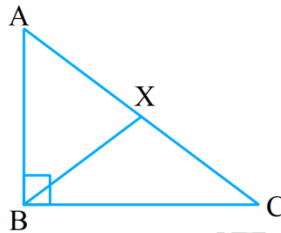
Group 'D'

4 × 4 = 16

30. A person pays a loan of Rs 975 in monthly installments, each installment being less than a former by Rs 5. The amount of first installment is Rs 100. In how many installments will the entire amount be paid? Given reason. **Ans: 15**

31. A (2, 4) is a vertex of an equilateral triangle ABC in which the equation of base BC is $\sqrt{3}x + y + 2 = 0$. Find the equations of its sides AB and AC. **Ans: $y - 4 = 0$ and $\sqrt{3}x - y + (4 - 2\sqrt{3}) = 0$**

32. In the given ΔABC ; $\angle ABC = 90^\circ$ and X is the mid-point of side AC. Prove by vector method that $AX = BX = CX$.



33. A(1, 1), B(3, 5) and C(0, -2) are the vertices of ΔABC . r_1 is reflection about the line $x = 2$ and r_2 is reflection about the line $x = -1$. Find the single transformation equivalent to $r_1 \circ r_2$. Transform ΔABC by $r_1 \circ r_2$. Represent the object and image triangles in same graph.

Ans: $T = \begin{pmatrix} 6 \\ 0 \end{pmatrix}$ A' (7, 1), B' (9, 5) and C' (6, -2)

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Advanced Set-I

SEE MODEL SET-5

Optional- I (Mathematics)

Time: 3 hours

F.M: 75

Answer all the questions.

Group-A

10 × 1 = 10

1. Define transcendental function.

Ans: A function which is not algebraic function i.e., that cannot be expressed as a finite combination of basic algebraic operations like addition, subtraction, multiplication, division, raising to a power, and extracting a root is called transcendental function. For example: $y = f(x) = \sin x$.

2. State the nature of size of graph of parabola $y = ax^2 + bx + c$, when 'a' becomes larger? **Ans: The graph becomes narrower**

3. Show the interval $[2, \infty)$ on a number line.

4. What is the determinant of an identity matrix of order 2×2 ? **Ans: 1**

5. Write the single equation representing both the coordinate axes.

Ans: $xy = 0$

6. Name the conic section so formed when an intersecting plane is parallel neither base nor axis nor generator of the cone. **Ans: Ellipse**

7. Express $\tan \frac{A}{2}$ in terms of $\cos A$.

Ans: $\tan \frac{A}{2} = \sqrt{\frac{1 - \cos A}{1 + \cos A}}$

8. Without using calculator, what is the value of $4\sin 105^\circ \cdot \sin 15^\circ$? **Ans: 1**

9. If $\vec{a} = k\vec{b}$ for a scalar number k , what is the angle between \vec{a} and \vec{b} ?

Ans: 0°

10. If a point P is at the centre of inversion circle, where does its inverse lie? Ans: To the infinity

Group 'B'

[8 × 2 = 16]

11. What should be added to the polynomial $f(x) = x^3 - 6x^2 + 12x - 7$ to make a factor $(x - 3)$? Ans: -2
12. If the arithmetic mean between 2 and x is 5, find the geometric mean. Ans: 4
13. In the equations $3x + y = 7$ and $x + 2y = 4$, according to Cramer's rule, the value of D is 5, find the value of x. Ans: 2

14. Find the separate lines represented by $x^2 + 2xy \sec \theta + y^2 = 0$

Ans: $x - y(\sec \theta - \tan \theta) = 0, x - y(\sec \theta + \tan \theta) = 0$

15. If $\sin \frac{A}{2} = \frac{1}{2} \left(t + \frac{1}{t} \right)$ then show that: $\cos A + \frac{1}{2} \left(t^2 + \frac{1}{t^2} \right) = 0$.

16. Solve: $\tan x + \cot x = 2$ $(0^\circ \leq x \leq 90^\circ)$ Ans: 45°

17. If $\vec{a} + \vec{b} + \vec{c} = 0, |\vec{a}| = 6, |\vec{b}| = 7$ and $|\vec{c}| = \sqrt{127}$, find the angle between \vec{a} and \vec{b} . Ans: 60°

18. In a continuous series, the coefficient of quartile deviation is $\frac{1}{3}$ and its upper quartile is 60, find the quartile deviation. Ans: 15

Group 'C'

11 × 3 = 33

19. The sum of first four terms is 40 and the sum of the first two terms is 4 of a geometric series whose common ratio is positive, find the sum of first 8 terms the series. Ans: 3280

20. Solve graphically: $y = x^2$ and $y = 2 - x$ Ans: $x = (1, 1), (-2, 4)$

21. For what value of k, will the function $f(x) = \begin{cases} x^2 - 2 & \text{for } x < 3 \\ 2x + 1 & \text{for } x = 3 \\ 8 - k & \text{for } x > 3 \end{cases}$ be continuous at $x = 3$? Ans: 1

22. Solve the following system of equations by inverse matrix method:
 $\frac{x-1}{y+1} = \frac{3}{4}$ and $\frac{x+2}{y-2} = \frac{4}{3}$ Ans: $x = 10, y = 11$

23. A (3, 5) and C (7, 9) are the opposite vertices of a rhombus ABCD, find the equation of the diagonal BD. Ans: $x + y = 12$

24. Prove that: $\frac{1 - \cos A + \cos B - \cos(A+B)}{1 + \cos A - \cos B - \cos(A-B)} = \tan \frac{A}{2} \cdot \cot \frac{B}{2}$

25. If $A + B + C = \pi$ then prove that:

$\sin \frac{A}{2} + \sin \frac{B}{2} + \sin \frac{C}{2} = 1 + 4 \sin \left(\frac{\pi - A}{4} \right) \sin \left(\frac{\pi - B}{4} \right) \sin \left(\frac{\pi - C}{4} \right)$

26. The angle of the top of a tower observed from 75 m far is double of the angle observed from 200 m far from the tower. What is the height of the tower? Ans: 100 m

27. A unit square MNOP is transformed under the matrix transformation through $y = -x$ and write the vertices of the images quadrilateral M'N'O'P' so formed. Ans: $M'(0, 0), N'(0, -1), O'(-1, -1)$ and $P'(-1, 0)$

28. Calculate the mean deviation from the median. Also, find its coefficient. Ans: 11.8, 0.295

Marks obtained	10-20	20-30	10-30	30-40	20-60	10-70
No. of students	6	8	25	11	41	50

29. The snack expenditure of 100 civil servants working in an office is as below.

Expenditure (Rs)	60-63	63-66	66-69	69-72	72-75
Servants civil	50	46	40	30	10

From the above data, find the coefficient of standard deviation and coefficient of variation.

Ans: 0.0429, 4.29 %

Group 'D'

$4 \times 4 = 16$

30. A stone is thrown into a pond, creating a circular ripple that spreads over the pond in such a way that the radius is increasing at the rate of 3 ft/sec.
- (a) Find a function $r(t)$ for the radius in terms of t .
- (b) Find a function $A(r)$ for the area of the ripple in terms of the radius r .
- (c) Find (Aor) (t) . Explain the meaning of this function.

Ans: (a) $r(t) = 3t$

(b) $A(r) = \pi r^2$

(c) $(Aor) (t) = 9\pi t^2$

The function $(Aor)(t)$ represents the area of the ripple as a function of time. It shows how the area of the ripple changes over time as the radius increases at a constant rate.

31. Find the equation of a circle with centre (3, 2) and passing through the centre of the circle $x^2 + y^2 = 2(x - 2y + 2)$.

Ans: $x^2 + y^2 - 6x - 4y - 7 = 0$

32. Prove by vector method that the angle in a semi-circle is right angle.
33. A triangle having vertices A(2, 5), B(-1, 3) and C(4, 1) is rotated about origin through $+90^\circ$. The image so obtained is reflected on the line $x = 0$. Find the vertices of image triangles. Show all the

triangles in the same graph paper and also write the single transformation to represent these two transformations.

Ans: Reflection in the line $x = y$

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