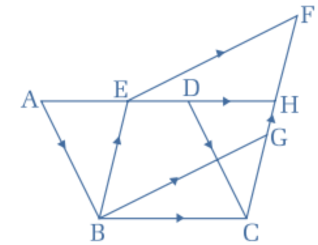
Important Higher Ability Questions (Geometry: Area of triangles and quadrilaterals)

QUESTION 1

In the given figure, AB // DC, AH // BC, BE // CF and EF // BG.
Prove that: ▱ABCD = ▱GBEF.



Solution

Given: AB // DC, AH // BC, BE // CF and EF // BG.

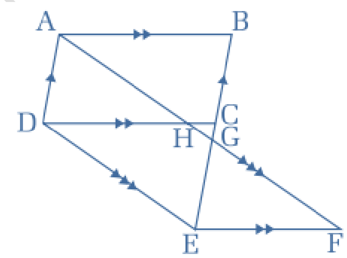
To prove: ▱ABCD = ▱GBEF

Proof:

S.N.	Statements	S.N.	Reasons
1.	▱ABCD = ▱EBCH	1.	Both are standing on the same base BC and between AH // BC.
2.	▱EBCH = ▱GBEF	2.	Both are standing on the same base EB and between EB // FC.
3.	▱ABCD = ▱GBEF	3.	From statements (1) and (2).
Hence, proved			

QUESTION 2

In the given figure, AB // DC // EF, AD // BE and AF // DE.
Prove that ▱DEFH = ▱ABCD.



Solution

Given: AB // DC // EF, AD // BE and AF // DE.

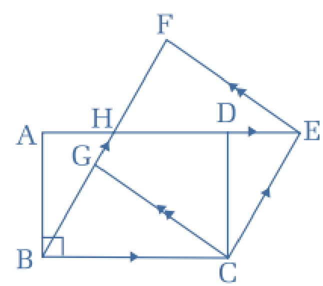
To prove: ▱DEFH = ▱ABCD

Proof:

S.N.	Statements	S.N.	Reasons
1.	▱DEFH = ▱DEGA	1.	Both are standing on the same base DE and between AF // DE.
2.	▱DEGA = ▱ABCD	2.	Both are standing on the same base AD and between BE // AD.
3.	▱DEFH = ▱ABCD	3.	From statements (1) and (2).
Hence, proved			

QUESTION 3

In the given figure, AE // BC, BF // CE, CG // EF and AB ⊥ BC.
Prove that rectangle ABCD = ▱EFGC.



Solution

Given: AE // BC, BF // CE, CG // EF and AB ⊥ BC.

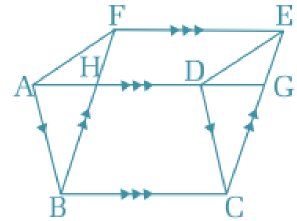
To prove: Rectangle ABCD = ▱EFGC.

Proof:

S.N.	Statements	S.N.	Reasons
1.	□ABCD = ▱HBCE	1.	Both are standing on the same base BC and between AE // BC.
2.	▱HBCE = ▱EFGC	2.	Both are standing on the same base CE and between BF // CE.
3.	□ABCD = ▱EFGC	3.	From statements (1) and (2).
Hence, proved			

QUESTION 4

In the given figure, AB // DC, BF // CE and FE // AG // BC.
Prove that: ▱BCEF = ▱ABCD + ▱ADEF



Solution

Given: AB // DC, BF // CE and FE // AG // BC

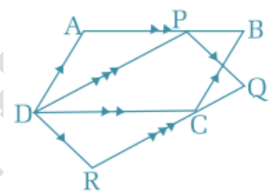
To prove: ▱BCEF = ▱ABCD + ▱ADEF

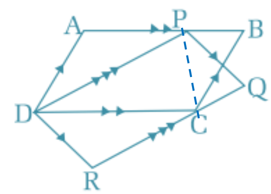
Proof:

S.N.	Statements	S.N.	Reasons
1.	▱ABCD = ▱HBCG	1.	Both are standing on the same base BC and between AG // BC.
2.	▱FADE = ▱FHGE	2.	Both are standing on the same base FE and between AG // FE.
3.	▱BCEF = ▱HBCG + ▱FHGE	3.	By whole part axiom
4.	▱BCEF = ▱ABCD + ▱ADEF	4.	From statements (1), (2) and (3).
Hence, proved			

QUESTION 5

In the given figure, prove that the area of parallelograms ABCD and PQRD are equal.





Solution

Given: ABCD and PQRD are parallelograms.

To prove: ▱ABCD = ▱PQRD

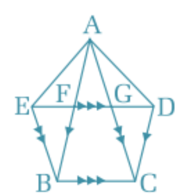
Construction: P and C are joined.

Proof:

S.N.	Statements	S.N.	Reasons
1.	$\Delta PDC = \frac{1}{2}$ ▱ABCD	1.	Both are standing on the same base DC and between AB // DC.
2.	$\Delta PDC = \frac{1}{2}$ ▱PQRD	2.	Both are standing on the same base PD and between QR // PD.
3.	▱ABCD = ▱PQRD	3.	From statements (1) and (2).
Hence, proved			

QUESTION 6

In the given figure, AB // DC, BC // ED and EB // AC. Prove that: Ar. (ΔAEB) =Ar. (ΔACD).



Solution

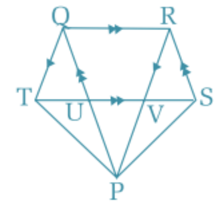
Given: AB // DC, BC // ED and EB // AC

To prove: Ar. (ΔAEB) =Ar. (ΔACD).

Proof:

S.N.	Statements	S.N.	Reasons
1.	$\Delta AEB = \frac{1}{2}$ ▱EBCG	1.	Both are standing on the same base EB and between AC // EB.
2.	$\Delta ACD = \frac{1}{2}$ ▱FBCD	2.	Both are standing on the same base CD and between BA // CD.
3.	▱EBCG = ▱FBCD	3.	Both are standing on the same base BC and between ED // BC.
4.	ΔAEB = ΔACD	4.	From statements (1), (2) and (3)
Hence, proved			

QUESTION 7



In the given figure, QR // TS, QT // RP and RS // QP. Prove that: Ar. ΔPQT = Ar. ΔPRS.

Solution

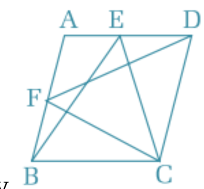
Given: QR // TS, QT // RP and RS // QP

To prove: Area of ΔPQT = Area of ΔPRS

Proof:

S.N.	Statements	S.N.	Reasons
1.	ΔPQT= $\frac{1}{2}$ ▱RQTV	1.	Both are standing on the same base QT and between RP // QT.
2.	ΔPRS = $\frac{1}{2}$ ▱RQUS	2.	Both are standing on the same base RS and between PQ // SR.
3.	▱RQTV = ▱RQUS	3.	Both are standing on the same base QR and between TS // QR.
4.	ΔPQT = ΔPRS	4.	From statements (1), (2) and (3
Hence, proved			

QUESTION 8



In the given figure, ABCD is a parallelogram. E and F are any points on AD and AB respectively. Prove that: ΔCDF = ΔABE + ΔCDE

Solution

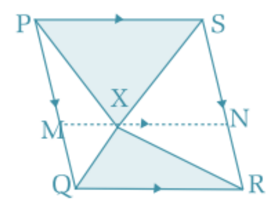
Given: ABCD is a parallelogram. E and F are any points on AD and AB respectively.

To prove: ΔCDF = ΔABE + ΔCDE

Proof:

S.N.	Statements	S.N.	Reasons
1.	ΔCDF = $\frac{1}{2}$ ▱ABCD	1.	Both are standing on the same base CD and between AB// CD.
2.	ΔEBC = $\frac{1}{2}$ ▱ABCD	2.	Both are standing on the same base BC and between AD // BC.
3.	ΔABE + ΔCDE = $\frac{1}{2}$ ▱ABCD	3.	From statement (2), being remaining parts of ▱ABCD
4.	ΔCDF = ΔABE + ΔCDE	4.	From statements (1) and (3)

QUESTION 9



In the given figure, PQRS is a parallelogram. M and N are any points on PQ and RS respectively such that PS // MN // QR. Prove that:▱PQRS = 2(ΔPXS + ΔQXR)

Solution

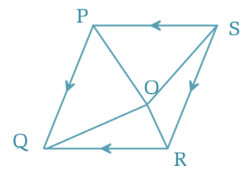
Given: PQRS is a parallelogram, PS // MN // QR.

To prove: ▱PQRS = 2(ΔPXS + ΔQXR)

Proof:

S.N.	Statements	S.N.	Reasons
1.	▱PMNS = 2ΔPXS	1.	Both are standing on the same base PS and between PS// MN.
2.	▱MQRN = 2ΔQXR	2.	Both are standing on the same base QR and between MN // QR.
3.	▱PQRS = ▱PMNS + ▱MQRN	3.	By whole part axiom
4.	▱PQRS = 2(ΔPXS + ΔQXR)	4.	From statements (1), (2) and (3)
Hence, proved			

QUESTION 10

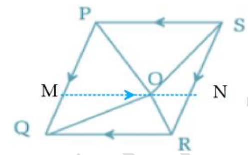


In the given figure, O is any point within the parallelogram PQRS. Prove that the sum of area of ΔPOS and ΔQOR is equal to half of the area of parallelogram PQRS.

Solution

Given: O is any point within the parallelogram PQRS

To prove: $\Delta POS + \Delta QOR = \frac{1}{2}$ ▱PQRS

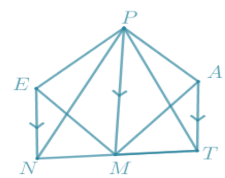


Construction: MN // QR is drawn.

Proof:

S.N.	Statements	S.N.	Reasons
1.	$\Delta POS = \frac{1}{2}$ ▱PMNS	1.	Both are standing on the same base PS and between PS// MN.
2.	$\Delta QOR = \frac{1}{2}$ ▱MQRN	2.	Both are standing on the same base QR and between MN // QR.
3.	$\Delta POS + \Delta QOR = \frac{1}{2}$ (▱PMNS + ▱MQRN)	3.	Adding statements (1) and (2)
4.	$\Delta POS + \Delta QOR = \frac{1}{2}$ ▱PQRS	4.	From statements (1), (2) and (3) and by whole part axiom
Hence, proved			

QUESTION 11



In a pentagon PENTA; M is any point on side NT so that EN//PM//AT.

Prove that: area of triangle NPT = area of quadrilateral PEMA.

Solution

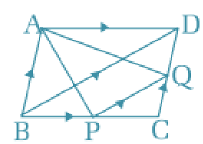
Given: In pentagon PENTA; M is any point on side NT so that EN//PM//AT

To prove: Area of ΔNPT = Area of quadrilateral PEMA.

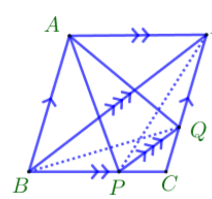
Proof:

S.N.	Statements	S.N.	Reasons
1.	ΔPNM= ΔPEM	1.	Both are on the same base PM and between EN//PM.
2.	ΔPMT = ΔPAM	2.	Both are on the same base PM and between AT//PM.
3.	ΔPNM +ΔPMT = ΔPEM +ΔPAM	3.	Adding statements (1) and (2).
4.	ΔNPT = PEMA	4.	From statement (3), by whole part axiom.

QUESTION 12



In parallelogram ABCD; P and Q are any points on BC and CD respectively such that BD//PQ. Prove that the area of triangles ABP and AQD are equal.



Solution

Given: ABCD is a parallelogram, BD//PQ.

To prove: Area of ΔABP = Area of ΔAQD

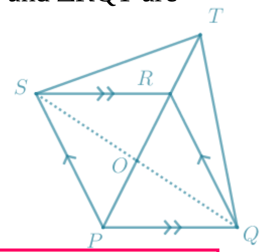
Construction: B, Q and P, D are joined.

Proof:

S.N.	Statements	S.N.	Reasons
1.	ΔABP = ΔDBP	1.	Both are standing on same base BP and between AD//BP.
2.	ΔDBP = ΔDBQ	2.	Both are standing on same base BD and between PQ//BD.
3.	ΔDBQ = ΔAQD	3.	Both are standing on same base QD and between AB//QD.
4.	ΔABP = ΔAQD	4.	From statements (1), (2) and (3)
Hence, proved			

QUESTION 13

In parallelogram PQRS; diagonal PR is produced to the point T. Prove that the ΔRST and ΔRQT are equal in area.



Solution

Given: In parallelogram PQRS; diagonal PR is produced to the point T.

To prove: Area of ΔRST = Area of ΔRQT

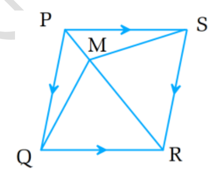
Construction: S and Q are joined so that the diagonals PR and SQ intersect at O.

Proof:

S.N.	Statements	S.N.	Reasons
1.	O is the mid-point of SQ.	1.	The diagonals of parallelogram bisect each other.
2.	ΔSOT = ΔQOT	2.	The median OT bisects the ΔSQT.
3.	ΔSOR = ΔQOR	3.	The median OR bisects the ΔSQR.
4.	ΔSOT – ΔSOR = ΔQOT– ΔQOR	4.	Subtracting statement (2) from statement (1)
5.	ΔRST = ΔRQT	5.	From statement (4)
Proved			

QUESTION 14

In the given figure, PQRS is a parallelogram. If M is any point on diagonal PR then prove that ΔPQM and ΔPSM are equal in area.

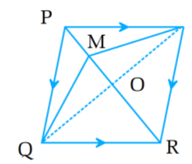


Solution

Given: In parallelogram PQRS; M is any point on diagonal PR.

To prove: Area of ΔQMR = Area of ΔSMR

Construction: S and Q are joined so that the diagonals PR and SQ intersect at O.

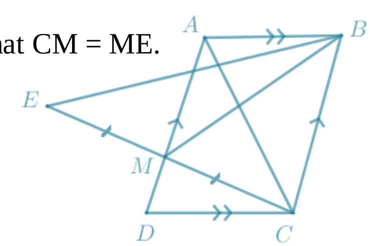


Proof:

S.N.	Statements	S.N.	Reasons
1.	O is the mid-point of QS.	1.	The diagonals of parallelogram bisect each other.
2.	ΔQOM = ΔSOM	2.	The median OM bisects the ΔSQM.
3.	ΔQOR = ΔSOR	3.	The median OR bisects the ΔSQR.
4.	ΔQOM + ΔQOR = ΔSQM + ΔSOR	4.	Adding statements (2) and (3)
5.	ΔQMR = ΔSMR	5.	From statement (4)
Proved			

QUESTION 15

In parallelogram ABCD; M is any point on side AD. CM is produced to E such that CM = ME. Prove that: area of triangle BEM = area of triangle ADC.



Solution

Given: In parallelogram ABCD; M is any point on side AD. CM is produced to E such that CM = ME.

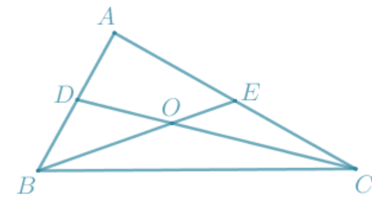
To prove: Area of ΔBEM = Area of ΔADC

Proof:

S.N.	Statements	S.N.	Reasons
1.	ΔBEM = ΔBMC	1.	The median BM bisects the ΔBEC.
2.	ΔBMC = $\frac{1}{2}$ ▱ABCD	2.	Both are standing on same base BC and between AD//BC.
3.	ΔADC = $\frac{1}{2}$ ▱ABCD	3.	The diagonal AC bisects the parallelogram ABCD.
4.	ΔBEM = ΔADC	4.	From statements (1), (2) and (3)
Proved			

QUESTION 16

In ΔABC; medians BE and CD intersect at O. Prove that ΔBOC and quadrilateral ADOE are equal in area.



Solution

Given: In ΔABC; medians BE and CD intersect at O.

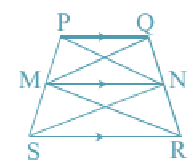
To prove: Area of ΔBOC = Area of quadrilateral ADOE

Proof:

S.N.	Statements	S.N.	Reasons
1.	ΔBCD = $\frac{1}{2}$ ΔABC	1.	The median CD bisect ΔABC.
2.	ΔABE = $\frac{1}{2}$ ΔABC	2.	The median BE bisect ΔABC.
3.	ΔBCD = ΔABE	3.	From statements (2) and (3).
4.	ΔBCD = ΔBOC + ΔBOD ΔABE = Quad. ADOE + ΔBOD	4.	By whole part axiom
5.	ΔBOC + ΔBOD = Quad. ADOE + ΔBOD ∴ Quad. ADOE = ΔBOC	5.	From (3) and (4)
Proved			

QUESTION 17

In the given figure, PQRS is the trapezium where PQ // MN // SR. Prove that: ΔPNS = ΔQMR.



Solution

Given: PQRS is the trapezium where PQ // MN // SR

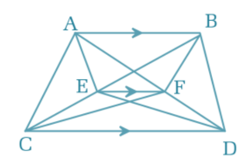
To prove: ΔPNS = ΔQMR

Proof:

S.N.	Statements	S.N.	Reasons
1.	ΔPMN = ΔQMN	1.	Both are standing on same base MN and between PQ//MN.
2.	ΔMNS = ΔMNR	2.	Both are standing on same base MN and between MN//SR.
3.	ΔPMN + ΔMNS = ΔQMN + ΔMNR	3.	Adding (1) and (2)
4.	ΔPNS = ΔQMR	4.	By whole part axiom
Proved			

QUESTION 18

In the figure, AB // CD // EF. Prove that: ΔAED = ΔBFC



Solution

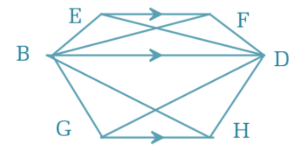
Given: AB // CD // EF

To prove: ΔAED = ΔBFC

Proof:

S.N.	Statements	S.N.	Reasons
1.	ΔAEF = ΔBEF	1.	Both are standing on same base EF and between AB//EF.
2.	ΔDEF = ΔCEF	2.	Both are standing on same base EF and between CD//EF.
3.	ΔAEF + ΔDEF = ΔBEF + ΔCEF	3.	Adding (1) and (2)
4.	ΔAED = ΔBFC	4.	By whole part axiom
Proved			

QUESTION 19



In the figure, EF // BD // GH. Prove that:
Area of quadrilateral BHDE = Area of quadrilateral BGDF

Solution

Given: EF // BD // GH

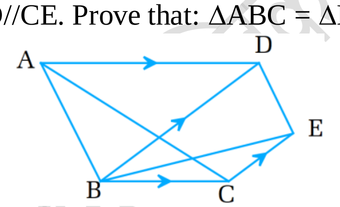
To prove: Area of quadrilateral BHDE = Area of quadrilateral BGDF

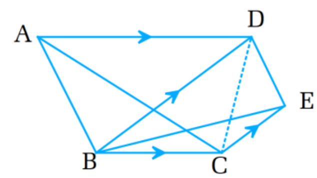
Proof:

S.N.	Statements	S.N.	Reasons
1.	ΔEBD = ΔFBD	1.	Both are standing on same base BD and between EF//BD.
2.	ΔHBD = ΔGBD	2.	Both are standing on same base BD and between GH//BD.
3.	ΔEBD + ΔHBD = ΔFBD + ΔGBD	3.	Adding (1) and (2)
4.	Quad. HBDE = Quad. BGDF	4.	By whole part axiom
Proved			

QUESTION 20

In the adjoining figure, it is given that AD//BC and BD//CE. Prove that: ΔABC = ΔBDE.





Solution

Given: AD//BC and BD//CE

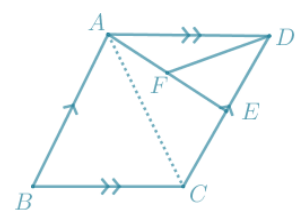
To prove: Area of ΔABC = Area of ΔBDE

Construction: D and C are joined.

Proof:

S.N.	Statements	S.N.	Reasons
1.	ΔABC = ΔDBC	1.	Both are standing on same base BC and between AD//BC.
2.	ΔDBC = ΔDBE	2.	Both are standing on same base BD and between CE//BD.
3.	ΔABC = ΔBDE	3.	From statements (1) and (2)
Proved			

QUESTION 21



In parallelogram ABCD; E is the mid-point of side CD and F is the mid-point of AE.
Prove that area of parallelogram ABCD = 8 × area of ΔAFD.

Solution

Given: In ▱ABCD; E is the mid-point of side CD and F is the mid-point of AE

To prove: Area of parallelogram ABCD = 8 × Area of ΔAFD.

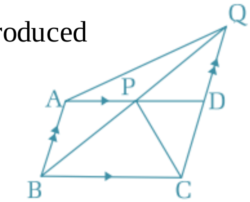
Construction: A and C are joined.

Proof:

S.N.	Statements	S.N.	Reasons
1.	$\Delta AFD = \frac{1}{2}\Delta AED$	1.	The median FD bisects the ΔAED.
2.	$\Delta AED = \frac{1}{2}\Delta ACD$	2.	The median AE bisects the ΔACD.
3.	$\Delta ACD = \frac{1}{2}$ ▱ABCD	3.	The diagonal AC bisects the parallelogram ABCD.
4.	$\Delta AFD = \frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}$ ▱ABCD	4.	From statements (1), (2) and (3).
5.	▱ABCD = 8ΔAFD	5.	From statement (4)
Proved			

QUESTION 22

In quadrilateral ABCD; AB//CD and BC//AD. P is any point on the side AD and BP is produced to meet CD produced at Q. Prove that the area of triangles APQ and PDC are equal.



Solution

Given: In quadrilateral ABCD; AB//CD and BC//AD. P is any point on the side AD.

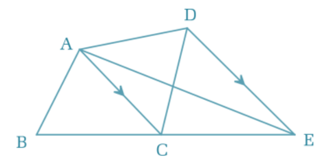
To prove: Area of ΔAPQ = Area of ΔPDC

Proof:

S.N.	Statements	S.N.	Reasons
1.	$\Delta PBC = \frac{1}{2}$ ▱ABCD	1.	Both are standing on same base BC and between AD//BC.
2.	$\Delta PAB + \Delta PDC = \frac{1}{2}$ ▱ABCD	2.	Remaining parts of parallelogram ABCD.
3.	$\Delta ABQ = \frac{1}{2}$ ▱ABCD	3.	Both are standing on same base AB and between AB//QC.
4.	ΔPAB + ΔPDC = ΔABQ	4.	From statements (2) and (3)
5.	ΔPAB + ΔPDC = ΔAPQ + ΔPAB ∴ ΔPDC = ΔAPQ	5.	ΔABQ = ΔAPQ + ΔPAB
Proved			

QUESTION 23

In a quadrilateral ABCD, side BC is extended to point E in such a way that AC // DE. Prove that: area of ΔABE = area of quad. ABCD



Solution

Given: AC // DE

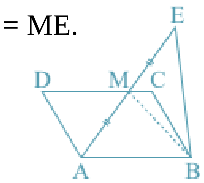
To prove: area of ΔABE = area of quad. ABCD

Proof:

S.N.	Statements	S.N.	Reasons
1.	ΔDAC = ΔEAC	1.	Both are standing on AC and between DE//AC.
2.	ΔDAC + ΔABC = ΔEAC + ΔABC	2.	Adding ΔABC on both sides of statement (1)
3.	ΔABE = Quad. ABCD	3.	By whole part axiom
Proved			

QUESTION 24

In the figure, M is any point of side DC of ▱ABCD and AM is produced to E such that AM = ME. Prove that the area of ΔABE = area of parallelogram ABCD.



Solution

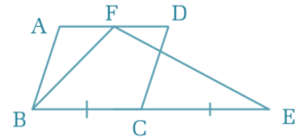
Given: M is any point of side DC of ▱ABCD and AM = ME.

To prove: area of ΔABE = area of ▱ABCD

Proof:

S.N.	Statements	S.N.	Reasons
1.	$\Delta MAB = \frac{1}{2}$ ▱ABCD	1.	Both are standing on AB and between DC//AB.
2.	$\Delta MAB = \frac{1}{2}\Delta ABE$	2.	Median MB bisects ΔABE
3.	ΔABE = ▱ABCD	3.	From statements (1) and (2)

QUESTION 25



In the given figure, side BC of parallelogram ABCD is extended to a point M such that BC = CE. Write with reason that ΔBEF and parallelogram ABCD are equal in area.

Solution

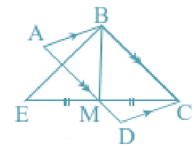
Given: BC of parallelogram ABCD is extended to a point M such that BC = CE.

To prove: area of ΔABE = area of ▱ABCD

Proof:

S.N.	Statements	S.N.	Reasons
1.	ΔBEF = ▱ABCD	1.	The base of triangle is twice the base of parallelogram and they are lying between AD//BE.
Proved			

QUESTION 26



ABCD is a parallelogram; M is the mid-point of the side EC of triangle BEC.
Prove that: Area of ΔEBM = Area of Δ ADC.

Solution

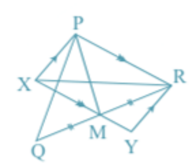
Given: ABCD is a parallelogram; EM = MC

To prove: Area of ΔEBM = Area of Δ ADC

Proof:

S.N.	Statements	S.N.	Reasons
1.	ΔEBM = ΔCBM	1.	Median BM bisects ΔBEC
2.	ΔCBM = $\frac{1}{2}$ ▱ABCD	2.	Both are standing on BC and between AD//BC.
3.	ΔADC = $\frac{1}{2}$ ▱ABCD	3.	Diagonal AC bisects ▱ABCD.
4.	ΔEBM = Δ ADC	4.	From statements (1), (2) and (3).
Proved			

QUESTION 27



M is the mid-point of the side QR of ΔPQR. If PX//RY and PR//XY, prove that ΔXYR = $\frac{1}{2}$ Δ PQR.

Solution

Given: QM = MR, PX//RY and PR//XY

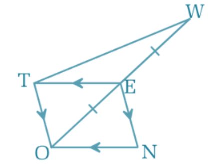
To prove: ΔXYR = $\frac{1}{2}$ Δ PQR

Proof:

S.N.	Statements	S.N.	Reasons
1.	ΔXYR = $\frac{1}{2}$ ▱PXYR	1.	Diagonal XR bisects parallelogram PXYR.
2.	ΔPMR = $\frac{1}{2}$ ▱PXYR	2.	Both are standing on PR and between XY//PR.
3.	ΔPMR = $\frac{1}{2}$ ΔPQR	3.	Median PM bisects ΔPQR
4.	ΔXYR = $\frac{1}{2}$ Δ PQR	4.	From statements (1), (2) and (3).

QUESTION 28

In the given figure; TONE is a parallelogram and E is the mid-point of side OW of triangle TWO. Prove that: ΔTWO = 2ΔONE.



Solution

Given: TONE is a parallelogram and OE = EW

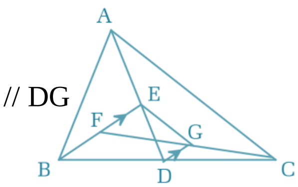
To prove: ΔTWO = 2ΔONE

Proof:

S.N.	Statements	S.N.	Reasons
1.	ΔTWO = 2ΔTOE	1.	Median TE bisects ΔTWO
2.	ΔTOE = ΔONE	2.	Diagonal OE bisects ▱TONE.
3.	ΔTWO = 2ΔONE	3.	From statements (1) and (2).
Proved			

QUESTION 29

In the given ΔABC; D, E and F are the mid-points of BC, AD and BE respectively where BE // DG. Prove: ΔABC = 8ΔEFG



Solution

Given: In ΔABC; D, E and F are the mid-points of BC, AD and BE respectively, BE // DG

To prove: ΔABC = 8ΔEFG

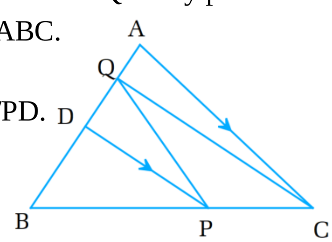
Construction: Join F and D

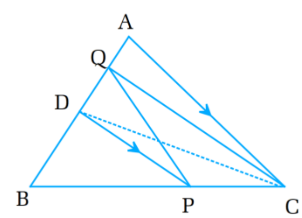
Proof:

S.N.	Statements	S.N.	Reasons
1.	ΔABC = 2ΔABD	1.	Median AD bisects ΔABC.
2.	ΔABD = 2ΔBED	2.	Median BE bisects ΔABD.
3.	ΔBED = 2ΔDEF	3.	Median FD bisects ΔBED.
4.	ΔDEF = ΔEFG	4.	Both are standing on FE and between DG // FE
5.	ΔABC = 2 × 2 × 2 × ΔEFG = 8ΔEFG	5.	From statements (1), (2), (3) and (4)
Proved			

QUESTION 30

In ΔABC; D is the mid-point of side AB. If P is any point on BC and Q is any point on AD such that CQ//PD, prove that the area of ΔBPQ is half of the area of ΔABC.





Solution

Given: In ΔABC; D is the mid-point of side AB, CQ//PD.

To prove: Area of ΔBPQ = $\frac{1}{2}$ Area of ΔABC

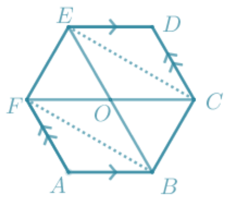
Construction: D and C are joined.

Proof:

S.N.	Statements	S.N.	Reasons
1.	ΔQDP = ΔCDP	1.	Both are standing on DP and between QC//DP.
2.	ΔQDP +ΔBPD = ΔCDP +ΔBPD	2.	Adding ΔBPD on both sides of statement (1)
3.	ΔBPQ = ΔBDC	3.	By whole part axiom
4.	ΔBDC = $\frac{1}{2}$ ΔABC	4.	The median DC bisects the ΔABC.
5.	ΔBPQ = $\frac{1}{2}$ ΔABC	5.	From statements (3) and (4).

QUESTION 31

In hexagon ABCDEF; AB // FC // ED and AF // BE // CD. If the diagonals BE and CF intersect at O such that parallelograms ABOF and OCDE are equal in area, prove that BC // FE.



Solution

Given: AB//FC//ED and AF//BE//CD, diagonals BE and CF intersect at O.

To prove: BC//FE

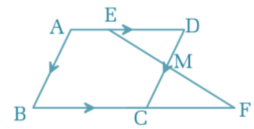
Construction: B, F and C, E are joined.

Proof:

S.N.	Statements	S.N.	Reasons
1.	$\square$ABOF = $\square$OCDE	1.	Given
2.	$\Delta BOF = \frac{1}{2}\square ABOF$	2.	The diagonals bisect the parallelogram
3.	$\Delta COE = \frac{1}{2}\square OCDE$	3.	The diagonals bisect the parallelogram
4.	ΔBOF = ΔCOE	4.	From statements (1) and (2)
5.	ΔFBC = ΔEBC	5.	Adding ΔBOC in statement (3)
6.	BC//FE	6.	From statement (4), Δ^s on the same base BC have equal areas.
Proved			

QUESTION 32

In the parallelogram ABCD given alongside, M is the mid-point of CD. Prove that: area of parallelogram ABCD and trapezium ABFE are equal in area.



Solution

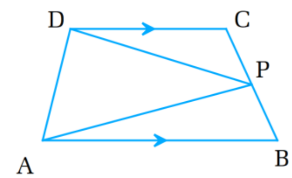
Given: ABCD is a parallelogram; M is the mid-point of CD

To prove: Area of $\square$ABCD = Area of trapezium ABFE

Proof:

S.N.	Statements	S.N.	Reasons
1.	In ΔMED and ΔMCF (i) $\angle$MED = $\angle$CFM (A) (ii) $\angle$EDM = $\angle$MCF (A) (iii) DM = CM (S)	1.	(i) ED // CF, alternate angles (ii) ED // CF, alternate angles (iii) Given
2.	ΔMED $\cong$ ΔMCF	2.	By A.A.S. axiom
3.	ΔMED = ΔMCF	3.	The areas of congruent triangles are equal.
4.	ΔMED + Pent. ABCME = ΔMCF + Pent. ABCME	4.	Adding Pentagon ABCME in (3)
5.	$\square$ABCD = Trapezium ABFE	5.	By whole part axiom
Proved			

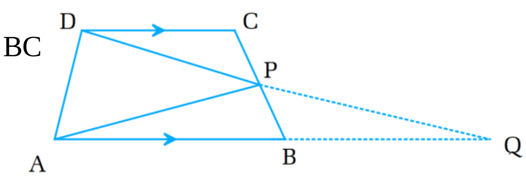
QUESTION 33



In the trapezium ABCD, AB // DC and P is the midpoint of BC.

Prove that $\Delta APD = \frac{1}{2}$ trap. ABCD

Solution



Given: In the trapezium ABCD, AB // DC and P is the midpoint of BC

To prove: $\Delta APD = \frac{1}{2}$ trap. ABCD

Construction: Produce DP and AB to meet at Q.

Proof:

S.N.	Statements	S.N.	Reasons
1.	In ΔDCP and ΔPBQ (i) ∠PDC = ∠PQB (A) (ii) ∠DCP = ∠PBQ (A) (iii) CP = BP (S)	1.	 (i) DC // AQ, alternate angles (ii) DC // AQ, alternate angles (iii) Given
2.	ΔDCP ≅ ΔPBQ	2.	By A.A.S. axiom
3.	ΔDCP = ΔPBQ	3.	The areas of congruent triangles are equal.
4.	ΔAPQ = ΔAPB + ΔPBQ	4.	By whole part axiom
5.	ΔAPQ = ΔAPB + ΔDCP	5.	From statements (3) and (4)
6.	ΔAPD = ΔAPQ	6.	Median AP bisects ΔDAQ
7.	ΔAPD = ΔAPB + ΔDCP	7.	From statements (5) and (6)
8.	ΔAPD + ΔAPB + ΔDCP = Trap. ABCD	8.	By whole part axiom
9.	2ΔAPD = Trap. ABCD $\therefore$ ΔAPD = $\frac{1}{2}$ trap. ABCD	9.	From statements (7) and (8)
Proved			