

## SEE MODEL QUESTION SET

## Optional-I (Mathematics)

Time: 3 hours

Max. Marks: 100

Attempt all the questions.

## Group-A (10 × 1 = 10)

- Write down the period of  $f(x) = \sin x$ .
  - What is the geometric mean between the numbers 'p' and 'q'?
- Write down the condition under which the function  $f(x)$  is continuous at  $x = a$ .
  - Find the determinant of  $A = \begin{pmatrix} m & t \\ h & a \end{pmatrix}$
- If  $\theta$  is the angle between two straight lines having slopes  $m_1$  and  $m_2$ , find the value of  $\tan \theta$ .
  - Which geometrical figure will form if a plane intersects a cone parallel to its base?
- Write down the formula of  $\cos 2\alpha$  in terms of  $\sin \alpha$ .
  - Define angle of elevation.
- Write down the condition in terms of dot product under which the vectors  $\vec{a}$  and  $\vec{b}$  are orthogonal to each other.
  - To what transformation is the matrix  $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$  associated?

## Group-B (13 × 2 = 26)

- State remainder theorem. If  $p(x) = 4x^3 + 5x^2 - 6x + 7$  is a dividend and  $(x + 2)$  a divisor, what is the remainder?
  - If the polynomial  $4x^3 - 3x^2 + 2bx + b - 11$  has a factor  $(x - 3)$ , find the value of  $b$ .

- Find the points of intersection of the curve  $f(x) = x^2 - 2$  and  $f(x) = 7$ .

- If  $M = \begin{pmatrix} 2 & 5 \\ 1 & 3 \end{pmatrix}$  and  $N = \begin{pmatrix} -4 & -6 \\ 3 & 2 \end{pmatrix}$ , find  $|MN|$ .
  - By using Cramer's rule, find the value of  $x$  from the given linear equations  $3x - y = 7$  and  $x + 2y = 4$ .
- Prove that the lines  $y + (2 - \sqrt{3})x = 7$  and  $y - (2 + \sqrt{3})x = 4$  are orthogonal to each other.
  - Find the separate equation of lines represented by the equation  $(x - 2) - y(y - 2) = 0$ .
- Express  $\frac{\sin \frac{\alpha}{2} + \sin \alpha}{1 + \cos \frac{\alpha}{2} + \cos \alpha}$  in terms of sub-multiple of tangent.
  - Find the value of  $\sin 75^\circ + \sin 15^\circ$ .
  - Solve:  $\sec^2 \theta = 2 \tan^2 \theta$  ( $0^\circ \leq \theta \leq 90^\circ$ )
- If  $\vec{a} + 2\vec{b}$  and  $5\vec{a} - 4\vec{b}$  are perpendicular to each other, and  $\vec{a}$  and  $\vec{b}$  are unit vectors, find the angle between  $\vec{a}$  and  $\vec{b}$ .
  - If the position vectors of M and N are  $7\vec{i} + 2\vec{j}$  and  $\vec{i} + 4\vec{j}$ , find the position vector of a point P such that  $\vec{MP} = \vec{PN}$ .
  - In a continuous series, the quartile deviation and its coefficient are 25 and 0.5 respectively then find the upper quartile.

## Group-C (11 × 4 = 44)

- Two real-valued functions  $f$  and  $g$  are defined by  $f(x) = 3x - 11$ ,  $g(x) = \frac{2x - 3}{5}$ . If  $ff(x) = g^{-1}(x)$ , find the value of  $x$ .
- Optimize objective function  $P = 2x + 3y$  under the given constraints:  $x + 2y \leq 10$ ,  $2x + y \leq 14$ ,  $x \geq 0$ ,  $y \geq 0$

13. For a real valued function  $f(x) = 3x + 1$ .
- Find the values of  $f(x)$  at  $x = 1.9, 1.99, 1.999, 2.1, 2.01, 2.001$ .
  - What are the values of  $\lim_{x \rightarrow 2^-} f(x)$ ,  $\lim_{x \rightarrow 2^+} f(x)$  and  $f(2)$ ?
  - Is this function continuous at  $x = 2$ ?
14. By using matrix method, solve the following system of equations:  
 $10x + y = 3(x + y)$ ,  $10y + x + 9 = 10x + y$
15. Find the single equation of pair of straight lines passing through the point  $(1, 0)$  and perpendicular to the pair of straight lines represented by  $x^2 + 3xy - 4y^2 = 0$ .
16. Prove that:  $\cos \frac{\pi^c}{7} \cdot \cos \frac{2\pi^c}{7} \cdot \cos \frac{3\pi^c}{7} = \frac{1}{8}$
17. If A, B and C are the angles of a triangle ABC, prove that  $\sin^2 A + \sin^2 B + \sin^2 C = 2(1 + 2\cos A \cdot \cos B \cdot \cos C)$
18. The angle of elevation of the top of an incomplete tower from a distance of 100 m is  $45^\circ$ . What height should it be raised so that the angle of elevation of the may change to  $60^\circ$ ?
19. Find the inverse of the point  $(4, 5)$  with respect to the circle having equation  $x^2 + y^2 - 4x - 6y = 3$ .
20. The table given below represents the number of people infected from Corona virus from a community. Calculate the mean deviation and its coefficient of the data from median.

Age (in years)	0-10	10-20	20-30	30-40	40-50
No. of patients	3	5	7	3	4

21. Calculate the standard deviation and coefficient of variation from the data given below.

Marks obtained	30-40	40-50	50-60	60-70	70-80
No. of students	2	3	6	5	4

**Group-D****(4 × 5 = 20)**

22. The IQ scores of 10 students in a test are as the rule that the score of second student is the double of score of first student. The score of third student is double the score of second and so on.
- If the score of the first student is 2, find the score of 10<sup>th</sup> student.
  - Find total score of all 10 students.
23. A  $(-2, 0)$ , B  $(2, 4)$  and C  $(3, -1)$  are vertices of  $\Delta ABC$ .
- Find the equation of straight line which passes through the centroid of the  $\Delta ABC$  and parallel to side BC.
  - Find the equation of circle in which AB is a diameter.
24. Prove by vector method that the diagonals of rectangle are equal.
25. P  $(2, 2)$ , Q  $(1, -1)$  and R  $(3, 0)$  are vertices of  $\Delta PQR$ . If  $r_1$  represents the reflection about x-axis and  $r_2$  represents the rotation about origin through  $+90^\circ$ , find the rule that is equivalent to  $r_2 \circ r_1$ . Transform  $\Delta PQR$  by  $r_2 \circ r_1$  and represent the object and image in same graph paper.

**The End**