

Class: IX

Maximum Marks: 100

Subject: Optional-I (Mathematics)

Time: 3 hours

Candidates are required to answer in their own words as far as practicable. Credit shall be given to originality in expression, creativity and neatness in hand, not to rote learning.

Attempt all the questions.

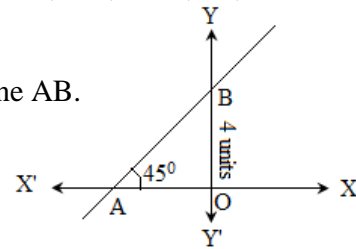
Group-A (10 × 1 = 10)

1. (a) Define Cartesian product of any two non-empty sets.
(b) What is the degree of the polynomial $p(x) = x^3 + 2x(x^2 - x) + 5x^2 - 3x - 1$?
2. (a) A glass is full of 200 ml milk. If a boy drinks 100 ml milk in the first drink, 50 ml in the second drink, 25 ml in the third drink and so on, what is the limiting amount of milk in the glass at last?
(b) Write down the type of the matrix $\begin{pmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{pmatrix}$.
3. (a) Write the co-ordinates of the centroid of a triangle whose vertices are (x_1, y_1) , (x_2, y_2) and (x_3, y_3) .
(b) Define locus of a moving point.
4. (a) If an arc of length l units subtends an angle of θ° at the centre of circle with radius r unit, what is the relation among l , θ° and r ?
(b) Write down the formula of $\tan(180^\circ - A)$
5. (a) Define unit vector.
(b) List out the isometric transformations.

Group-B (13 × 2 = 26)

6. (a) If $f: A \rightarrow B$ is defined by $f(x) = 2x - 1$ and $A = \{1, 2, 3\}$, find the range of the function.
(b) If $p(x) = 4x^3 - x + 2$ and $q(x) = 3x^2 + 2x - 1$, find $p(x) \cdot q(x)$. Also, find its degree.
(c) Find the value of $\sum_{n=1}^4 (-1)^{n+1} \cdot 2^{n-1}$ by expanding.
7. (a) Define identity matrix. For what values of p , q , r and s , the matrix $\begin{pmatrix} p-1 & q-2 \\ r-3 & s-4 \end{pmatrix}$ will be an identity matrix? Find.

- (b) For the matrix $A = \begin{pmatrix} 2 & 1 \\ -2 & -1 \end{pmatrix}$, prove that $A^2 = A$.
8. (a) In what ratio does X-axis divide the line joining the points (2, -3) and (5, 8)? Find.



- (b) From the figure given alongside, find the equation of line AB.

9. (a) Prove that: $\sqrt{\frac{1-\sin\theta}{1+\sin\theta}} = \sec\theta - \tan\theta$

(b) Find the value of $\cos\frac{\pi^c}{8} + \cos\frac{3\pi^c}{8} + \cos\frac{5\pi^c}{8} + \cos\frac{7\pi^c}{8}$

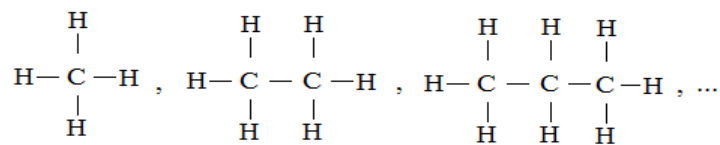
(c) If $\alpha + \beta = \frac{\pi^c}{4}$, prove that: $(1 + \tan\alpha)(1 + \tan\beta) = 2$.

10. (a) If \overline{AB} displaces the point A (6, 1) to the point B (3, 4), find its magnitude and the direction.
- (b) For what value of k, the vectors $\vec{a} = 4\vec{i} + 5\vec{j}$ and $\vec{b} = 8\vec{i} + k\vec{j}$ are parallel to each other?
- (c) Find the values of D_6 and P_{37} from the following data:
40, 20, 18, 36, 24, 44, 28, 38, 50, 42, 30, 22, 16, 34, 46, 60, 58, 70, 64

Group-C

(11 × 4 = 44)

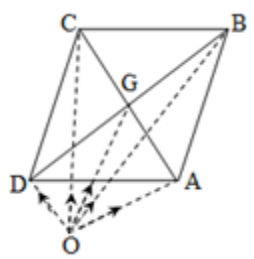
11. If $f(x) = mx + c$, $f(2) = 7$ and $f(3) = 10$, find the value of m and c. Also, find $f(x)$.
12. Each of the members of a family of chemical compounds contains Carbon atoms (C) and Hydrogen atoms (H). Some of the members of a family are represented in the diagrams below.



- (i) Draw next diagram in the same pattern.
- (ii) Find the nth term formula of the sequence of numbers of Hydrogen atoms (H).
- (iii) Find the 10th term of the sequence of the numbers of Hydrogen atoms (H).
13. By tabulating values for x, find the value of $\lim_{x \rightarrow 2} \frac{x^2 - 4}{x - 2}$
14. If $M = \begin{pmatrix} 2 & 4 & 5 \\ 6 & 3 & 1 \end{pmatrix}$ and $N = \begin{pmatrix} -1 & 5 & 7 \\ 8 & 4 & -2 \end{pmatrix}$ are two matrices, verify that $(M + N)^T = M^T + N^T$
15. The coordinates of A, B and C are (6, 3), (-3, 5) and (4, -2) respectively and P (x, y) is any arbitrary point. Show that: $\frac{\text{Area of } \Delta PBC}{\text{Area of } \Delta ABC} = \frac{x + y - 2}{7}$

16. Three angles of triangle are $\left(\frac{2x}{3}\right)^g$, $\left(\frac{3x}{2}\right)^o$ and $\left(\frac{\pi x}{75}\right)^c$. Express all angles in degree as well as circular measures.
17. Prove that: $(3 - 4\sin^2 A)(1 - 3\tan^2 A) = (3 - \tan^2 A)(4\cos^2 A - 3)$
18. Solve for x: $x\cot\theta \cdot \tan(90^\circ + \theta) = \tan(90^\circ + \theta) \cdot \cot(180^\circ - \theta) + x\sec(90^\circ + \theta) \cdot \operatorname{cosec}\theta$
19. Reflect a trapezium PQRS with vertices P (-3, 5), Q (1, 5), R (3, 1) and S (-2, 1) about the line $x = 1$ to get the trapezium P'Q'R'S'. Find the vertices of trapezium P'Q'R'S'. Also, present both the trapeziums on the same graph paper.
20. Compute the mean deviation from median of the data given below.
- | | | | | | | |
|-----------------|----|----|----|----|----|----|
| Marks obtained | 40 | 50 | 60 | 70 | 80 | 90 |
| No. of students | 2 | 3 | 5 | 2 | 4 | 4 |
21. Compute the standard deviation of the data given below.
10, 20, 30, 40, 50

Group-D **(4 × 5 = 20)**

22. Let $A = \{1, 2, 3\}$, express the relation $R = \{(x, y) : x + y < 4\}$ on A by
- (i) Set of ordered pairs
 - (ii) Tabulation methods
 - (iii) Arrow diagram method
 - (iv) Graphical method
23. Find the equation of straight lines which pass through the point (3, 4) and have the intercepts on the axes such that their sum is 14 units.
24. In the adjoining figure, G is the point of intersection of diagonals AC and BD of the parallelogram. Prove that:
 $\vec{OA} + \vec{OB} + \vec{OC} + \vec{OD} = 4\vec{OG}$
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25. $\triangle ABC$ with vertices A (4, 0), B (4, 2) and C (1, 3) mapped into the $\triangle A'B'C'$ with vertices A' (6, -1), B' (6, 3) and C' (0, 5) by an enlargement with centre (a, b) and scale factor k. Find the centre (a, b) and scale factor k of the enlargement.

The End