Symbol no. ……………………….

Set-B

Second Term Exam 2081 (2024)

Optional- I (Mathematics)

Class: 10	Time: 3 hrs	F.M:75

Attempt **all** the questions.

Group 'A' **($10 \times 1= 10$)**

1. What is period of the function $f(x) = \cos x$?
2. Write the equation of boundary line of the inequality $ax + by \leq c$.
3. Write the left hand limit of $f(x)$ at $x = a$ in mathematical statement.
4. If AB = BA= I then what types of matrices are A and B?
5. What is the angle between two straight lines $x = 0$ and $y = 0$?
6. The semi vertical angle of a cone is α and the angle made by the plane with the axis of cone is θ. If $\alpha = \theta$, what is the name of the conic section formed by the intersection of a plane surface and cone?
7. Express cosA in term of $\tan\frac{A}{2}$.
8. If $\cos(360° – \theta) = \sin\theta$ then find the value of $\theta < 90°$.
9. If $\vec{i}$ and $\vec{j}$ and are unit vectors along X- axis and Y-axis respectively, what is the value of $\vec{i}.\vec{j}$?
10. Define inversion point.

Group 'B' **($8 \times 2 = 16$)**

11. When a polynomial $f(x) = x^3 – 3x^2 + kx + k$ is divided by $g(x) = x + 2$ then the remainder 5 is obtained. Using remainder theorem, find the value of k.
12. A polynomial is expressed as $3x^3 + x^2 + x + 1 = (x –1).Q(x) + R$, find the values of Q(x) and R.
13. If $A = \begin{pmatrix} p & 3 \\ 2 & 4 \end{pmatrix}$ and $B = \begin{pmatrix} 4 & 3 \\ 5 & 2 \end{pmatrix}$ such that AB is a singular matrix, find the value of p.
14. Find the acute angle between two straight lines $y – (2+\sqrt{3}) x – 5= 0$ and $y – (2 – \sqrt{3}) x – 2= 0$.
15. If $\cos\frac{A}{3} = \frac{3}{5}$, find the value of cosA.
16. Solve: $\text{cosec}\theta = \sec 45°.\tan 45°$. $[0° \leq \theta \leq 90°]$
17. If $\vec{a} = p\vec{i} + 3\vec{j}$, $\vec{b} = 5\vec{i} – \vec{j}$, and $\vec{a}.\vec{b} = 7$, find the value of p.
18. The inter-quartile range of a continuous data is 30 and upper quartile is 40. What is the value of quartile deviation and coefficient of quartile deviation?

Group 'C' **($11 \times 3 = 3$)**

19. The image of a function f is 4 times of its pre-image plus 5. The image of composite function of g and f is 8 times of its pre image plus 13. If $g_of(x) = 28$, find is the value of x.

20. Find the maximum value of $P = 3x + 2y$ under the conditions: $x + y \geq 0$, $x - y \leq 0$, $x \geq -1$, $y \leq 2$.

21. Examine the continuity or discontinuity of $f(x) = \begin{cases} 4x - 1 & \text{for } x < 1 \\ 7x & \text{for } x \geq 1 \end{cases}$ at $x = 1$ by calculating left hand limit, right hand limit and functional values.

22. If the men are sitting on the chair in a hall. The sum of legs of men and legs of chairs is 102 and 3 chairs are empty. Find the number of men and number of chairs by making linear equation and by using matrix method.

23. Find the equation of the straight lines passing through the point (– 1, 2) and making an angle of 45° with the line $x - y = 1$.

24. Prove that: $(\cos\alpha - \cos\beta)^2 + (\sin\alpha + \sin\beta)^2 = 4\sin^2\left(\frac{\alpha + \beta}{2}\right)$

25. If A + B + C = 90° then prove that: sinA.cosA + sinB.cosB + sinC.cosC = 2cosA.cosB.cosC

26. The angle of elevation of the top of the tower observed from 27 m and 75 m away from its foot on the same side are found to be complementary. Find the height of the tower.

27. ΔABC with the vertices A(3, 6), B(4, 2) and C(3, 3) is mapped onto ΔA'B'C' such that A' (6, –3), B'(2, –4) and C'(3, –3). Find the 2 ×2 transformation matrix that represents this transformation. Also, which is the single transformation for this mapping?

28. Find the mean deviation from mean and its coefficient.

Age (in years)	0-4	4-8	8-12	12-16	16-20
No. of boys (f)	12	8	10	6	4

29. Find the standard deviation from the given data.

Marks	0-50	0-40	0-30	0-20	0-10
Frequency	50	44	28	13	5

Group 'D' (4 × 4 = 16)

30. If a, b and c are in A.P and x, y and z are in G.P. Prove that: $x^{b-c} \times y^{c-a} \times z^{a-b} = (3x)^\circ$

31. Find the equation of a circle having the center at (–1, – 2) and which has the same radius as the circle $x^2 + y^2 + 3x - 7y - 2 = 0$.

32. Prove by vector method: The diagonals of rhombus bisect to each other at right angle.

33. A quadrilateral having vertices P(0, 3), Q(–6, 1), R(–6, 5) and S(–1, 5) is reflected in the line $y = x$ followed by rotation about origin through +90°. Find the coordinates of image of quadrilateral PQRS. Also present the object and final image on same graph paper.

The End

Set-B

COMPLETE SOLUTION

Group 'A' **($10 \times 1 = 10$)**

1. ***Solution:***

Here, the period of the function $f(x) = \cos x$ is $2\pi^c$.

2. ***Solution:***

Here, the equation of boundary line of the inequality $ax + by \le c$ is $ax + by = c$.

3. ***Solution:***

Here, the left hand limit of $f(x)$ at $x = a$ in mathematical statement is $\lim\limits_{x \to a^-} f(x)$.

4. ***Solution:***

Here, AB = BA= I. Thus, A and B are inverse matrices to each other.

5. ***Solution:***

Here, the angle between two straight lines $x = 0$ and $y = 0$ i.e. the angle between x-axis and y-axis is 90^o.

6. ***Solution:***

Here, if the semi vertical angle of a cone α and the angle made by the plane with the axis of cone θ are equal i.e., $\theta = \alpha$ then the conic so formed is parabola.

7. ***Solution:***

Here, $\cos A = \dfrac{1 - \tan^2\frac{A}{2}}{1 + \tan^2\frac{A}{2}}$

8. ***Solution:***

Here,

$\cos(360° - \theta) = \sin\theta$

or, $\cos\theta = \sin\theta$

or, $\dfrac{\cos\theta}{\cos\theta} = \dfrac{\sin\theta}{\cos\theta}$

or, $1 = \tan\theta$

or, $\tan 45^o = \tan\theta$

Hence, $\theta = 45^o$

9. ***Solution:***

Here, $\vec{i}$ and $\vec{j}$ and are unit vectors along X- axis and Y-axis respectively. Thus, the value of $\vec{i}.\vec{j} = 0$.

10. ***Solution:***

Here, the image point P′ of a point P with respect to a circle C with centre O and radius ‘r’ is called inversion point if $OP \times OP' = r^2$.

Group ‘B’ $(8 \times 2 = 16)$

11. ***Solution:***

Here, $f(x) = x^3 - 3x^2 + kx + k$ and divisor, $g(x) = x + 2$ and remainder = 5.

Comparing $x + 2$ with $x - a$, we get $a = -2$

Now, remainder (R) = $f(a)$

or, $5 = (-2)^3 - 3(-2)^2 + k(-2) + k$

or, $5 = -8 - 3(4) - 2k + k$

or, $5 = -8 - 12 - k$

or, $k = -20 - 5$

or, $k = -25$

Hence, the required value of k is = – 25.

12. ***Solution:***

Here, $3x^3 + x^2 + x + 1 = (x - 1).Q(x) + R$

Comparing $x - 1$ with $x - a$, we get $a = 1$

Now, using synthetic division method, we get

1	3	1	1	1
		3	4	5
	3	4	5	6

Hence, quotient, $Q(x) = 3x^2 + 4x + 5$ and remainder (R) = 6.

13. ***Solution:***

Here, $A = \begin{pmatrix} p & 3 \\ 2 & 4 \end{pmatrix}$ and $B = \begin{pmatrix} 4 & 3 \\ 5 & 2 \end{pmatrix}$

Now, $AB = \begin{pmatrix} p & 3 \\ 2 & 4 \end{pmatrix}\begin{pmatrix} 4 & 3 \\ 5 & 2 \end{pmatrix}$

$= \begin{pmatrix} 4p + 15 & 3p + 6 \\ 8 + 20 & 6 + 8 \end{pmatrix}$

$= \begin{pmatrix} 4p + 15 & 3p + 6 \\ 28 & 14 \end{pmatrix}$

Since, AB is a singular matrix.

So, $\begin{vmatrix} 4p + 15 & 3p + 6 \\ 28 & 14 \end{vmatrix} = 0$

or, $14(4p + 15) - 28(3p + 6) = 0$

or, $56p + 210 - 84p - 168 = 0$

or, $42 - 28p = 0$

or, $42 = 28p$

or, $p = \frac{42}{28}$

or, $p = \frac{3}{2}$

Hence, the required value of p is $\frac{3}{2}$.

14. ***Solution:***

Here, the equations of lines are;

$y - (2+\sqrt{3})x - 5 = 0$ … (i)

$y - (2-\sqrt{3})x - 2 = 0$ … (ii)

Now, slope of line (i) is $m_1 = -\frac{\text{Coefficient of } x}{\text{Coefficient of } y}$

$= -\frac{-(2+\sqrt{3})}{1}$

$= (2+\sqrt{3})$

Also, slope of line (ii) is $m_2 = -\frac{\text{Coefficient of } x}{\text{Coefficient of } y}$

$= -\frac{-(2-\sqrt{3})}{1}$

$= (2-\sqrt{3})$

Let, θ be the angle of between the lines (i) and (ii).

Then, $\tan\theta = \pm\frac{m_1 - m_2}{1 + m_1.m_2}$

$= \pm\frac{(2+\sqrt{3}) - (2-\sqrt{3})}{1 + (2+\sqrt{3})(2-\sqrt{3})}$

$= \pm\frac{2+\sqrt{3} - 2 + \sqrt{3}}{1 + 2^2 - (\sqrt{3})^2}$

$= \pm\frac{2\sqrt{3}}{1 + 4 - 3}$

$= \pm\sqrt{3}$

For acute angle, taking (+) sign, we get

$\tan\theta = \sqrt{3}$

or, $\tan\theta = \tan 60^o$

or, $\theta = 60^o$

Hence, the required acute angle is 60^o.

15. *Solution:*

Here, $\cos\frac{A}{3} = \frac{3}{5}$

We have, $\cos A = 4\cos^3\frac{A}{3} - 3\cos\frac{A}{3}$

$= 4\left(\frac{3}{5}\right)^3 - 3\times\frac{3}{5}$

$= 4\times\frac{27}{125} - \frac{9}{5}$

$= -\frac{117}{125}$

Hence, the required value of cosA is $-\frac{117}{125}$

16. *Solution:*

Here, $\text{cosec}\theta = \sec45°.\tan45°$ $[0° \le \theta \le 90°]$

or, $\text{cosec}\theta = \sqrt{2}\times 1$

or, $\text{cosec}\theta = \text{cosec}45^o$

or, $\theta = 45^o$

Hence, the required value of θ is 45^o.

17. *Solution:*

Here, $\vec{a} = p\,\vec{i} + 3\vec{j} = \begin{pmatrix} p \\ 3 \end{pmatrix}$

$\vec{b} = 5\,\vec{i} - \vec{j} = \begin{pmatrix} 5 \\ -1 \end{pmatrix}$

Now, $\vec{a}.\vec{b} = 7$

or, $\begin{pmatrix} p \\ 3 \end{pmatrix}.\begin{pmatrix} 5 \\ -1 \end{pmatrix} = 7$

or, $5p - 3 = 7$

or, $5p = 10$

or, $p = 2$

Hence, the required value of p is 2.

18. *Solution:*

Here, the inter-quartile range = 30 i.e., $Q_3 - Q_1 = 30$ and $Q_3 = 40$

We have, $Q_3 - Q_1 = 30$

or, $40 - Q_1 = 30$

or, $10 = Q_1$

Now, quartile deviation (Q.D.) $= \frac{Q_3 - Q_1}{2}$

$= \frac{30}{2}$

$= 15$

Again, coefficient of quartile deviation $= \frac{Q_3 - Q_1}{Q_3 + Q_1}$

$= \frac{40 - 10}{40 + 10}$

$= \frac{30}{50}$

$= 0.6$

Hence, required quartile deviation is 15 and coefficient of quartile deviation is 0.6

Group 'C' (11 × 3 = 3)

19. ***Solution:***

Here, $f(x) = 4x + 5$, $f_o g(x) = 8x + 13$ and $g_o f(x) = 28$

Let, $g(x) = ax + b$

Now, $f_o g(x) = 8x + 13$

or, $f(ax + b) = 8x + 13$

or, $4(ax + b) + 5 = 8x + 13$

or, $4ax + 4b = 8x + 8$

Equating the coefficient of like terms, we get

$4a = 8 \quad \therefore a = 2$

$4b = 8 \quad \therefore b = 2$

Thus, $g(x) = 2x + 2$

Again, $g(4x + 5) = 28$

or, $2(4x + 5) + 2 = 28$

or, $8x + 10 + 2 = 28$

or, $8x = 16$

or, $x = 2$

Hence, the required value of x is 2

20. *Solution:*

Here,

The equation of boundary line corresponding to $x + y \geq 0$ is $x + y = 0$

$\therefore y = -x$

X	1	2	3
Y	–1	–2	–3

Taking (1, 0) as the testing point, we get

$1 + 0 > 0$

or, $1 > 0$ which is true.

Hence the half- plane of $x + y \geq 0$ contains the testing point (1, 0).

Also, the equation of boundary line corresponding to $x - y \leq 0$ is $x - y = 0$

$\therefore y = x$

X	1	2	3
Y	1	2	3

Taking (1, 0) as the testing point, we get

$1 - 0 < 0$

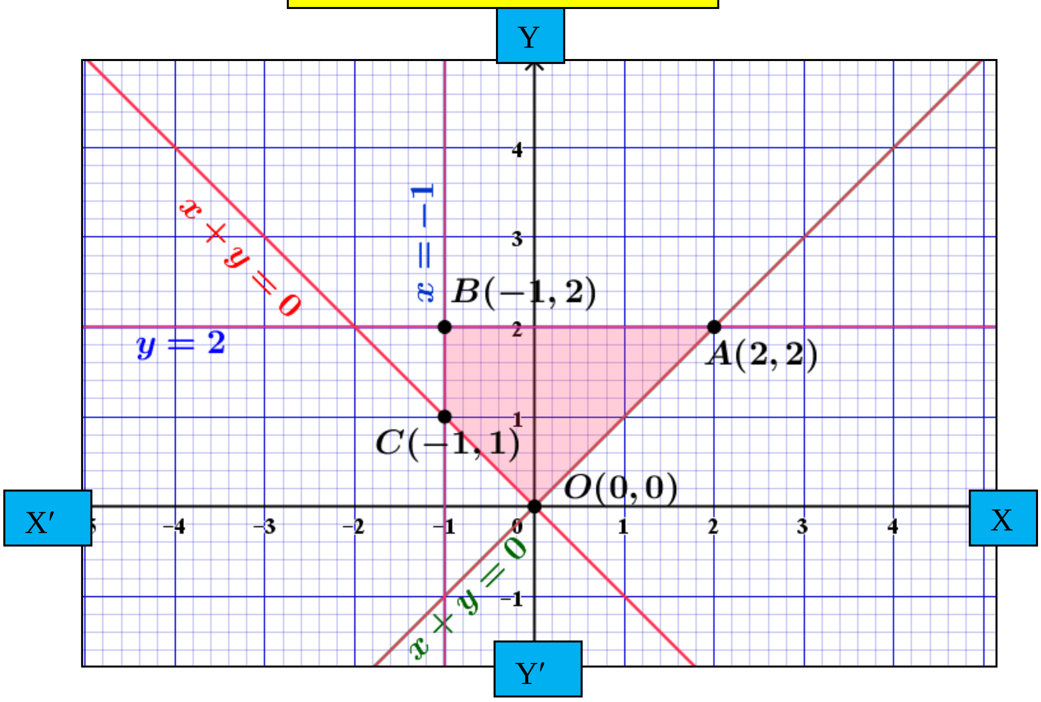
or, $1 < 0$ which is false.

Hence the half- plane of $x - y \leq 0$ does not contain the testing point (1, 0).

For, $x \geq -1$ $\therefore x = \{-1, 0, 1, \ldots\}$ with boundary line $x = -1$

For, $y \leq 2$ $\therefore y = \{\ldots 0, 1, 2\}$ with boundary line $y = 2$

Let, 10 small divisions = 1



From the above graph, the shaded region OABC is the feasible region with vertices O (0, 0), A (2, 2), B (– 1, 2) and C (–1, 1).

Again, tabulating the values of given objective function P = $3x + 2y$ at each vertex of feasible region.

S.N.	Vertex	P = $3x + 2y$	Remarks
1.	O (0, 0)	P = 3.0 + 2.0 = 0	
2.	A (2, 2)	P = 3.2 + 2.2 = 10	Maximum
3.	B (– 1, 2)	P = 3(– 1) + 2.2 = 1	
4.	C (– 1, 1)	P = 3(– 1) + 2.1 = – 1	Minimum

Hence, the maximum value of P $(x, y) = 3x + 2y$ is 10 at A (2, 2)

21. *Solution:*

Here, the given function is $f(x) = \begin{cases} 4x - 1 & \text{for } x < 1 \\ 7x & \text{for } x \geq 1 \end{cases}$ at $x = 1$

Now, for functional value at $x = 1$, $f(x) = 7x$

Thus, $f(1) = 7 \times 1 = 7$

Also, for $x < 1$, $f(x) = 4x - 1$ and taking x = 0.9, 0.99, 0.999, …

x	0.9	0.99	0.999	…	$x \to 1^-$
$f(x) = 4x - 1$	2.6	2.96	2.996	…	$f(x) \to 3$

As x tends to 1 from left, $f(x)$ tends to 3.

Thus, left hand limit at x = 1 is 3 i.e., $\lim_{x \to 1^-} f(x) = 3$

Also, for $x > 1$, $f(x) = 7x$ and taking x = 1.1, 1.01, 1.001, …

x	1.1	1.01	1.001	…	$x \to 1^+$
$f(x) = 2x + 3$	7.7	7.07	7.007	…	$f(x) \to 7$

As x tends to 1 from right, $f(x)$ tends to 7.

Thus, right hand limit at $x = 1$ is 7 i.e., $\lim_{x \to 1^+} f(x) = 7$

Since, $\lim_{x \to 1^+} f(x) = f(1)$ but $\lim_{x \to 1^+} f(x) \neq \lim_{x \to 1^-} f(x)$.

Hence, the given function is discontinuous at $x = 1$.

22. *Solution:*

Let, the number of men be x and the number of chairs be y.

From the first condition; $2x + 4y = 102$ $\quad \therefore x + 2y = 51$ $\quad$ … (i)

From the second condition; $x = y - 3$ $\quad \therefore x - y = -3$ $\quad$ … (ii)

Now, expressing equations (i) and (ii) in matrix form. We get

$$\begin{pmatrix} 1 & 2 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 51 \\ -3 \end{pmatrix}$$

or, $AX = B$ where $A = \begin{pmatrix} 1 & 2 \\ 1 & -1 \end{pmatrix}$, $B = \begin{pmatrix} 51 \\ -3 \end{pmatrix}$ and $X = \begin{pmatrix} x \\ y \end{pmatrix}$

$\therefore \quad X = A^{-1}B$ … (iii)

<u>For A^{-1}</u>

Determinant of A $= \begin{vmatrix} 1 & 2 \\ 1 & -1 \end{vmatrix}$

$= -1 - 2$

$= -3$

Since, $|A| \neq 0$; A^{-1} exists and the given system has a unique solution.

Again, $A^{-1} = \dfrac{1}{|A|}$ Adjoint of A

$= \dfrac{1}{-3}\begin{pmatrix} -1 & -2 \\ -1 & 1 \end{pmatrix}$

Putting the value of A^{-1} in equation (iii), we get

$$\begin{pmatrix} x \\ y \end{pmatrix} = \frac{1}{-3}\begin{pmatrix} -1 & -2 \\ -1 & 1 \end{pmatrix}\begin{pmatrix} 51 \\ -3 \end{pmatrix}$$

or, $\begin{pmatrix} x \\ y \end{pmatrix} = \dfrac{1}{-3}\begin{pmatrix} -51 + 6 \\ -51-3 \end{pmatrix}$

or, $\begin{pmatrix} x \\ y \end{pmatrix} = \dfrac{1}{-3}\begin{pmatrix} -45 \\ -54 \end{pmatrix}$

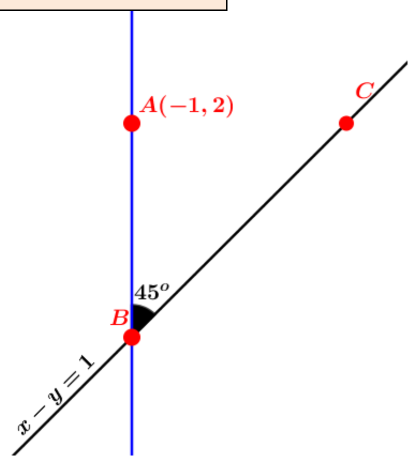
or, $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 15 \\ 18 \end{pmatrix}$

Equating the corresponding elements, we get

$x = 15$ and $y = 18$

Hence, the required number of men is 15 and the number of chairs is 18.

23. ***Solution:***



Here,

The equation of given line BC is $x - y = 1$ … (i)

$\therefore$ Slope $(m_1) = -\dfrac{\text{coefficient of } x}{\text{coefficeint of } y}$

$= -\dfrac{1}{-1}$

$= 1$

Let, m_2 be the slope of the line AB.

$\angle ABC\ (\theta) = 45^o$

We have, $\tan\theta = \pm \frac{m_1 - m_2}{1 + m_1 m_2}$

or, $\tan 45^o = \pm \frac{1 - m_2}{1 + 1.m_2}$

or, $1 = \pm \frac{1 - m_2}{1 + m_2}$

or, $1 + m_2 = \pm (1 - m_2)$

Taking (+) ve sign, we get

$1 + m_2 = 1 - m_2$

or, $2m_2 = 0$

$\therefore m_2 = 0$

Taking (-) ve sign, we get

$1 + m_2 = -(1 - m_2)$

or, $1 + m_2 = -1 + m_2$

or, $0.m_2 = -2$

$\therefore m_2 = -\frac{2}{0}$

Case-I

Slope (m_2) = 0 and passing point = A (– 1, 2) → (x_1, y_1)

Equation of required line AB is given by

$y - y_1 = m_2 (x - x_1)$

or, $y - 2 = 0(x + 1)$

or, $y - 2 = 0$

Case-II

Slope (m_2) = $-\frac{2}{0}$ and passing point = A (– 1, 2) → (x_1, y_1)

Equation of required line AB is given by

$y - y_1 = m_2 (x - x_1)$

or, $y - 2 = -\frac{2}{0}(x + 1)$

or, $0 = x + 1$

or, $x + 1 = 0$

Hence, the equation of line AB is $y - 2 = 0$ or $x + 1 = 0$

24. *Solution:*

Here,

L.H.S. = $(\cos\alpha - \cos\beta)^2 + (\sin\alpha + \sin\beta)^2$

$= \left(2\sin\frac{\alpha+\beta}{2}.\sin\frac{\alpha-\beta}{2}\right)^2 + \left(2\sin\frac{\alpha+\beta}{2}.\cos\frac{\alpha-\beta}{2}\right)^2$

$= 4\sin^2\left(\frac{\alpha+\beta}{2}\right).\sin^2\left(\frac{\alpha-\beta}{2}\right) + 4\sin^2\left(\frac{\alpha+\beta}{2}\right).\cos^2\left(\frac{\alpha-\beta}{2}\right)$

$= 4\sin^2\left(\frac{\alpha+\beta}{2}\right)\left[\sin^2\left(\frac{\alpha-\beta}{2}\right) + \cos^2\left(\frac{\alpha-\beta}{2}\right)\right]$

$= 4\sin^2\left(\frac{\alpha+\beta}{2}\right) \times 1$

$= 4\sin^2\left(\frac{\alpha+\beta}{2}\right)$

= R.H.S.

Hence, proved

25. *Solution:*

Here, A + B + C = 90°

or, A + B = 90° – C

Taking sin and cos on both sides successively, we get

sin(A + B) = sin(90° – C) = cosC

cos(A + B) = cos(90° – C) = sinC

Now, L.H.S. = sinA.cosA + sinB.cosB + sinC.cosC

$= \frac{1}{2}$(2sinA.cosA + 2sinB.cosB) + sinC.cosC

$= \frac{1}{2}$(sin2A + sin2B) + sinC.cosC

$= \frac{1}{2} \times 2\sin\left(\frac{2A+2B}{2}\right).\cos\left(\frac{2A-2B}{2}\right)$ + sinC.cosC

= sin (A + B). cos (A – B) + sinC.cosC

= cosC. cos (A – B) + sinC.cosC

= cosC [cos (A – B) + sinC]

= cosC [cos (A – B) + cos (A + B)]

= cosC (cosA.cosB + sinA.sinB + cosA.cosB – sinA.sinB)

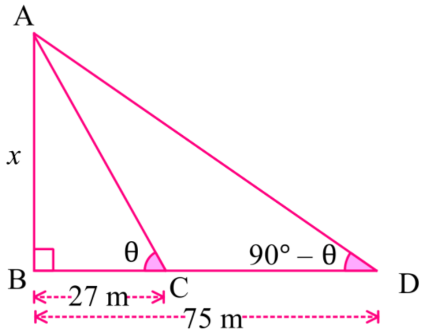
= cosC × 2cosA.cosB

= 2cosA.cosB.cosC

= R.H.S.

Hence, proved

26. *Solution:*



Let, AB be the height of tower, C and D be the observation points. ∠BCA and ∠BDA be the angles of elevation of the top of the tower observed from the places C and D respectively.

Then, BC = 27 m

BD = 75 m

Suppose, ∠BCA =θ, then ∠BDA = 90° – θ

AB = x m (say)

Now,

From right angled triangle ABC; $\tan\theta = \frac{AB}{BC}$

$$\text{or, } \tan\theta = \frac{x}{27} \qquad \ldots\text{(i)}$$

Again,

From right angled triangle ADC; $\tan(90° - \theta) = \frac{AB}{BD}$

$$\text{or, } \cot\theta = \frac{x}{75} \qquad \ldots\text{(ii)}$$

Multiplying (i) and (ii), we get

$$\tan\theta \times \cot\theta = \frac{x}{27} \times \frac{x}{75}$$

$$\text{or, } 1 = \frac{x^2}{2025}$$

$$\text{or, } x^2 = 2025$$

$$\text{or, } x = \sqrt{2025} = 45$$

Hence, the height of the tower is 45 m.

27. *Solution:*

Here,

$$\text{Object} = \Delta ABC = \begin{pmatrix} A & B & C \\ 3 & 4 & 3 \\ 6 & 2 & 3 \end{pmatrix}$$

$$\text{Image} = \Delta A'B'C' = \begin{pmatrix} A' & B' & C' \\ 6 & 2 & 3 \\ -3 & -4 & -3 \end{pmatrix}$$

Let, 2 × 2 transformation matrix be $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$

We have, image = T.M. × Object

$$\text{or, } \begin{pmatrix} 6 & 2 & 3 \\ -3 & -4 & -3 \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} 3 & 4 & 3 \\ 6 & 2 & 3 \end{pmatrix}$$

$$\text{or, } \begin{pmatrix} 6 & 2 & 3 \\ -3 & -4 & -3 \end{pmatrix} = \begin{pmatrix} 3a+6b & 4a+2b & 3a+3b \\ 3c+6d & 4c+2d & 3c+3d \end{pmatrix}$$

Equating the corresponding elements, we get

(i) $3a + 6b = 6 \qquad \therefore a = \frac{6-6b}{3} = 2 - 2b$

(ii) $4a + 2b = 2$

(iii) $3c + 6d = -3 \qquad \therefore c = \frac{-3-6d}{3} = -1 - 2d$

(iv) $4c + 2d = -4$

Now, putting the value of ‘a’ from (i) in (ii), we get

$4(2 - 2b) + 2b = 2$

or, $8 - 8b + 2b = 2$

or, $6 = 6b$

or, $b = 1$

Putting the value of ‘b’ in (i), we get

$a = 2 - 2 \times 1 = 0$

Again, putting the value of ‘c’ from (iii) in (iv), we get

$4(-1 - 2d) + 2d = -4$

or, $-4 - 8d + 2d = -4$

or, $0 = 6d$

or, $d = 0$

Putting the value of ‘d’ in (i), we get

$c = -1 - 2 \times 0 = -1$

Hence, required transformation matrix is $\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$

Again, $\begin{pmatrix} x \\ y \end{pmatrix} \xrightarrow{\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}} \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} y \\ -x \end{pmatrix}$

i.e., $P(x, y) \longrightarrow P'(y, -x)$

Hence, the single transformation for this mapping is the rotation through -90^o about origin.

28. Solution:

Here, computation of the mean deviation from the mean:

Age (in yrs)	No. of boys (f)	m	fm	$\lvert m-\overline{X}\rvert$	$f\lvert m-\overline{X}\rvert$
0-4	12	2	24	6.2	74.4
4-8	8	6	48	2.2	17.6
8-12	10	10	100	1.8	18
12-16	6	14	84	5.8	34.8
16-20	4	18	72	9.8	39.2
	N = 40		$\Sigma fm = 328$		$\Sigma f\lvert m-\overline{X}\rvert = 184$

Now,

Mean $(\overline{X}) = \frac{\Sigma fm}{N} = \frac{328}{40} = 8.2$

Also, M.D. from mean $= \frac{\Sigma f\lvert m-\overline{X}\rvert}{N}$

$= \frac{184}{40}$

$= 4.6$

Again, coefficient of M.D. $= \frac{\text{M.D. from mean}}{\text{Mean}}$

$= \frac{4.6}{8.2}$

$= 0.5609$

Hence, the mean deviation is 4.6 and its coefficient is 0.5609.

29. Solution:

Here, changing the given data in continuous series and calculating standard deviation:

Marks	Frequency (f)	m	fm	m^2	fm^2
0-10	5	5	25	25	125
10-20	13 – 5 = 8	15	120	225	1800
20-30	28 –13 = 15	25	375	625	9375
30-40	44 – 28 = 16	35	560	1225	19600
40-50	50 – 44 = 6	45	270	2025	12150
	N = 50		$\Sigma fm = 1350$		$\Sigma fm^2 = 43050$

Now, S.D. (σ) $= \sqrt{\frac{fm^2}{N} - \left(\frac{fm}{N}\right)^2}$

$= \sqrt{\frac{43050}{50} - \left(\frac{1350}{50}\right)^2}$

$= \sqrt{861 - 729}$

$= \sqrt{132}$

$= 11.489$

Hence, the required standard deviation is 11.489.

Group 'D' (4 × 4 = 16)

30. *Solution:*

Here, a, b and c are in A.P $\therefore b = \frac{a+c}{2}$ or, $a = 2b - c$

and x, y and z are in G.P. $\therefore y = \sqrt{xz}$ or, $y^2 = xz$

L.H.S. $= x^{b-c} \times y^{c-a} \times z^{a-b}$

$= x^{b-c} \times y^{c-(2b-c)} \times z^{2b-c-b}$

$= x^{b-c} \times y^{c-2b+c} \times z^{b-c}$

$= (xz)^{b-c} \times y^{2c-2b}$

$= (y^2)^{b-c} \times y^{2c-2b}$

$= y^{2b-2c} \times y^{2c-2b}$

$= y^{2b-2c+2c-2b}$

$= y^{\circ}$

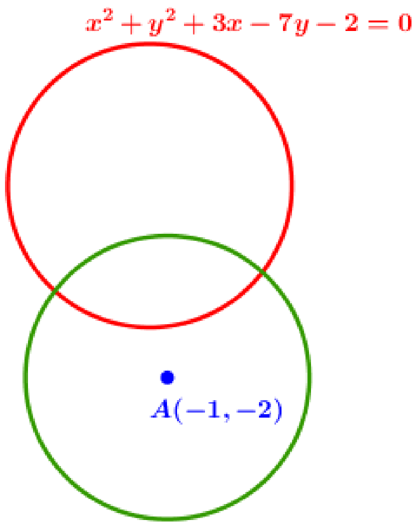
$= 1$

RH.S. $= (3x)^{\circ}$

$= 1$

Hence, $x^{b-c} \times y^{c-a} \times z^{a-b} = (3x)^{\circ}$

31. *Solution:*



Here, the equation of a given circle is $x^2 + y^2 + 3x - 7y - 2 = 0$

Comparing it with $x^2 + y^2 + 2gx + 2fy + c = 0$, we get

$2g = 3$ $\therefore g = \frac{3}{2}$

$2f = -7$ $\therefore f = -\frac{7}{2}$

$c = -2$

Now, radius (r) = $\sqrt{g^2 + f^2 - c}$

$$= \sqrt{\left(\frac{3}{2}\right)^2 + \left(-\frac{7}{2}\right)^2 + 2}$$

$$= \sqrt{\frac{9}{4} + \frac{49}{4} + 2}$$

$$= \sqrt{\frac{33}{2}}$$

For required circle, center (h, k) = (–1, – 2) and radius (r) = $\sqrt{\frac{33}{2}}$

Thus, the equation of circle is given by $(x - h)^2 + (y - k)^2 = r^2$

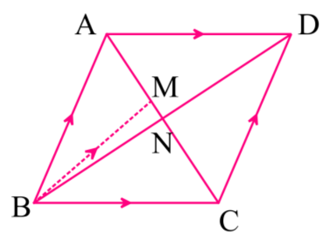
$$(x + 1)^2 + (y + 2)^2 = \left(\sqrt{\frac{33}{2}}\right)^2$$

or, $x^2 + 2x + 1 + y^2 + 4y + 4 = \frac{33}{2}$

or, $2x^2 + 4x + 2 + 2y^2 + 8y + 8 = 33$

i.e., $2x^2 + 2y^2 + 4x + 8y - 23 = 0$, which is required equation.

32. ***Solution:***



Here,

Given: In rhombus ABCD; AC and BD are the diagonals.

To prove: (i) Diagonals AC and BD bisect each other

(ii) AC and BD are perpendicular to each other

i.e. $\overrightarrow{AC}.\overrightarrow{BD} = 0$

Assumption: M is the mid-point of diagonal AC and N is the mid-point of diagonal BD

Proof:

(i) $\overrightarrow{BM} = \frac{1}{2}(\overrightarrow{BA} + \overrightarrow{BC})$ [By mid-point theorem]

(ii) $\overrightarrow{BN} = \frac{1}{2}(\overrightarrow{BD}$ [Being N the mid-point of diagonal BD]

$= \frac{1}{2}(\overrightarrow{BA} + \overrightarrow{BC})$ [By parallelogram law of vector addition]

From (i) and (ii), we get $\overrightarrow{BM} = \overrightarrow{BN}$ i.e., mid-point M of diagonal AC and N of diagonal BD coincide. Therefore, the diagonals of rhombus bisect each other.

Also,

(iii) In ΔABC; $\overrightarrow{AC} = \overrightarrow{AB} + \overrightarrow{BC}$ [By Δ law of vector addition]

(iv) In ΔBCD; $\overrightarrow{BD} = (\overrightarrow{BC} + \overrightarrow{CD}$ [By Δ law of vector addition]

Again, $\overrightarrow{AC} \cdot \overrightarrow{BD} = (\overrightarrow{AB} + \overrightarrow{BC}).(\overrightarrow{BC} + \overrightarrow{CD})$

$= (\overrightarrow{AB} + \overrightarrow{BC}).(\overrightarrow{BC} + \overrightarrow{BA})$

$= (\overrightarrow{AB} + \overrightarrow{BC}).(\overrightarrow{BC} - \overrightarrow{AB})$

$= BC^2 - AB^2$

$= BC^2 - BC^2$ [AB = BC]

$= 0$

Since, $\overrightarrow{AC} \cdot \overrightarrow{BD} = 0$. So, AC ⊥ BD

Hence, the diagonals of rhombus bisect each other at right angle.

33. *Solution:*

Here,

The vertices of a quadrilateral PQRS are P (0, 3), Q (–6, 1), R (–6, 5) and S (–1, 5).

Now,

Reflecting the quadrilateral PQRS about the line $y = x$, we get

P (x, y) $\xrightarrow{y = x}$ P′ (y, x)

P (0, 3) ⟶ P′ (3, 0)

Q (–6, 1) ⟶ Q′ (1, –6)

R (–6, 5) ⟶ R′ (5, –6)

S (–1, 5) ⟶ S′ (5, –1)

Thus, P′ (3, 0), Q′ (1, –6), R′ (5, –6) and S′ (5, –1) are the vertices of image quadrilateral P′Q′R′S′.

Again, rotating the quadrilateral P′Q′R′S′ through $+90^o$ about origin, we get

P (x, y) $\xrightarrow{[+90^o, (0, 0)]}$ P′ ($-y$, x)

P′ (3, 0) ⟶ P″ (0, 3)

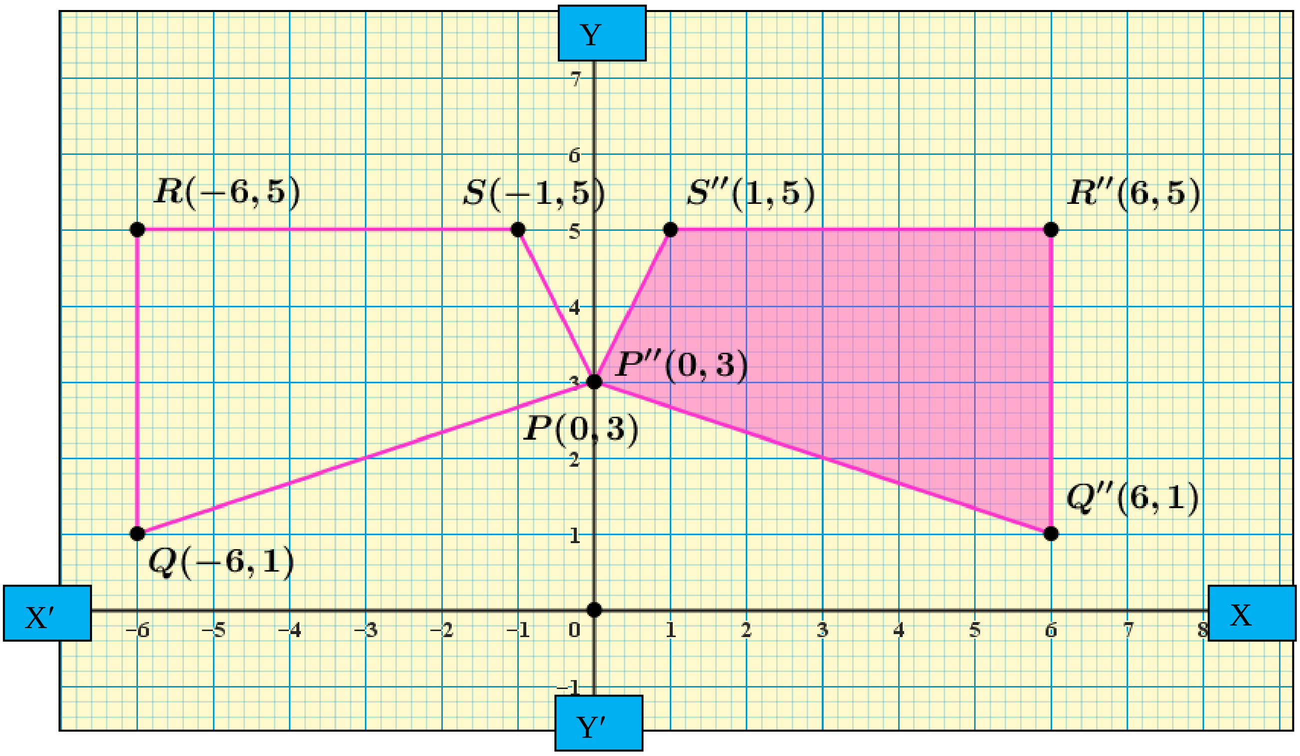
Q′ (1, –6) ⟶ Q″ (6, 1)

R′ (5, –6) ⟶ R″ (6, 5)

S′ (5, –1) ⟶ S″ (1, 5)

At last, representing the quadrilateral PQRS and its final image quadrilateral P″, Q″, R″ and S″ on the same graph paper

Let, 10 small divisions = 1 unit



The End