A Model Set along with its Complete Solution

OPT-I (Mathematics)

Group 'A' $[5 \times (1 + 1) = 10]$

1. (a) **Write the maximum and minimum values of sine function.**

Solution:

Here,

The maximum value of sine function is 1 and its minimum value is – 1.

(b) **What is the arithmetic mean between two numbers 'a' and 'b'?**

Solution:

Here, A.M. $= \dfrac{a + b}{2}$

2. (a) **If $A = \begin{pmatrix} m & t \\ h & a \end{pmatrix}$ is a given matrix, find the value of $|A|$.**

Solution:

Here, $|A| = \begin{vmatrix} m & t \\ h & a \end{vmatrix} = ma - th$

(b) **Does the set of natural numbers form the continuity or discontinuity? Give reason.**

Solution:

Here,

The set of natural numbers does not form the continuity because there is a jump between any two natural numbers on number line (i.e., there are infinitely many rational and irrational numbers between any two consecutive natural numbers).

3. (a) **What is the angle between pair of lines represented by the homogeneous equation $ax^2 + 2hxy + by^2 = 0$? Write it.**

Solution:

Here,

The angles between the pair of lines represented by the homogeneous equation $ax^2 + 2hxy + by^2 = 0$ are given by $\tan\theta = \pm \dfrac{2\sqrt{h^2 - ab}}{a + b}$

(b) **What type of the geometrical shape is formed when a cone is intersected by a plane surface which is parallel to the generator of a cone?**

Solution:

Here,

Parabola is formed when a cone is intersected by a plane surface being parallel to the generator of the cone.

4. (a) **Express 2cosA.cosB in the form of the sum of cosine.**

Solution:

Here, $2\cos A.\cos B = \cos(A + B) + \cos(A - B)$

(b) **If $\sin A = \frac{1}{2}$ and A is an acute angle, what will be the value of A?**

Solution:

Here, $\sin A = \frac{1}{2} = \sin 30^\circ$

Thus, $A = 30^\circ$

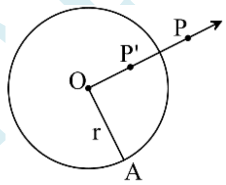
5. (a) **What is the scalar product of vectors $\vec{a}$ and $\vec{b}$ if θ is the angle between them?**

Solution:

Here,

The scalar product of vectors $\vec{a}$ and $\vec{b}$ is given by $\vec{a}.\vec{b} = |\vec{a}||\vec{b}|\cos\theta$

(b) **In the figure, O is the centre of a circle with radius r and P′ is the inversion point of P, write down the relation among OP, OP′ and OA.**



Solution:

Here, the relation among OP, OP′ and OA is $OP \times OP' = OA^2$

Group 'B' [3 × (2 + 2 + 2) + 2 × (2 + 2) = 26]

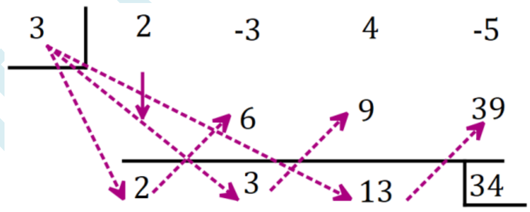
6. (a) **Find the quotient Q(x) and remainder R using synthetic division in the relation $2x^3 - 3x^2 + 4x - 5 = (x - 3) \times Q(x) + R$.**

Solution:

Let, $f(x) = 2x^3 - 3x^2 + 4x - 5$ and $d(x) = x - 3$

Comparing x – 3 with x – a, we get a = 3

Now, by using synthetic division method, we get



Hence, $Q(x) = 2x^2 + 3x + 13$ and remainder (R) = 34

(b) **Is (x – 2) a factor of a polynomial $f(x) = 3x^3 - 6x^2 + 4x - 8$ or not?**

Solution:

Let, $f(x) = 3x^3 - 6x^2 + 4x - 8$ and $d(x) = x - 2$

Comparing x – 2 with x – a, we get a = 2

Now, remainder (R) $= f(a)$

$= f(2)$

$= 3(2)^3 - 6(2)^2 + 4(2) - 8$

$= 24 - 24 + 8 - 8$

$= 0$

Since, f (2) = 0. So, (x – 2) a factor of $f(x) = 3x^3 - 6x^2 + 4x - 8$.

(c) **Which term of the sequence 21, 18, 15, ….. is 0?**

Solution:

Here, in the given A.P. 21, 18, 15, …

First term (a) = 84

Common difference (d) = $t_2 - t_1 = 18 - 21 = -3$

Let, last term (t_n) = 0

Now, $t_n = a + (n - 1) d$

or, $0 = 21 + (n - 1)(-3)$

or, $0 = 21 - 3n + 3$

or, $3n = 24$

$\therefore n = 8$

Hence, 0 is the 8th term of the sequence.

7. (a) **For what value of a, the inverse of $A = \begin{pmatrix} 4 & 3 \\ a & 3 \end{pmatrix}$ is not defined? Find it.**

Solution:

Here, the given matrix is $A = \begin{pmatrix} 4 & 3 \\ a & 3 \end{pmatrix}$

Since, the inverse of A is undefined.

So, $|A| = 0$

or, $\begin{vmatrix} 4 & 3 \\ a & 3 \end{vmatrix} = 0$

or, $12 - 3a = 0$

or, $12 = 3a$

$\therefore a = 4$

Hence, the required value of 'a' is 4.

(b) **If $A^{-1} = \begin{pmatrix} -3 & 5 \\ 2 & -3 \end{pmatrix}$, $B = \begin{pmatrix} 21 \\ 13 \end{pmatrix}$ and $X = \begin{pmatrix} x \\ y \end{pmatrix}$, find the value of x and y such that $X = A^{-1}B$ by using matrix method.**

Solution:

Here, $A^{-1} = \begin{pmatrix} -3 & 5 \\ 2 & -3 \end{pmatrix}$, $B = \begin{pmatrix} 21 \\ 13 \end{pmatrix}$ and $X = \begin{pmatrix} x \\ y \end{pmatrix}$

Now, $X = A^{-1}B$

or, $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -3 & 5 \\ 2 & -3 \end{pmatrix}\begin{pmatrix} 21 \\ 13 \end{pmatrix}$

or, $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -63 + 65 \\ 42 - 39 \end{pmatrix}$

or, $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$

Equating corresponding elements, we get

$x = 2$ and $y = 3$

Hence, $x = 2$ and $y = 3$.

8. (a) **The angle between two straight lines having equations $2x - y + 1 = 0$ and $x + my - 7 = 0$ is right angle, find the value of m.**

Solution:

Here,

Slope of line $2x - y + 1 = 0$ is $m_1 = -\frac{\text{Coeff. of y}}{\text{Coeff. of x}} = -\frac{2}{-1} = 2$

Slope of line $x + my - 7 = 0$ is $m_2 = -\frac{\text{Coeff. of y}}{\text{Coeff. of x}} = -\frac{1}{m}$

Since, the angle between the lines is a right angle.

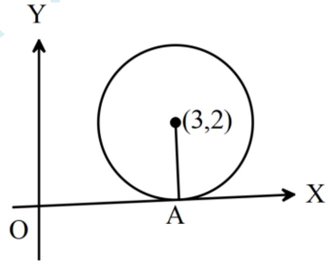
So, $m_1 \times m_2 = -1$

or, $2 \times \left(-\frac{1}{m}\right) = -1$

or, $m = 2$

Hence, the value of m is 2.

(b) **From the adjoining the figure, find the equation of a circle.**



Solution:

Here,

The centre of circle (h, k) = P (3, 2).

Since, the circle touches X-axis at Q. So, r = k = 2

Now,

The equation of the circle is given by $(x - h)^2 + (y - k)^2 = r^2$

or, $(x - 3)^2 + (y - 2)^2 = 2^2$

or, $x^2 - 6x + 9 + y^2 - 4y + 4 = 4$

or, $x^2 + y^2 - 6x - 4y + 9 = 0$ which is the required equation.

9. (a) **If $\sin\frac{A}{2} = \frac{4}{5}$, find the value of cosA.**

Solution:

Here, $\sin\frac{A}{2} = \frac{4}{5}$

Now, $\cos A = 1 - 2\sin^2\frac{A}{2}$

$= 1 - 2 \times \left(\frac{4}{5}\right)^2$

$= 1 - 2 \times \frac{16}{25}$

$= -\frac{7}{25}$

(b) **Prove that: $\frac{\sin 8A + \sin 2A}{\cos 8A + \cos 2A} = \tan 5A$.**

Solution:

Here,

$$\text{LHS} = \frac{\sin 8A + \sin 2A}{\cos 8A + \cos 2A}$$

$$= \frac{2\sin\left(\frac{8A + 2A}{2}\right)\cos\left(\frac{8A - 2A}{2}\right)}{2\cos\left(\frac{8A + 2A}{2}\right)\cos\left(\frac{8A - 2A}{2}\right)}$$

$$= \frac{\sin 5A.\cancel{\cos 3A}}{\cos 5A.\cancel{\cos 3A}}$$

$$= \tan 5A$$

$$= \text{RHS}$$ Proved

(c) Solve for θ: $\sin\theta.\cos\theta = \frac{1}{2}$. **[$0° \le \theta \le 180°$]**

Solution:

Here, $\sin\theta.\cos\theta = \frac{1}{2}$

or, $2\sin\theta.\cos\theta = 0$

or, $\sin 2\theta = 0$

or, $\sin 2\theta = \sin 90^o$

or, $2\theta = 90^o$

Hence, $\theta = 45^o$

10. (a) If $|\vec{a} + \vec{b}| = |\vec{a} - \vec{b}|$, then prove that $\vec{a} \perp \vec{b}$.

Solution:

Here, $|\vec{a} + \vec{b}| = |\vec{a} - \vec{b}|$

On squaring both sides, we get

$(|\vec{a} + \vec{b}|)^2 = (|\vec{a} - \vec{b}|)^2$

or, $(\vec{a} + \vec{b})^2 = (\vec{a} - \vec{b})^2$

or, $a^2 + 2\vec{a}.\vec{b} + b^2 = a^2 - 2\vec{a}.\vec{b} + b^2$

or, $4\vec{a}.\vec{b} = 0$

or, $\vec{a}.\vec{b} = 0$

Hence, $\vec{a} \perp \vec{b}$ Proved

(b) If the position vectors of A and B are $2\vec{i} - 3\vec{j}$ and $5\vec{i} + 4\vec{j}$ respectively, find the position vector of P dividing AB internally in the ratio 3:2.

Solution:

Here,

The position vector of A $(\vec{a}) = 2\vec{i} - 3\vec{j}$

The position vector of B $(\vec{b}) = 5\vec{i} + 4\vec{j}$

$m_1 : m_2 = 3:2$

The position vector of internal point P ($\vec{p}$) =?
Now,
Using internal section formula, we get

$$\vec{p} = \frac{m_1\vec{b} + m_2\vec{a}}{m_1 + m_2}$$

$$= \frac{3(5\vec{i} + 4\vec{j}) + 2(2\vec{i} - 3\vec{j})}{3 + 2}$$

$$= \frac{15\vec{i} + 12\vec{j} + 4\vec{i} - 6\vec{j}}{5}$$

$$= \frac{19\vec{i} + 6\vec{j}}{5}$$

Hence, the position vector of point P is $\frac{19\vec{i} + 6\vec{j}}{5}$.

(c) If the first quartile of a grouped data is 15 and the quartile deviation is 30, then find the coefficient of the quartile deviation.

Solution:

Here,
The first quartile (Q_1) = 15 and Q.D. = 30
Coefficient of quartile deviation =?

Now, Q.D. $= \frac{Q_3 - Q_1}{2}$

or, $30 = \frac{Q_3 - 15}{2}$

or, $Q_3 - 15 = 60$

$\therefore Q_3 = 75$

Again, coefficient of Q.D. $= \frac{Q_3 - Q_1}{Q_3 + Q_1}$

$= \frac{75 - 15}{75 + 15}$

$= \frac{60}{90}$

$= 0.67$

Hence, the coefficient of quartile deviation is 0.67

Group 'C' [11 × 4 = 44]

11. $f(x)$ and $g(x)$ are the two functions defined by $f(x) = x + 2$ and $g(x) = \frac{3x - 2}{4}$

Find the value of x when $f(x) = g^{-1}(x)$.

Solution:

Here,

The given functions are $f(x) = x + 2$ and $g(x) = \frac{3x - 2}{4}$

Let, g (x) = y then $y = \frac{3x - 2}{4}$

Now, interchanging the role of x and y, we get

$$x = \frac{3y - 2}{4}$$

$$\text{or, } 3y - 2 = 4x$$

$$\text{or, } 3y = 4x + 2$$

$$\text{or, } y = \frac{4x + 2}{3}$$

$$\therefore g^{-1}(x) = \frac{4x + 2}{3}$$

Again, $f(x) = g^{-1}(x)$

$$\text{or, } x + 2 = \frac{4x + 2}{3}$$

$$\text{or, } 4x + 2 = 3x + 6$$

$$\text{or, } x = 4$$

Hence, the required value of x is 4.

12. A function f is defined by $f(x) = \begin{cases} 2x + 3 & \text{for } x > 1 \\ 6x - 1 & \text{for } x \leq 1 \end{cases}$.

Examine the continuity or discontinuity of the function $f(x)$ at x = 1 by calculating the left hand limit, right hand limit and the value of the function.

Solution:

Here, the given function is $f(x) = \begin{cases} 2x + 3 & \text{for } x > 1 \\ 6x - 1 & \text{for } x \leq 1 \end{cases}$.

Now, for left hand limit at x = 1, $f(x) = 6x - 1$

Taking x = 0.9, 0.99, 0.999, …

x	0.9	0.99	0.999	…	$x \to 1^-$
$f(x) = 6x - 1$	4.4	4.94	4.994	…	$f(x) \to 5$

As x tends to 1 from left, $f(x)$ tends to 5. Thus, left hand limit at x = 1 is 5

i.e., $\lim_{x \to 1^-} f(x) = 5$

Also, for right hand limit at x = 1, $f(x) = 2x + 3$

Taking x = 1.1, 1.01, 1.001, …

x	1.1	1.01	1.001	…	$x \to 1^+$
$f(x) = 2x + 3$	5.2	5.02	5.002	…	$f(x) \to 5$

As x tends to 1 from right, $f(x)$ tends to 5. Thus, right hand limit at x = 1 is 5.

i.e., $\lim_{x \to 1^+} f(x) = 5$

Again, functional value at x = 1, $f(1) = 6 \times 1 - 1 = 5$

Since, $\lim_{x \to 1^-} f(x) = \lim_{x \to 1^+} f(x) = f(1)$

Hence, the given function is continuous at x = 1.

13. Solve the following system of linear equations by Cramer's rule:

$2x + \frac{6}{y} = 5$ and $5x - \frac{8}{y} = 1$

Solution:

Here, the given equations are $2x + \frac{6}{y} = 5$ ... (i)

and $5x - \frac{8}{y} = 1$ ... (ii)

Now,

Coefficient of x	Coefficient of $\frac{1}{y}$	Constant
2	6	5
5	-8	1

Also,

$D = \begin{vmatrix} 2 & 6 \\ 5 & -8 \end{vmatrix} = -16 - 30 = -46$

$D_x = \begin{vmatrix} 5 & 6 \\ 1 & -8 \end{vmatrix} = -40 - 6 = -46$

$D_y = \begin{vmatrix} 2 & 5 \\ 5 & 1 \end{vmatrix} = 2 - 25 = -23$

Again, by using Cramer's rule

$x = \frac{D_x}{D} = \frac{-46}{-46} = 1$

and $\frac{1}{y} = \frac{D_y}{D}$

or, $\frac{1}{y} = \frac{-23}{-46}$

or, $\frac{1}{y} = \frac{1}{2}$

or, y = 2

Hence, the value of x is 1 and that of y is 2.

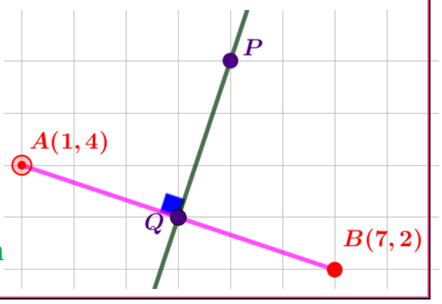
14. Find the equation of the perpendicular bisector of the line joining the points (1, 4) and (7, 2).

Solution:

Let PQ be the perpendicular bisector of the line joining the points A (1, 4) and B (7, 2).

Now,

Coordinates of mid-point Q (x, y) = $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$



$= \left(\frac{1+7}{2}, \frac{4+2}{2}\right)$

$= (4, 3)$

Also, slope of line AB $(m_1) = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2-4}{7-1} = \frac{-2}{6} = -\frac{1}{3}$

Since, $PQ \perp AB$

So, $m_1 \times m_2 = -1$ where m_2 is the slope of PQ.

or, $-\frac{1}{3} \times m_2 = -1$

$\therefore m_2 = 3$

Again, the equation of line PQ is given by $y - y_1 = m_2 (x - x_1)$

or, $y - 3 = 3 (x - 4)$

or, $y - 3 = 3x - 12$

or, $3x - y - 9 = 0$

15. Find the single equation of pair of straight lines passing through the origin and perpendicular to the lines represented by $2x^2 - 5xy + 2y^2 = 0$.

Solution:

Here,

The given equation is $2x^2 - 5xy + 2y^2 = 0$

or, $2x^2 - (4 + 1)xy + 2y^2 = 0$

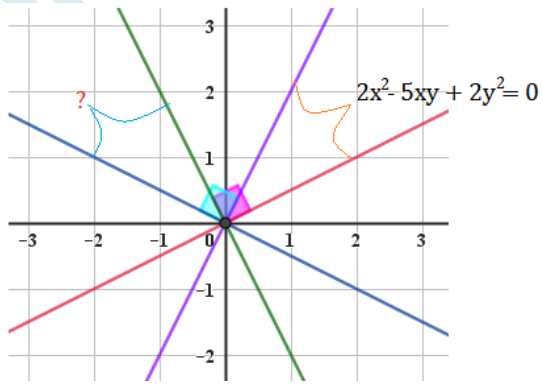
or, $2x^2 - 4xy - xy + 2y^2 = 0$

or, $2x (x - 2y) - y(x - 2y) = 0$

or, $(x - 2y)(2x - y) = 0$

Either, $x - 2y = 0$ ... (i)

OR, $2x - y = 0$ ... (ii)



Now,

The equation of straight line $\perp$ to line (i) is $2x + y + c = 0$

Since, it passes through origin i.e., (0, 0). So, $2\times 0 + 0 + c = 0 \therefore c = 0$

Putting the value of c, we get. $2x + y = 0$ ... (iii)

Also,

The equation of straight line $\perp$ to line (ii) is $x + 2y + k = 0$

Since, it passes through origin i.e., (0, 0). So, $0 + 2\times 0 + k = 0 \therefore k = 0$

Putting the value of k, we get. $x + 2y = 0$ ... (iv)

Again, combining equations (iii) and (iv), we get

$(2x + y)(x + 2y) = 0$

or, $2x^2 + 4xy + xy + 2y^2 = 0$

or, $2x^2 + 5xy + 2y^2 = 0$ which is required equation.

16. Prove that: $\cos\frac{\pi^c}{7} . \cos\frac{2\pi^c}{7} . \cos\frac{3\pi^c}{7} = \frac{1}{8}$

Solution:

Here,

LHS $= \cos\frac{\pi^c}{7}.\cos\frac{2\pi^c}{7}.\cos\frac{3\pi^c}{7}$

$= \frac{1}{2\sin\frac{\pi^c}{7}} \times 2\sin\frac{\pi^c}{7}.\cos\frac{\pi^c}{7}.\cos\frac{2\pi^c}{7}.\cos\frac{3\pi^c}{7}$

$= \frac{1}{2\sin\frac{\pi^c}{7}} \times \sin\frac{2\pi^c}{7}.\cos\frac{2\pi^c}{7}.\cos\frac{3\pi^c}{7}$

$= \frac{1}{2\times 2\sin\frac{\pi^c}{7}} \times 2\sin\frac{2\pi^c}{7}.\cos\frac{2\pi^c}{7}.\cos\frac{3\pi^c}{7}$

$= \frac{1}{4\sin\frac{\pi^c}{7}} \times \sin\frac{4\pi^c}{7}.\cos\frac{3\pi^c}{7}$

$= \frac{1}{4\sin\frac{\pi^c}{7}} \times \sin\left(\pi^c - \frac{3\pi^c}{7}\right).\cos\frac{3\pi^c}{7}$

$= \frac{1}{4\sin\frac{\pi^c}{7}} \times \sin\frac{3\pi^c}{7}.\cos\frac{3\pi^c}{7}$

$= \frac{1}{2\times 4\sin\frac{\pi^c}{7}} \times 2\sin\frac{3\pi^c}{7}.\cos\frac{3\pi^c}{7}$

$= \frac{1}{8\sin\frac{\pi^c}{7}} \times \sin\frac{6\pi^c}{7}$

$= \frac{1}{8\sin\frac{\pi^c}{7}} \times \sin\left(\pi^c - \frac{\pi^c}{7}\right)$

$= \frac{1}{8\sin\frac{\pi^c}{7}} \times \sin\frac{\pi^c}{7}$

$= \frac{1}{8}$

= RHS

Hence, proved

17. If A + B + C = 180°, prove that: sin2A + sin2B + 2sinC.cosC = 4sinA.sinB.sinC.

Solution:

Here,

$A + B + C = 180^0$

$\therefore A + B = 180^0 - C$

Taking sin and cos on both side successively, we get

$\sin(A + B) = \sin(180^0 - C) = \sin C$

$\& \cos(A + B) = \cos(180^0 - C) = -\cos C$

Now,

LHS $= \sin 2A + \sin 2B + \sin 2C$

$= 2\sin\left(\frac{2A + 2B}{2}\right).\cos\left(\frac{2A - 2B}{2}\right) + \sin 2C$

$= 2\sin(A + B).\cos(A - B) + \sin 2C$

$= 2\sin C.\cos(A - B) + 2\sin C.\cos C$

$= 2\sin C[\cos(A - B) + \cos C]$

$= 2\sin C[\cos(A - B) - \cos(A + B)]$

$= 2\sin C[\cos A.\cos B + \sin A.\sin B - (\cos A.\cos B - \sin A.\sin B)]$

$= 2\sin C[\cos A.\cos B + \sin A.\sin B - \cos A.\cos B + \sin A.\sin B]$

$= 2\sin C(2\sin A.\sin B)$

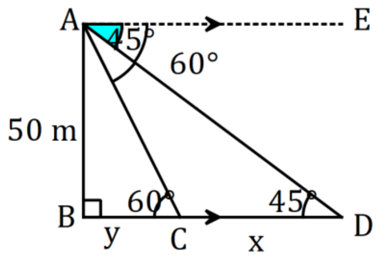
$= 4\sin A.\sin B.\sin C$

$=$ RHS

Hence, proved

18. From the top of a cliff 50 metre high, a man observes two boats on the ocean coming towards the bottom of a cliff from the east side on the same line at angles of depression of 45° and 60° respectively, find the distance between the two boats.

Solution:



Let AB be the height of cliff, C and D be the positions of boats, ∠EAC and ∠EAD be the angles of depression of the boats observed from the top of the cliff respectively.

Then,

AB = 50 m, ∠EAC = ∠ACB = 60°, ∠EAD = ∠ADB = 45°

CD = x m and BC = y m (say) ∴ BD = (x + y) m

Now,

From right angled ΔABC, $\tan 60^\circ = \frac{50}{y}$

or, $\sqrt{3} = \frac{50}{y}$

or, $1.732y = 50$

$\therefore y = 28.87$

Also,

From right angled ΔABD, $\tan 45^\circ = \frac{50}{x + y}$

or, $1 = \frac{50}{x + 28.87}$

or, $x + 28.87 = 50$

$\therefore x = 21.13$

Hence the distance between the boats is 21.13 m

19. Find the 2 × 2 transformation matrix which transformed the unit square $\begin{pmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{pmatrix}$ into a parallelogram $\begin{pmatrix} 0 & 6 & 8 & 2 \\ 0 & 2 & 6 & 4 \end{pmatrix}$..

Solution:

Here,

Object = unit square = $\begin{pmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{pmatrix}$

Image = Parallelogram $\begin{pmatrix} 0 & 6 & 8 & 2 \\ 0 & 2 & 6 & 4 \end{pmatrix}$.

Let 2 × 2 transformation matrix (T.M.) be $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$.

We have,

Image = T.M. × Object

or, $\begin{pmatrix} 0 & 6 & 8 & 2 \\ 0 & 2 & 6 & 4 \end{pmatrix}. = \begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{pmatrix}$

or, $\begin{pmatrix} 0 & 6 & 8 & 2 \\ 0 & 2 & 6 & 4 \end{pmatrix}. = \begin{pmatrix} 0+0 & a+0 & a+b & 0+b \\ 0+0 & c+0 & c+d & 0+d \end{pmatrix}$

or, $\begin{pmatrix} 0 & 6 & 8 & 2 \\ 0 & 2 & 6 & 4 \end{pmatrix}. = \begin{pmatrix} 0 & a & a+b & b \\ 0 & c & c+d & d \end{pmatrix}$

Equating the corresponding elements, we get

$a = 6, b = 2, c = 2$ and $d = 4$

Hence, required 2 × 2 transformation matrix is $\begin{pmatrix} 6 & 2 \\ 2 & 4 \end{pmatrix}$

20. Find the mean deviation from mean and its coefficient from the following frequency distribution:

Marks obtained	0-10	10-20	20-30	30-40	40-50
No. of students	3	5	7	3	4

Solution:

Here,

Computation of the mean deviation from the mean:

Marks	No. of students (f)	m	Fm	$\lvert m - \overline{X} \rvert$	$f\lvert m - \overline{X} \rvert$
0-10	3	5	15	20	60
10-20	5	15	75	10	50
20-30	7	25	175	0	0
30-40	3	35	105	10	30
40-50	4	45	180	20	80
	N = 22		Σfm =550		$\Sigma f\lvert m - Md\rvert = 220$

Now,

Mean ($\overline{X}$) = $\frac{\Sigma fm}{N}$ = $\frac{550}{22}$ = 25

Also, M.D. from mean = $\frac{\Sigma f|m - Md|}{N}$ = $\frac{220}{22}$ =10

Again, coefficient of M.D. = $\frac{\text{M.D. from mean}}{\text{Mean}}$ = $\frac{10}{25}$ = 0.4

Hence, the mean deviation is 10 and its coefficient is 0.4

21. An association doing charity work decided to give old age pension to people over 60 years age. The scales of pay were fixed as follows:

Age group	60-65	65-70	70-75	75-80	80-85
No. of people	7	6	6	4	3

Calculate the standard deviation and find the coefficient of variation.

Solution:

Here,

Computation of the standard deviation:

Ages group	No. of people (f)	m	Fm	fm²
60-65	7	62.5	437.5	27343.75
65-70	6	67.5	405	27337.5
70-75	6	72.5	435	31537.5
75-80	4	77.5	310	24025
80-85	3	82.5	247.5	20418.75
	N = 26		Σfm =1835	Σ fm² = 130662.5

Now, S.D. (σ) = $\sqrt{\frac{fm^2}{N} - \left(\frac{fm}{N}\right)^2}$

$= \sqrt{\frac{130662.5}{26} - \left(\frac{1835}{26}\right)^2}$

$= \sqrt{5025.481 - 4981.102}$

$= \sqrt{44.379}$

= 6.66

Also, Mean ($\overline{X}$) = $\frac{\Sigma fm}{N}$ = $\frac{1835}{26}$ = 70.58

Again, coefficient of variation = $\frac{\sigma}{\overline{X}} \times 100\% = \frac{6.66}{70.58} \times 100\% = 9.44\%$

Group 'D' [$4 \times 5 = 20$]

22. **Study the given cases and answer the following questions.**
- **Ram gives Rs 200 to Sita in the first day and increases each subsequent day by Rs 10.**
- **Sita gives Ram Rs 3 on the first day and then every day twice the previous day.**

If this rule continues up to ten days, who will give more amount of money and by how much?

Solution:

Here,

For Ram;

The sequence of money given to Sita is Rs 200, Rs 210, Rs 220, …

First term (a) = Rs 200

No of days (n) = 10

Common difference (d) = Rs 210 – Rs 200 = Rs 10

Sum (S_n) =?

We have, $S_n = \frac{n}{2}[2a + (n-1)d]$

$= \frac{10}{2}[2\times 200 + (10-1)10]$

$= 5(400 + 90)$

= Rs 2450

Thus, Ram gives Rs 2,450 to Sita in 10 days.

For Sita;

The sequence of money given to Ram is Rs 3, Rs 6, Rs 12, …

First term (a) = Rs 200

No of days (n) = 10

Common ratio (r) = $\frac{\text{Rs } 6}{\text{Rs } 3} = 2$

Sum (S_n) =?

We have, $S_n = \frac{a(r^n - 1)}{r - 1}$

$= \frac{3(2^{10} - 1)}{2 - 1}$

$= \frac{3 \times 1023}{1}$

= Rs 3069

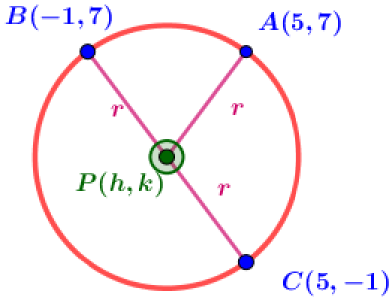
Thus, Sita gives Rs 3,069 to Ram in 10 days.

Again, difference = Rs 3069 – Rs 2450 = Rs 619

Hence, Sita gives Rs 619 more to Ram.

23. On a wheel, there are three points (5, 7), (–1, 7) and (5, –1) located such that the distance from a fixed point to these points is always equal. Find the coordinate of the fixed point and then derive the equation, representing the locus that contains all three points.

Solution:



Let P (h, k) be the coordinates of the fixed point and r be the radius of the wheel containing three points A (5, 7), B (–1, 7) and C (5, –1) on the circumference.

Then, AP = BP = CP= radius

Now, $AP = BP$

or, $AP^2 = BP^2$ [On squaring both sides]

or, $(h-5)^2 + (k-7)^2 = (h+1)^2 + (k-7)^2$

or, $h^2 - 10h + 25 = h^2 + 2h + 1$

or, $-12h = -24$

$\therefore h = 2$

Also, $BP = CP$

or, $BP^2 = CP^2$ [On squaring both sides]

or, $(h+1)^2 + (k-7)^2 = (h-5)^2 + (k+1)^2$

or, $(2+1)^2 + (k-7)^2 = (2-5)^2 + (k+1)^2$ [As, h = 2]

or, $9 + k^2 - 14k + 49 = 9 + k^2 + 2k + 1$

or, $-16k = -48$

$\therefore k = 3$

Thus, the coordinates of the fixed point is P (2, 3).

Again, radius (r) = AP

$$= \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$$
$$= \sqrt{(2-5)^2 + (3-7)^2}$$
$$= \sqrt{(-3)^2 + (-4)^2}$$
$$= \sqrt{9+16}$$
= 5 units

Lastly, equation of the locus is given by

$(x-h)^2 + (y-k)^2 = r^2$

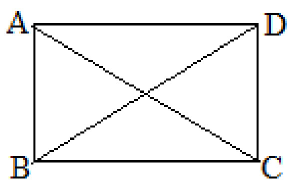
or, $(x-2)^2 + (y-3)^2 = 5^2$

or, $x^2 - 4x + 4 + y^2 - 6x + 9 = 25$

or, $x^2 + y^2 - 4x - 6x = 12$

24. By using vector method, prove that the diagonals rectangles are equal.

Solution



Here,

Given: In rectangle ABCD; AC and BD are diagonals.

To prove: AC = BD

Proof:

(i) In ΔABC; $\overrightarrow{AC} = \overrightarrow{AB} + \overrightarrow{BC}$ [By Δ law of vector addition]

Squaring on both sides, we get

$(\overrightarrow{AC})^2 = (\overrightarrow{AB} + \overrightarrow{BC})^2$

or, $AC^2 = AB^2 + 2\overrightarrow{AB}.\overrightarrow{BC} + BC^2$

or, $AC^2 = AB^2 + 2 \times 0 + BC^2$ [$AB \perp BC$]

$\therefore$ $AC^2 = AB^2 + BC^2$

(ii) In ΔBCD; $\overrightarrow{BD} = \overrightarrow{BC} + \overrightarrow{CD}$ [By Δ law of vector addition]

Squaring on both sides, we get

$(\overrightarrow{BD})^2 = (\overrightarrow{BC} + \overrightarrow{CD})^2$

or, $BD^2 = BC^2 + 2\overrightarrow{BC}.\overrightarrow{CD} + CD^2$

or, $BD^2 = BC^2 + 2 \times 0 + CD^2$ [$BC \perp CD$]

or, $BD^2 = BC^2 + AB^2$ [$CD = AB$]

$\therefore$ $BD^2 = AB^2 + BC^2$ From (i)

and (ii), we get

$AC^2 = BD^2$

i.e., $AC = BD$

Hence, the diagonals of a rectangle are equal. QED

25. A (2, 5), B (–1, 3) and C (4, 1) are the vertices of a triangle ABC. If r_1 is the reflection on y-axis and r_2 is the rotation through positive quarter turn. Transform the ΔABC by r_1 and then the image so obtained by r_2. Show all the triangles in the same paper. Also investigate the transformation which is equivalent to $r_2 o r_1$.

Solution:

Here,

The coordinates of vertices of ΔABC are A (2, 5), B (–1, 3) and C (4, 1).

r_1: reflection on y-axis

r_1: rotation through positive quarter turn i.e., R [+90°, (0, 0)]

Now,

We have, P (x, y) $\xrightarrow{r_1: \text{ y-axis}}$ P′(–x, y)

$\therefore$ A (2, 5) $\longrightarrow$ A′(–2, 5)

B (–1, 3) $\longrightarrow$ B′(1, 3)

C (4, 1) $\longrightarrow$ C′(–4, 1)

Thus, A′(–2, 5), B′(1, 3) and C′(–4, 1)are the vertices ΔA′B′C′.

Again,

We have, P (x, y) $\longrightarrow$ P′(–y, x)

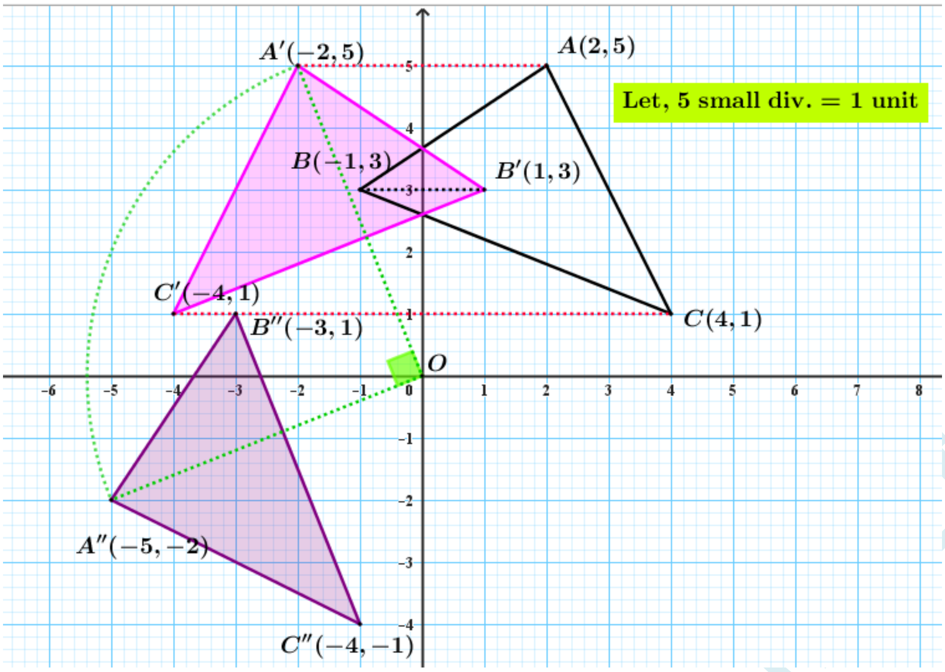
$\therefore$ A′(–2, 5) $\longrightarrow$ A″ (–5, –2)

B′(1, 3) $\longrightarrow$ B″ (–3, 1)

C′(–4, 1) $\longrightarrow$ C″ (–1, –4)

Thus, A″ (–5, –2), B″ (–3, 1) and C″ (–1, –4) are the vertices ΔA″B″C″.

Representing the ΔABC and its images on the same graph paper



At last, finding the single transformation

A (2, 5) $\xrightarrow{r_2 o r_1}$ A″ (−5, −2)

B (−1, 3) $\xrightarrow{r_2 o r_1}$ B″ (−3, 1)

C (4, 1) $\xrightarrow{r_2 o r_1}$ C″ (−1, −4)

∴ P (x, y) $\xrightarrow{r_2 o r_1}$ P″ (−y, −x)

Which is equivalent to the reflection about the line y = – x

Hence, the the transformation which is equivalent to $r_2 o r_1$ is the reflection about the line y = – x.

♣♣♣