

## Optional-I (Mathematics)

Time: 3 hours

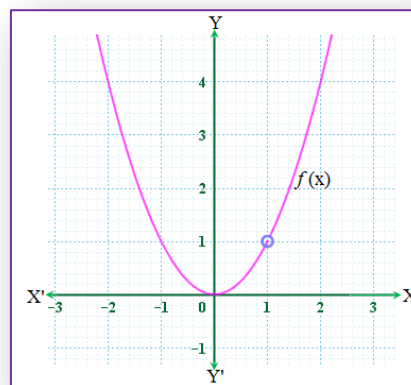
Maximum Marks: 100

Candidates are required to answer in their own words as far as practicable. Credit shall be given to originality in expression, creativity and neatness in hand, not to rote learning.

Attempt all the questions.

## Group-A (10 × 1 = 10)

- Write down the period of sine function.
  - State remainder theorem.
- Determine the continuity or discontinuity of the graph of function  $f(x)$  at  $x = 1$ .



- Define singular matrix.
- Write down the condition of coincident of pair of straight lines represented by the equation  $ax^2 + 2hxy + by^2 = 0$ .
    - Identify and name the conic section from the figure given alongside.
  - Write down the formula of  $\cos 2\alpha$  in terms of  $\tan \alpha$ .
    - Express  $2\sin A \cdot \sin B$  in terms of sum or difference of cosines.
  - If  $\vec{a}$  and  $\vec{b}$  are two vectors and  $\theta$  be angle between them then what will be the value of  $\cos \theta$ ?
    - In an inversion transformation, if  $P'$  is the image of the  $P$  and  $r$  is the radius of inversion circle with centre  $O$ , write the relation of  $OP$ ,  $OP'$  and  $r$ .



## Group-B (13 × 2 = 26)

- If  $(x + 3)$  is a factor of  $x^3 - (k - 1)x^2 + kx + 54$ , find the value of  $k$ .
  - Find the values of  $x$ ,  $y$  and  $z$  from the given GP:  $\frac{1}{8}, x, y, z, 2$

- (c) Find the vertex of the parabola  $y = x^2 - 2x - 3$ .
7. (a) If  $A = \begin{pmatrix} -1 & 0 \\ 2 & 4 \end{pmatrix}$ , find the determinant of  $AA^T$ .
- (b) Find the value of  $x$  from the linear equations  $3x - y = 7$  and  $x + 2y = 4$  by using Cramer's rule.
8. (a) Find the angle between the pair of straight lines represented by  $2x^2 + 7xy + 3y^2 = 0$ .
- (b) Find the equation of a circle whose centre is  $(4, 5)$  and  $y$ -axis is tangent to it.
9. (a) Prove that:  $\frac{\sin A - \cos A + 1}{\sin A + \cos A + 1} = \tan \frac{A}{2}$
- (b) Find the value of  $\cos 130^\circ + \cos 110^\circ + \cos 10^\circ$
- (c) Solve:  $\sin 2\theta = \sin \theta$   $[0^\circ \leq \theta \leq 90^\circ]$
10. (a) Find the angle between  $\vec{a} = \sqrt{3}\vec{i} + \vec{j}$  and  $X$ -axis.
- (b) The position vectors of  $A$  and  $B$  are  $\vec{i} + 2\vec{j}$  and  $6\vec{i} + 7\vec{j}$ . Find the position vector of a point  $P$  which divides  $AB$  internally in the ratio  $2:3$ .
- (c) In a continuous series, the lower quartile is 12 and the upper quartile exceeds twice the lower quartile by 4. Find the quartile deviation and its coefficient.

**Group-C**

**(11 × 4 = 44)**

11. If  $f = \{(x, 3x - 11) : x \in \mathbb{R}\}$ ,  $g = \left\{ \left( x, \frac{2x-3}{5} \right) : x \in \mathbb{R} \right\}$  and  $ff(x) = g^{-1}(x)$ , find the value of  $x$ .
12. If the third and eleventh terms of an arithmetic series are 8 and -8 respectively, find the sum of the first seven terms of the series.
13. For a real valued function  $f(x) = 3x + 5$ .
- (i) What are the values of  $f(x)$  at  $x = 1.9, 1.99$  and  $1.999$ ?
- (ii) What are the values of  $f(x)$  at  $x = 2.1$  and  $2.01$  and  $2.001$ ?
- (iii) What are the values of  $\lim_{x \rightarrow 2^-} f(x)$ ,  $\lim_{x \rightarrow 2^+} f(x)$  and  $f(2)$ ?
- (iv) Is this function continuous at  $x = 2$ ?
14. Solve by matrix method:  $\frac{3}{2}x + y = 7$ ,  $\frac{x}{3} - \frac{y}{3} = 1$
15.  $A(3, 5)$  and  $C(7, 9)$  are the opposite vertices of a rhombus  $ABCD$ , find the equation of the diagonal  $BD$ .
16. Calculate the value of  $\left(1 + \cos \frac{\pi^c}{8}\right) \left(1 + \cos \frac{3\pi^c}{8}\right) \left(1 + \cos \frac{5\pi^c}{8}\right) \left(1 + \cos \frac{7\pi^c}{8}\right)$
17. If  $A + B + C = 180^\circ$ , prove that  $\sin A \cdot \cos A + \sin B \cdot \cos B + \sin C \cdot \cos C = 2 \sin A \cdot \sin B \cdot \sin C$

18. The angle of depression and elevation of the top of a building 25m high from the top and bottom of a cliff are found to be  $60^\circ$  and  $30^\circ$  respectively, find the height of the cliff.

19. Find the  $2 \times 2$  matrix which transforms the unit square into a parallelogram

$$\begin{pmatrix} 0 & 2 & 5 & 3 \\ 0 & 1 & 0 & -1 \end{pmatrix}.$$

20. Calculate the mean deviation and its coefficient of the data from median.

Height (in cm)	0-10	10-20	20-30	30-40	40-50
No. of plants	5	8	15	10	6

21. The age of 30 boys of a school is given below.

Age (in years)	0-4	4-8	8-12	12-16	16-20
No. of boys	4	6	10	6	4

- (i) What is the average age of the boys?  
(ii) What is the standard deviation of the data?

**Group-D** **(4 × 5 = 20)**

22. Write one importance of linear programming. Optimize the objective function  $P = 3x + 2y$  subject to the following constraints:  $x + y \leq 5$ ,  $x + 4y \leq 8$ ,  $x \geq 0$ ,  $y \geq 0$

23. A cow tighten to a stake by a rope moves through the points (2, -2), (5, 7) and (6, 6) in a path always keeping the rope tight. Find the coordinates of the point at which the stake is fixed and the equation representing the locus of cow.

24. In a rhombus BEST, prove by vector method that the diagonals BS and ET bisect to each other at a right angle.

25. Rotate the quadrilateral ABCD with vertices A (1, 1), B (2, 3), C (4, 2) and D (3, -2) about origin through  $180^\circ$ . Then reflect the image of quadrilateral so obtained about the line  $y = -x$ . Find the coordinates of the vertices of images quadrilaterals and represent them in the same graph paper. Also, write the name of transformation which denotes the combined transformation of above two transformations.

**The End**