Unit 1 — Set

Teaching hours: 9

Competency

- Solving behavioural problems based on set operations and cardinality.

Learning Outcomes

- To find the set operations and represent in Venn-diagrams.
- To find the cardinality of sets.

Level-wise learning objectives

S.N.	Level	Objectives
1.	Knowledge (K)	▪ To list the operations on sets ▪ To define cardinality of sets ▪ To tell the formulae involving two sets
2.	Understanding (U)	▪ To write the set notations from the given set builder form ▪ To find the operations of sets ▪ To find the symmetric difference of sets ▪ To find the cardinality of a set.
3.	Application (A	▪ To find the set operations and show in Venn-diagrams. ▪ To verify the relations of set operations. ▪ To solve the verbal problems on operations (union, intersection, difference and complement) of sets by using Venn-diagram
4.	High Ability (HA)	▪ To answer the logical questions with proper reasons. ▪ To link various real life/ contemporary problems with sets and solve.

Required Teaching Materials/ Resources

➢ Definitions and formulae in colourful chart-paper, scissors, cello tape, different coloured markers, highlighter, models of Venn-diagrams, ICT tools etc.

Pre-knowledge:

➢ Check the Pre-knowledge on types of sets, relation of sets and subsets, Venn-diagrams etc.

Teaching Activities

➢ Make a group discussion on the definition of sets, set notation, types of sets, cardinality of sets and subsets by using Venn-diagram.
➢ Ask individually the basic concepts from set as revision.
➢ With real life examples as given in textbook, discuss about the operations of sets.
➢ Use ICT tools to make content more interactive.
➢ Make the group of students and give the questions on set operations from exercise.
➢ Verify the following basic properties of set operations with examples.

Property 1:   Commutative property

Intersection and union of sets satisfy the commutative property.

(i) $A \cap B = B \cap A$ (ii) $A \cup B = B \cup A$

Property 2:   Associative property

Intersection and union of sets satisfy the associative property.

(i) $(A \cap B) \cap C = A \cap (B \cap C)$ (ii) $(A \cup B) \cup C = A \cup (B \cup C)$

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Property 3:   Distributive property
Intersection and union of sets satisfy the distributive property.
(i) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ (ii) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
Property 4:   Identity property
(i) $A \cup \emptyset = A$ (ii) $A \cap U = A$
Property 5:   Complement property
(i) $A \cup A^C = U$ (ii) $A \cap A^C = \emptyset$
Property 6:   Idempotent property
(i) $A \cap A = A$ (ii) $A \cup A = A$
De Morgan's Laws for Set Difference
For any three sets A, B and C
(i) $A - (B \cup C) = (A - B) \cap (A - C)$ (ii) $A - (B \cap C) = (A - B) \cup (A - C)$
De Morgan's Laws for Complementation
For any two subsets A and B of a universal set U
(i) $(A \cup B)^C = A^C \cap B^C$ (ii) $(A \cup B)^C = A^C \cap B^C$

➤ Solve the problems and verify the relations from the exercise with discussion.
➤ Discuss about the cardinality of sets, list the following formula by using Venn-diagrams along with examples.
➤ Give the work to the students to write the formulae in chart paper after discussion and paste the best one in math corner of the classroom or math lab.

Case-I: When A is a subset of B
       (i) $n(A \cap B) = n(A)$ (ii) $n(A \cup B) = n(B)$ (iii) $n_o(B) = n(B) - n(A)$
Case- II: When A and B are disjoint sets
       (i) $n(A \cap B) = 0$ (ii) $n(A \cup B) = n(A) + n(B)$
Case- III: When A and B overlapping sets
       (i) $n(A \cup B) = n(A) + n(B) - n(A \cap B)$
       (ii) $n(\text{only } A) = no(A) = n(A) - n(A \cap B)$
       (iii) $n(\text{only } B) = no(B) = n(A) - n(A \cap B)$
       (iv) $n(\text{only one})$ or $n(\text{exactly one}) = no(A) + no(B)$
   (v) $n(A \cup B) = no(A) + no(B) + n(A \cap B)$

EXERCISE 1.1

General section

1. *Let P and Q are the subsets of a universal set U. Write the set operations defined by the following set-builder forms.*
    a) {x: x ∈ P or x∈ Q}   b) {x: x ∈ P and x ∈ Q}   c) {x: x ∈ P but x ∉ Q}
    d) {x: x ∈ Q but x∉ P}   e) {x: x ∈ U but x ∉ P}   f) {x: x ∈ U but x ∉ Q}
    g) {x: x ∈ U but x ∉ P or Q}   h) {x: x ∈ U but x ∉ P and Q}   i) {x: x ∈ U but x ∉ P – Q}

    Solution:
    a) $\{x: x \in P \text{ or } x \in Q\} = P \cup Q$ b) $\{x: x \in P \text{ and } x \in Q\} = P \cap Q$
    c) $\{x: x \in P \text{ but } x \notin Q\} = P - Q$ d) $\{x: x \in Q \text{ but } x \notin P\} = Q - P$
    e) $\{x: x \in U \text{ but } x \notin P\} = \overline{P}$ f) $\{x: x \in U \text{ but } x \notin Q\} = \overline{Q}$

    g) $\{x: x \in U \text{ but } x \notin P \text{ or } Q\} = \overline{P \cup Q}$ h) $\{x: x \in U \text{ but } x \notin P \text{ and } Q\} = \overline{P \cap Q}$

    i) $\{x: x \in U \text{ but } x \notin P - Q\} = \overline{P - Q}$

2. *Write the set operations represented by shaded regions shown in the following Venn-diagrams.*
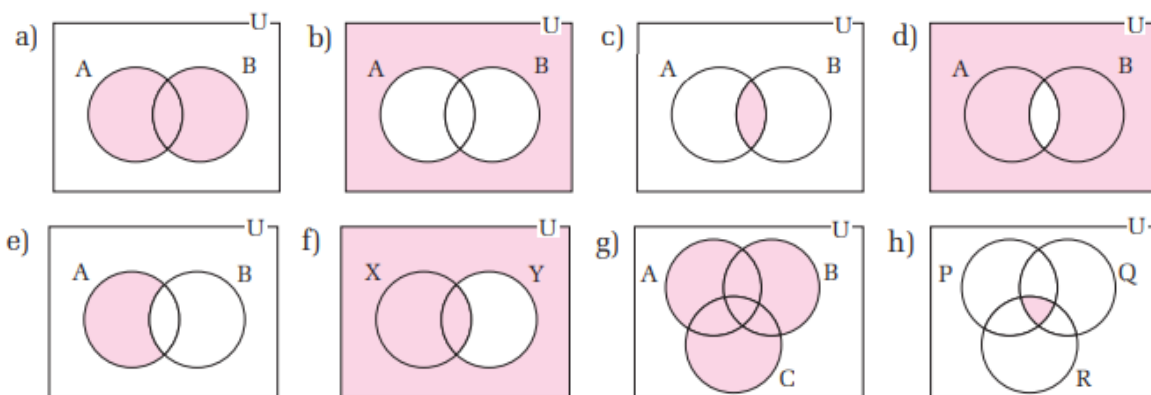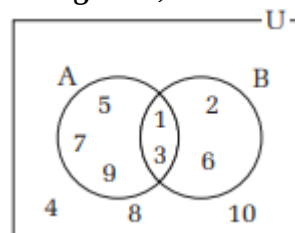

Solution:

a) The shaded region represents $A \cup B$ b) The shaded region represents $\overline{A \cup B}$

c) The shaded region represents $A \cap B$ d) The shaded region represents $\overline{A \cap B}$

e) The shaded region represents $A - B$ f) The shaded region represents $\overline{Y - X}$

g) The shaded region represents $A \cup B \cup C$ h) The shaded region represents $P \cap Q \cap R$

3. **a)** *A and B are the subsets of the universal set U. From the given diagrams, list the elements of the following set operations.*

   (i) $A \cup B$ *and* $\overline{A \cup B}$ **(ii)** $A \cap B$ *and* $\overline{A \cap B}$

   (iii) $A - B$ *and* $\overline{A - B}$ **(iv)** $B - A$ *and* $\overline{B - A}$
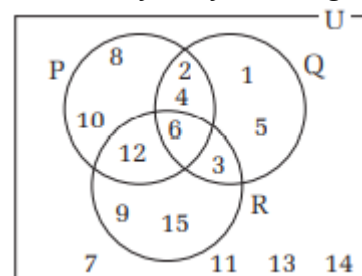
Solution:

   (i) $A \cup B = \{1, 2, 3, 5, 6, 7, 9\}$ and $\overline{A \cup B} = \{4, 8, 10\}$

   (ii) $A \cap B = \{1, 3\}$ and $\overline{A \cap B} = \{2, 4, 5, 6, 7, 8, 9, 10\}$

   (iii) $A - B = \{5, 7, 9\}$ and $\overline{A - B} = \{1, 2, 3, 4, 6, 8, 10\}$

   (iv) $B - A = \{2, 6\}$ and $\overline{B - A} = \{1, 3, 4, 5, 7, 8, 9, 10\}$

   b) *P, Q and R are the subsets of the universal set U. List the elements of the following set operations from the given diagram.*

   (i) $P \cup Q \cup R$ **(ii)** $P \cap Q \cap R$

   (iii) $\overline{P \cup Q \cup R}$ **(iv)** $\overline{P \cap Q \cap R}$

   (v) $(P \cup Q) \cap R$ **(vi)** $(P \cap Q) \cup R$

Solution:

   (i) $P \cup Q \cup R = \{1, 2, 3, 4, 5, 6, 8, 9, 10, 12, 15\}$

   (ii) $P \cap Q \cap R = \{6\}$

   (iii) $\overline{P \cup Q \cup R} = \{7, 11, 13, 14\}$

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(iv) $\overline{P \cap Q \cap R}$ = {1, 2, 3, 4, 5, 7, 8, 9, 10, 11, 12, 13, 14, 15}

(v) (P ∪ Q) ∩ R = {1, 2, 3, 4, 6, 8, 10, 12} ∩ {3, 6, 9, 12, 15} = {3, 6}

(vi) (P ∩ Q) ∪ R = {2, 4, 6} ∪ {3, 6, 9, 12, 15} = {2, 3, 4, 6, 9, 12, 15}

4. **a)** **If A = {n, e, p, a, l} and B = {b, h, u, t, a, n}, find**
   (i) A∪B   (ii) A∩B   (iii) A – B   (iv) B – A
   Also, represent them in Venn-diagrams.

   Solution:
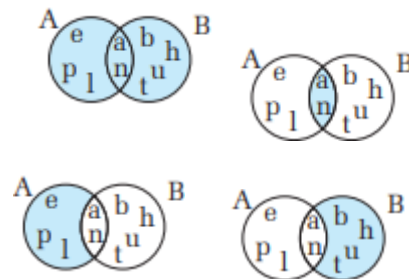   Here, A = {n, e, p, a, l} and B = {b, h, u, t, a, n}
   Now,

   (i)   A∪B   = {n, e, p, a, l} ∪ {b, h, u, t, a, n}
                   = {a, b, e, h, l, n, p, t, u}

   (ii) A∩B   = {n, e, p, a, l} ∪ {b, h, u, t, a, n}
                   = {a, n}

   (iii) A – B   = {n, e, p, a, l} – {b, h, u, t, a, n}
                   = {e, p, l}

   (iv) B – A   = {b, h, u, t, a, n} – {n, e, p, a, l}
                   = {b, h, t, u}

   b) **Let P = {x: x ∈ N and x < 10} and Q= {y: y is a factor of 8}, find**
   (i) P ∪ Q   (ii) P ∩ Q   (iii) P – Q   (iv) Q – P
   Also, show these operations in Venn-diagrams.

   Solution:
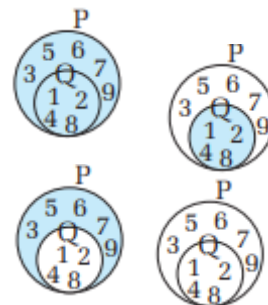   Here, P = {1, 2, 3, 4, 5, 6, 7, 8, 9} and Q = {1, 2, 4, 8}
   Now,

   (i)   P ∪ Q   = {1, 2, 3, 4, 5, 6, 7, 8, 9} ∪ {1, 2, 4, 8}
                    = {1, 2, 3, 4, 5, 6, 7, 8, 9}

   (ii)   P ∩ Q   = {1, 2, 3, 4, 5, 6, 7, 8, 9} ∩ {1, 2, 4, 8}
                    = {1, 2, 4, 8}

   (iii) P – Q   = {1, 2, 3, 4, 5, 6, 7, 8, 9} – {1, 2, 4, 8}
                    = {3, 5, 6, 7, 9}

   (iv)   Q – P   = {1, 2, 4, 8} – {1, 2, 3, 4, 5, 6, 7, 8, 9}
                    = { }

   c) **If M = {x: x is an odd numbers between 10 and 20} and Q= {y: y is a prime number between 15 and 25}, find and show the following operations in Venn-diagrams.**
   (i) M ∪ N   (ii) M ∩ N   (iii) M – N   (iv) N – M

   Solution:
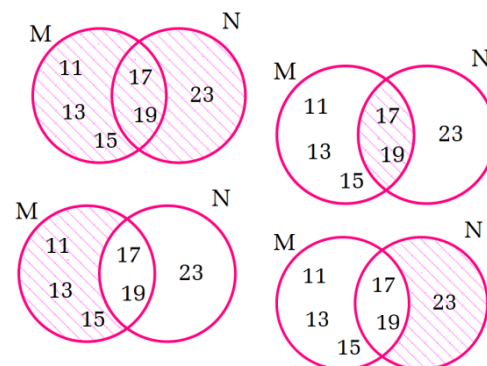   Here, M = {11, 13, 15, 17, 19} and N = {17, 19, 23}
   Now,

   (i)   M ∪ N   = {11, 13, 15, 17, 19} ∪ {17, 19, 23}
                    = {11, 13, 15, 17, 19, 23}

   (ii)   M ∩ N   = {11, 13, 15, 17, 19} ∩ {17, 19, 23}
                    = {17, 19}

   (iii) M – N   = {11, 13, 15, 17, 19} – {17, 19, 23}
                    = {11, 13, 15}
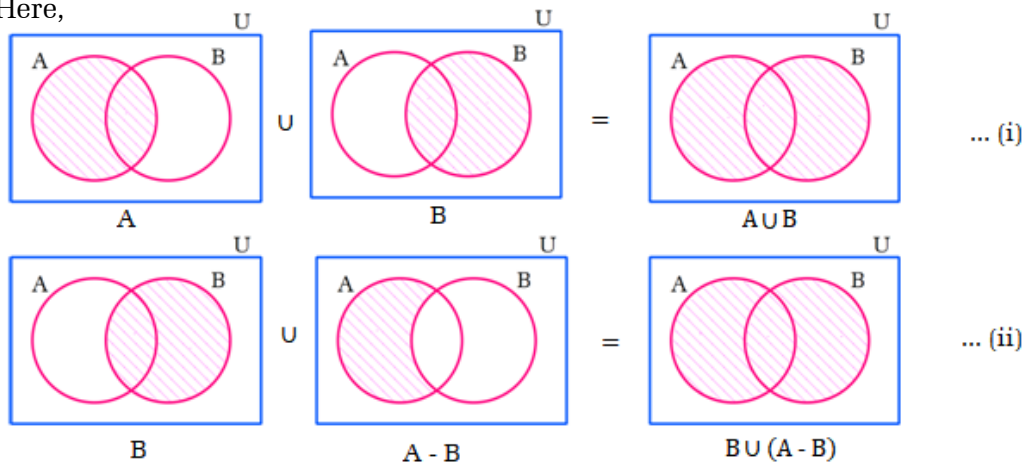
   (iv)   N – M   = {17, 19, 23} – {11, 13, 15, 17, 19}
                    = {23}

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5. **a)** *Assuming that A and B are two overlapping sets, draw two separate Venn-diagrams to verify A ∪ B = B ∪ (A – B) by shading.*
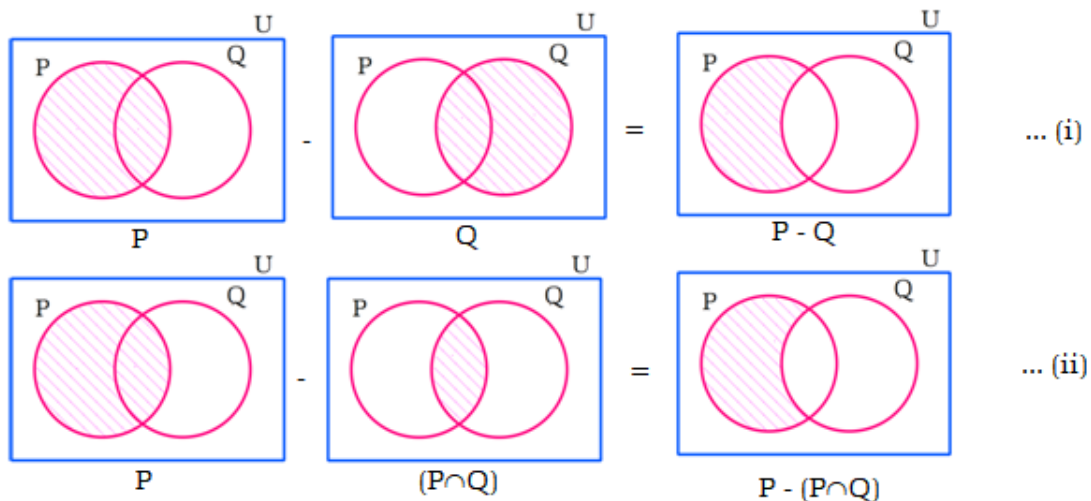
Solution:

Here,

From (i) and (ii), we get

$$A ∪ B = B ∪ (A – B)$$

b) *Let P and Q are two overlapping sets. Draw two separate Venn-diagrams of P – Q and P – (P ∩ Q) and verify P – Q = P – (P ∩ Q) by shading.*

From (i) and (ii), we get

$$P – Q = P – (P ∩ Q)$$

6. **a)** *If A = {1, 2, 4, 8} and B = {4, 6, 8, 10}, find (A – B) ∪ (B – A).*

Solution:

Here,

A = {1, 2, 4, 8} and B = {4, 6, 8, 10}

Now, A – B = {1, 2, 4, 8} – {4, 6, 8, 10} = {1, 2}

Again, B – A = {4, 6, 8, 10} – {1, 2, 4, 8} = {6, 10}

∴ (A – B) ∪ (B – A) = {1, 2} ∪ {6, 10} = {1, 2, 6, 10}

b) *Find the symmetric difference between the following sets.*

(i) A = {m, a, t, h} and B = {m, i, n, d, e, r}   (ii) P = {2, 3, 5, 7, 11} and Q={1, 3, 5, 11}

Solution:

Here,

(i) A = {m, a, t, h} and B = {m, i, n, d, e, r}

Now, A – B = {m, a, t, h} – {m, i, n, d, e, r} = {a, t, h}

Again, B – A = {m, i, n, d, e, r} – {m, a, t, h} = {i, n, d, e, r}

∴ A Δ B = (A – B) ∪ (B – A) = {a, t, h}∪ {i, n, d, e, r} = {a, d, e, h, i, n, r, t}

(ii) P = {2, 3, 5, 7, 11} and Q ={1, 3, 5, 11}

Now, P – Q = {2, 3, 5, 7, 11} – {1, 3, 5, 11} = {2, 7}

Again, Q – P = {1, 3, 5, 11} – {2, 3, 5, 7, 11} = {1}

∴ P Δ Q = (P – Q) ∪ (Q – P) = {2, 7}∪ {1} = {1, 2, 7}

7. **a) If U = {1, 2, 3, …, 10}, A ={1, 3, 5, 7, 9} and B = {2, 3, 5, 7}, find the following sets.**

(i) $\overline{A}$　　　　**(ii) $\overline{B}$**　　　　**(iii) $\overline{A} ∪ \overline{B}$**　　　　**(iv) $\overline{A} ∩ \overline{B}$**

(v) $\overline{A ∪ B}$　　　**(vi) $\overline{A ∩ B}$**　　　**(vii) $\overline{\overline{A}}$**　　　**(viii) $\overline{\overline{B}}$**

Solution:

Here, U = {1, 2, 3, …, 10}, A ={1, 3, 5, 7, 9} and B = {2, 3, 5, 7}

(i) $\overline{A}$ = {2, 4, 6, 8, 10}　　　　　　(ii) $\overline{B}$ = {1, 4, 6, 8, 9, 10}

(iii) $\overline{A} ∪ \overline{B}$ = {1, 2, 4, 6, 8, 9, 10}　　(iv) $\overline{A} ∩ \overline{B}$ = {4, 6, 8}

(v) A ∪ B ={1, 2, 3, 5, 7, 9}　　　　∴ $\overline{A ∪ B}$ = {4, 6, 8}

(vi) A ∩ B = {3, 5, 7}　　　　　　∴ $\overline{A ∩ B}$ = {1, 2, 4, 6, 8, 9, 10

(vii) $\overline{\overline{A}}$ = U - $\overline{A}$ = {1, 3, 5, 7, 9}　　(viii) $\overline{\overline{B}}$ = U - $\overline{B}$ = {2, 3, 5, 7}

b) If U = {1, 2, 3, …, 15} and A ={2, 4, 6, 8, 10, 12, 14}, find:

(i) $\overline{A}$　　　　**(ii) A ∪ $\overline{A}$**　　　　**(iii) A ∩ $\overline{A}$**　　　　**(iv) $\overline{U}$**

Solution:

Here, U = {1, 2, 3, …, 15} and A ={2, 4, 6, 8, 10, 12, 14}, Now,

(i) $\overline{A}$ = U – A = {1, 2, 3, …, 15} – {2, 4, 6, 8, 10, 12, 14} ={1, 3, 5, 7, 9, 11, 13, 15}

(ii) A ∪ $\overline{A}$ = {2, 4, 6, 8, 10, 12, 14}∪{1, 3, 5, 7, 9, 11, 13, 15}= {1, 2, 3, …, 15}=U

(iii) A ∩ $\overline{A}$ ={2, 4, 6, 8, 10, 12, 14}∩{1, 3, 5, 7, 9, 11, 13, 15}= ϕ

(iv) $\overline{U}$ = U – U = {1, 2, 3, …, 15} – {1, 2, 3, …, 15} = ϕ

Creative section

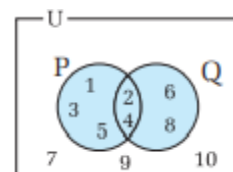**8.** **a) If U = {1, 2, 3, … 10}, P = {1, 2, 3, 4, 5}and Q = {2, 4, 6, 8}, list the elements of the following set operations and represent them by shading in Venn-diagrams.**

(i) P ∪ Q and $\overline{P ∪ Q}$　　　**(ii) P ∩ Q and $\overline{P ∩ Q}$**　　　**(iii) P – Q and $\overline{P – Q}$**

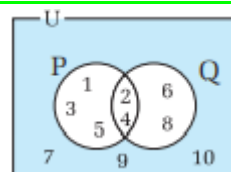**(iv) Q – P and $\overline{Q – P}$**　　　**(v) $\overline{P} ∪ \overline{Q}$**　　　　**(vi) $\overline{P} ∩ \overline{Q}$**

Solution:

Here, U = {1, 2, 3, … 10}, P = {1, 2, 3, 4, 5}and Q = {2, 4, 6, 8}

(i) P ∪ Q = {1, 2, 3, 4, 5} ∪ {2, 4, 6, 8}

= {1, 2, 3, 4, 5, 6, 8}

$$\overline{P \cup Q} = U - (P \cup Q)$$
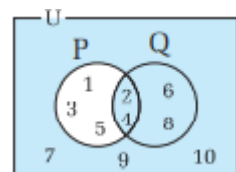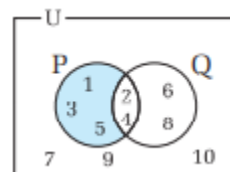$$= \{1, 2, 3, \dots 10\} - \{1, 2, 3, 4, 5, 6, 8\}$$
$$= \{7, 9, 10\}$$

(ii) $\quad P \cap Q = \{1, 2, 3, 4, 5\} \cap \{2, 4, 6, 8\}$
$$= \{2, 4\}$$

$$\overline{P \cap Q} = U - (P \cap Q)$$
$$= \{1, 2, 3, \dots 10\} - \{2, 4\}$$
$$= \{1, 3, 5, 6, 7, 8, 9, 10\}$$
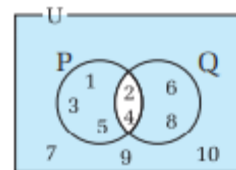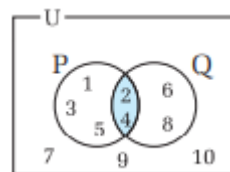
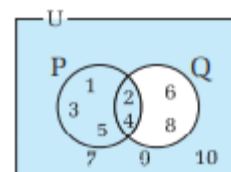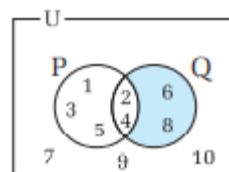**(iii)** $\quad P - Q = \{1, 2, 3, 4, 5\} - \{2, 4, 6, 8\}$
$$= \{1, 3, 5\}$$

$$\overline{P - Q} = U - (P - Q)$$
$$= \{1, 2, 3, \dots 10\} - \{1, 3, 5\}$$
$$= \{2, 4, 6, 7, 8, 9, 10\}$$

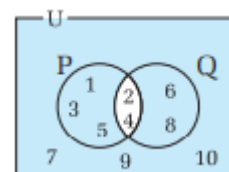**(iv)** $\quad Q - P = \{2, 4, 6, 8\} - \{1, 2, 3, 4, 5\}$
$$= \{6, 8\}$$

$$\overline{Q - P} = U - (Q - P)$$
$$= \{1, 2, 3, \dots 10\} - \{6, 8\}$$
$$= \{1, 2, 3, 4, 5, 7, 9, 10\}$$

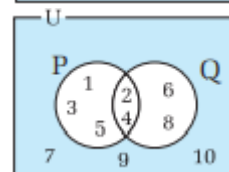**(v)** $\quad \overline{P} = U - P = \{6, 7, 8, 9, 10\}$ and

$$\overline{Q} = U - Q = \{1, 3, 5, 7, 9, 10\}$$

$$\overline{P} \cup \overline{Q} = \{6, 7, 8, 9, 10\} \cup \{1, 3, 5, 7, 9, 10\}$$
$$= \{1, 3, 5, 6, 7, 8, 9, 10\}$$

(vi) $\quad \overline{P} \cap \overline{Q} = \{6, 7, 8, 9, 10\} \cap \{1, 3, 5, 7, 9, 10\}$
$$= \{7, 9, 10\}$$

b) $A = \{1, 3, 5, 7, 9, 11\}$, $B = \{1, 2, 3, 4, 5, 6, 7\}$ *and* $C = \{3, 6, 9, 12, 15\}$ *are the subsets of the universal set* $U = \{1, 2, 3, \dots, 15\}$. *List the elements of the following set operations and illustrate them in Venn-diagrams by shading.*

(i) $A \cup B \cup C$ *and* $\overline{A \cup B \cup C}$ $\qquad$ *(ii)* $A \cap B \cap C$ *and* $\overline{A \cap B \cap C}$

(iii) $(A \cup B) \cap C$ *and* $\overline{(A \cup B) \cap C}$ $\qquad$ *(iv)* $A \cap (B \cup C)$ *and* $\overline{A \cap (B \cup C)}$

(v) $(A - B) \cup C$ *and* $\overline{(A - B) \cup C}$ $\qquad\qquad$ *(vi)* $A \cup (B - C)$ *and* $\overline{A \cup (B - C)}$
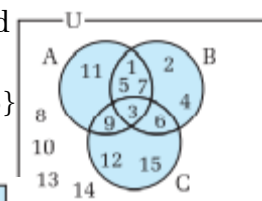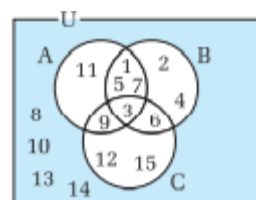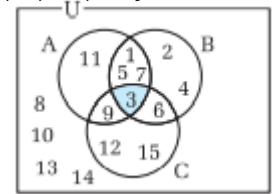
Solution:

Here, $U = \{1, 2, 3, \dots 15\}$, $A = \{1, 3, 5, 7, 9, 11\}$, $B = \{1, 2, 3, 4, 5, 6, 7\}$ and
$C = \{3, 6, 9, 12, 15\}$

(i) $\quad A \cup B \cup C = \{1, 3, 5, 7, 9, 11\} \cup \{1, 2, 3, 4, 5, 6, 7\} \cup \{3, 6, 9, 12, 15\}$
$$= \{1, 2, 3, 4, 5, 6, 7, 9, 11, 12, 15\}$$

$$\overline{A \cup B \cup C} = U - (A \cup B \cup C) = \{8, 10, 13, 14\}$$

(ii) A ∩ B ∩ C= {1, 3, 5, 7, 9, 11}∩{1, 2, 3, 4, 5, 6, 7}∩ {3, 6, 9, 12, 15}

   = {3}

$\overline{A \cap B \cap C}$ = U – (A ∩ B ∩ C)

   = {1, 2, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15}
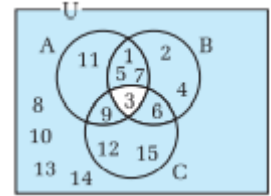
(iii) A ∪ B   = {1, 3, 5, 7, 9, 11}∪ {1, 2, 3, 4, 5, 6, 7}

   = {1, 2, 3, 4, 5, 6, 7, 9, 11}

(A ∪ B) ∩ C = {1, 2, 3, 4, 5, 6, 7, 9, 11}∩ {3, 6, 9, 12, 15}

   = {3, 6, 9}
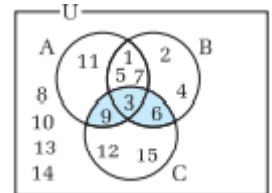
$\overline{(A \cup B) \cap C}$ = U – [(A ∪ B) ∩ C]

   = {1, 2, 3, ... 15} – {3, 6, 9}

   = {1, 2, 4, 5, 7, 8, 10, 11, 12, 13, 14, 15}
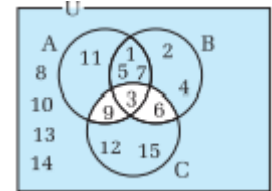
(iv) B ∪ C   = {1, 2, 3, 4, 5, 6, 7}∪ {3, 6, 9, 12, 15}

   = {1, 2, 3, 4, 5, 6, 7, 9, 12, 15}

A ∩ (B ∪ C) = {1, 3, 5, 7, 9, 11}∩ {1, 2, 3, 4, 5, 6, 7, 9, 12, 15}

   = {1, 3, 5, 7, 9}
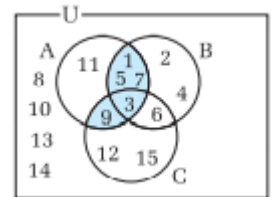
$\overline{(A \cup B) \cap C}$ = U – [A ∩ (B ∪ C)]

   = {1, 2, 3, ... 15} – {1, 3, 5, 7, 9}

   = {2, 4, 6, 8, 10, 11, 12, 13, 14, 15}
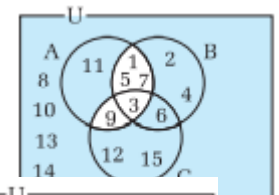
(v) (A – B)   = {1, 3, 5, 7, 9, 11} – {1, 2, 3, 4, 5, 6, 7}

   = {9, 11}

(A – B) ∪ C = {9, 11} ∪ {3, 6, 9, 12, 15}

   = {3, 6, 9, 11, 12, 15}
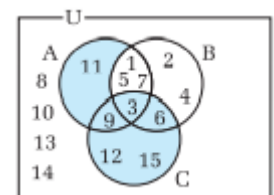
$\overline{(A – B) \cup C}$ = {1, 2, 3, ..., 15} – {3, 6, 9, 11, 12, 15}

   = {1, 2, 4, 5, 7, 8, 10, 13, 14}
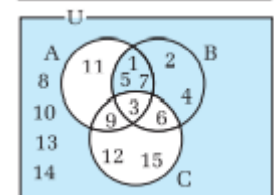
(vi) (B – C)   = {1, 2, 3, 4, 5, 6, 7} – {3, 6, 9, 12, 15}

   = {1, 2, 4, 5, 7}

A ∪ (B – C) = {1, 3, 5, 7, 9, 11} ∪ {1, 2, 4, 5, 7}

   = {1, 2, 3, 4, 5, 7, 9, 11}

8

$$\overline{A \cup (B - C)} = \{1, 2, 3, …, 15\} - \{1, 2, 3, 4, 5, 7, 9, 11\}$$

$$= \{6, 8, 10, 12, 13, 14, 15\}$$
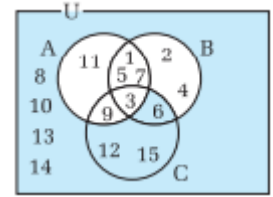
9. **a)** **If $U = \{0, 1, 2, …., 10\}$, $A = \{2, 3, 5, 7\}$ and $B = \{1, 3, 5, 7, 9\}$, verify the following operations. (i) $\overline{A \cup B} = \overline{A} \cap \overline{B}$ (ii) $\overline{A \cap B} = \overline{A} \cup \overline{B}$**

Solution:

Here, $U = \{0, 1, 2, …., 10\}$, $A = \{2, 3, 5, 7\}$ and $B = \{1, 3, 5, 7, 9\}$

(i) To verify: $\overline{A \cup B} = \overline{A} \cap \overline{B}$

Now, $A \cup B = \{2, 3, 5, 7\} \cup \{1, 3, 5, 7, 9\} = \{1, 2, 3, 5, 7, 9\}$

$\therefore \quad \overline{A \cup B} = U - (A \cup B) = \{0, 1, 2, …., 10\} - \{1, 2, 3, 5, 7, 9\} = \{0, 4, 6, 8, 10\}$ …

(1)

Again, $\overline{A} = U - A = \{0, 1, 2, …., 10\} - \{2, 3, 5, 7\} = \{0, 1, 4, 6, 8, 9, 10\}$

$\overline{B} = U - B = \{0, 1, 2, …., 10\} - \{1, 3, 5, 7, 9\} = \{0, 2, 4, 6, 8, 10\}$

$\therefore \overline{A} \cap \overline{B} = \{0, 1, 4, 6, 8, 9, 10\} \cap \{0, 2, 4, 6, 8, 10\} = \{0, 4, 6, 8, 10\}$ …

(2)

From (1) and (2), we get $\overline{A \cup B} = \overline{A} \cap \overline{B}$ verified.

(ii) To verify: $\overline{A \cap B} = \overline{A} \cup \overline{B}$
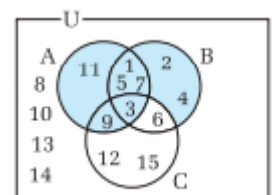
Now, $A \cap B = \{2, 3, 5, 7\} \cap \{1, 3, 5, 7, 9\} = \{3, 5, 7\}$

$\therefore \quad \overline{A \cap B} = U - (A \cap B) = \{0, 1, 2, …., 10\} - \{3, 5, 7\} = \{0, 1, 2, 4, 6, 8, 9, 10\}$ …

(1)

Again, $\overline{A} = U - A = \{0, 1, 2, …., 10\} - \{2, 3, 5, 7\} = \{0, 1, 4, 6, 8, 9, 10\}$

$\overline{B} = U - B = \{0, 1, 2, …., 10\} - \{1, 3, 5, 7, 9\} = \{0, 2, 4, 6, 8, 10\}$

$\therefore \overline{A} \cup \overline{B} = \{0, 1, 4, 6, 8, 9, 10\} \cup \{0, 2, 4, 6, 8, 10\} = \{0, 1, 2, 4, 6, 8, 9, 10\}$ …

(2)

From (1) and (2), we get $\overline{A \cap B} = \overline{A} \cup \overline{B}$ verified.

b) **If a universal set $U = \{x : x \in N, x \le 10\}$, $A = \{y : y = 2n, n \in N, n < 5\}$ and $B = \{z : z = 3n, n \in N, n < 4\}$, prove that**

(i) $A - B = \overline{B} - \overline{A}$ **(ii) $A \, \Delta \, B = \overline{A} \, \Delta \, \overline{B}$**

Solution:

Here, $U = \{x : x \in N, x \le 10\} = \{1, 2, …., 10\}$, $A = \{y : y = 2n, n \in N, n < 5\} = \{2, 4, 6, 8,\}$ and $B = \{z : z = 3n, n \in N, n < 4\} = \{3, 6, 9\}$

(i) To verify: $A - B = \overline{B} - \overline{A}$

Now, $A - B = \{2, 4, 6, 8\} - \{3, 6, 9\} = \{2, 4, 8\}$ … (1)

Again, $\overline{A} = U - A = \{1, 2, …., 10\} - \{2, 4, 6, 8\} = \{1, 3, 5, 7, 9, 10\}$

$$\overline{B} = U - B = \{1, 2, ..., 10\} - \{3, 6, 9\} = \{1, 2, 4, 5, 7, 8, 10\}$$

$$\therefore \ \overline{B} - \overline{A} \ = \{1, 2, 4, 5, 7, 8, 10\} - \{1, 3, 5, 7, 9, 10\} = \{2, 4, 8\} \qquad ... (2)$$

From (1) and (2), we get A – B = $\overline{B} - \overline{A}$ \qquad\qquad\qquad verified.

(ii) \qquad To verify: A ∆ B = $\overline{A} \, \Delta \, \overline{B}$

Now, \quad A – B = {2, 4, 6, 8} – {3, 6, 9} = {2, 4, 8}
B – A = {3, 6, 9} – {2, 4, 6, 8} = {3, 9}

$\therefore$ \ A ∆ B = (A – B) ∪ (B – A) = {2, 4, 8} ∪ {3, 9} = {2, 3, 4, 8, 9} \qquad ... (1)

Again, $\overline{A}$ = U – A = {0, 1, 2, ..., 10} – {2, 3, 5, 7} = {1, 4, 6, 8, 9, 10}

$\overline{B}$ = U – B = {0, 1, 2, ..., 10} – {1, 3, 5, 7, 9} = {2, 4, 6, 8, 10}

$\overline{A} - \overline{B}$ = {1, 3, 5, 7, 9, 10} - {1, 2, 4, 5, 7, 8, 10} = {3, 9}

$\overline{B} - \overline{A}$ \ = {1, 2, 4, 5, 7, 8, 10} – {1, 3, 5, 7, 9, 10} = {2, 4, 8}

$\therefore \ \overline{A} \, \Delta \, \overline{B}$ \ = ($\overline{A} - \overline{B}$) ∪ ($\overline{B} - \overline{A}$) = {3, 9} ∪ {2, 4, 8} = {2, 3, 4, 8, 9} \qquad ... (2)

From (1) and (2), we get A ∆ B = $\overline{A} \, \Delta \, \overline{B}$ \qquad\qquad verified.

---

10. \quad **a)** \quad *If P = {1, 2, 3, 4, 5, 6}, Q = {2, 4, 6, 8}, and R = {3, 6, 9, 12}, explore the relationship between the following operations.*
(i) P ∪ (Q ∩ R) and (P ∪ Q) ∩ (P ∪ R) \qquad *(ii) P ∩ (Q ∪ R) and (P ∩ Q) ∪ (P ∩ R)*
(iii) P – (Q ∪ R) and (P – Q) ∩ (P – R) \qquad *(iv) P – (Q ∩ R) and (P – Q) ∪ (P – R)*

Solution:

Here, P = {1, 2, 3, 4, 5, 6}, Q = {2, 4, 6, 8}, and R = {3, 6, 9, 12}

(i) \qquad Q ∩ R = {2, 4, 6, 8} ∩ {3, 6, 9, 12} = {6}

$\therefore$ P ∪ (Q ∩ R) = {1, 2, 3, 4, 5, 6} ∪ {6} = {1, 2, 3, 4, 5, 6} \qquad\qquad\qquad ...

(A)

Also, \qquad P ∪ Q = {1, 2, 3, 4, 5, 6} ∪ {2, 4, 6, 8} = {1, 2, 3, 4, 5, 6, 8}

\qquad\qquad P ∪ R \ = {1, 2, 3, 4, 5, 6} ∪ {3, 6, 9, 12} = {1, 2, 3, 4, 5, 6, 9, 12}

$\therefore$ (P ∪ Q) ∩ (P ∪ R) = {{1, 2, 3, 4, 5, 6, 8}} ∩ {1, 2, 3, 4, 5, 6, 9, 12} = {1, 2, 3, 4, 5, 6} \ ...

(B)

From (A) and (B), we observed that P ∪ (Q ∩ R) = (P ∪ Q) ∩ (P ∪ R).

(ii) \qquad Q ∪ R = {2, 4, 6, 8} ∩ {3, 6, 9, 12} = {2, 3, 4, 6, 8, 9, 12}
$\therefore$ P ∩ (Q ∪ R) = {1, 2, 3, 4, 5, 6} ∩ {2, 3, 4, 6, 8, 9, 12} = {2, 3, 4, 6}
\qquad ... (A)
Also, \qquad P ∩ Q = {1, 2, 3, 4, 5, 6} ∩ {2, 4, 6, 8} = {2, 4, 6}
\qquad\qquad P ∩ R \ = {1, 2, 3, 4, 5, 6} ∩ {3, 6, 9, 12} = {3, 6}
$\therefore$ (P ∩ Q) ∪ (P ∩ R) = {2, 4, 6} ∩ {3, 6} = {2, 3, 4, 6} \qquad\qquad\qquad ... (B)
From (A) and (B), we observed that P ∩ (Q ∪ R) = (P ∩ Q) ∪ (P ∩ R)

(iii) \quad Q ∪ R = {2, 4, 6, 8} ∪ {3, 6, 9, 12} = {2, 3, 4, 6, 8, 9, 12}
$\therefore$ P – (Q ∪ R) = {1, 2, 3, 4, 5, 6} – {2, 3, 4, 6, 8, 9, 12} = {1, 5} \qquad\qquad ... (A)

Also, $P - Q = \{1, 2, 3, 4, 5, 6\} - \{2, 4, 6, 8\} = \{1, 3, 5\}$
          $P - R = \{1, 2, 3, 4, 5, 6\} - \{3, 6, 9, 12\} = \{1, 2, 4, 5\}$
$\therefore (P - Q) \cap (P - R) = \{1, 3, 5\} \cap \{1, 2, 4, 5\} = \{1, 5\}$ ... (B)
From (A) and (B), we observed that $P - (Q \cup R) = (P - Q) \cap (P - R)$

(iv) $Q \cap R = \{2, 4, 6, 8\} \cap \{3, 6, 9, 12\} = \{6\}$
$\therefore P - (Q \cap R) = \{1, 2, 3, 4, 5, 6\} - \{6\} = \{1, 2, 3, 4, 5\}$ ... (A)
Also, $P - Q = \{1, 2, 3, 4, 5, 6\} - \{2, 4, 6, 8\} = \{1, 3, 5\}$
          $P - R = \{1, 2, 3, 4, 5, 6\} - \{3, 6, 9, 12\} = \{1, 2, 4, 5\}$
$\therefore (P - Q) \cup (P - R) = \{1, 3, 5\} \cup \{1, 2, 4, 5\} = \{1, 2, 3, 4, 5\}$ ... (B)
From (A) and (B), we observed that $P - (Q \cup R) = (P - Q) \cup (P - R)$

b)   **A, B and C are the subsets of a universal set U. If U = {x: x ∈ N, x≤12}, A = {odd numbers less than 10}, B = {prime numbers less than 12}, and R = {prime numbers less than 6}, verify the following operations.**

(i) $\overline{A \cap B \cap C} = \overline{A} \cup \overline{B} \cup \overline{C}$ **(ii)** $\overline{A \cup B \cup C} = \overline{A} \cap \overline{B} \cap \overline{C}$

Solution:

Here, $U = \{1, 2, ..., 12,\ A = \{1, 3, 5, 7, 9\}$, $B = \{2, 3, 5, 7, 11\}$ and $C = \{2, 3, 5\}$

(i)   To verify: $\overline{A \cap B \cap C} = \overline{A} \cup \overline{B} \cup \overline{C}$

Now, $A \cap B \cap C = \{3, 5\}$

$\therefore \overline{A \cap B \cap C} = U - (A \cap B \cap C) \qquad = \{1, 2, 4, 6, 7, 8, 9, 10, 11, 12\}$ ... (1)

Again, $\overline{A} = U - A = \{2, 4, 6, 8, 10, 12\}$, $\overline{B} = U - B = \{1, 4, 6, 8, 9, 10, 12\}$ and

$\overline{C} = U - C = \{1, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$

$\therefore \overline{A} \cup \overline{B} \cup \overline{C} = \{1, 2, 4, 6, 7, 8, 9, 10, 11, 12\}$ ... (2)

From (1) and (2), we get $\overline{A \cap B \cap C} = \overline{A} \cup \overline{B} \cup \overline{C}$ verified.

(ii)   To verify: $\overline{A \cup B \cup C} = \overline{A} \cap \overline{B} \cap \overline{C}$

Now, $A \cup B \cup C = \{1, 2, 3, 5, 7. 9, 11\}$

$\therefore \overline{A \cup B \cup C} = U - (A \cup B \cup C) = \{4, 6, 8, 10, 12\}$ ... (1)

Again, $\overline{A} \cap \overline{B} \cap \overline{C} = \{4, 6, 8, 10, 12\}$ ... (2)

From (1) and (2), we get $\overline{A \cup B \cup C} = \overline{A} \cap \overline{B} \cap \overline{C}$ verified.

11. **a)**   **If A = {2, 4, 6, 8, 10} and B = {1, 3, 5, 7, 9} are two disjoint sets verify that n(A∪B) = n(A) + n(B).**

Solution:

Here, $A = \{2, 4, 6, 8, 10\} \therefore n(A) = 5$ and $B = \{1, 3, 5, 7, 9\} \therefore n(B) = 5$

Now, $A \cup B = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\} \therefore n(A \cup B) = 10$

Again, $n(A \cup B) = 10 = 5 + 5 = n(A) + n(B)$
Hence, $n(A \cup B) = n(A) + n(B)$.   Proved

b)   **If A = {2, 3, 5, 7} and B = {1, 2, 3, 4, 6, 12} are two overlapping sets, show that n (A∪B) = n (A) + n (B) – n (A∩B)**

Solution:

Here, A = {2, 3, 5, 7} $\therefore$ n (A) = 4 and B = {1, 2, 3, 4, 6, 12} $\therefore$ n (B) = 6

Now, A∪B = {1, 2, 3, 4, 5, 6, 7, 12} $\therefore$ n (A∪B) = 8,   A∩B = {2, 3} $\therefore$ n (A∩B) = 2

$\therefore$ n (A∪B) = 8 = 4 + 6 − 2 = n (A) + n (B) − n (A∩B)

Hence, n (A∪B) = n(A) + n(B) − n(A∩B)   Proved

General section

1. **a) A and B are any two disjoint sets. If n(A) = x and n(B) = y, find n(A ∪ B).**
    Solution:
    Here, A and B are any two disjoint sets. n (A) = x and n (B) = y
    $\therefore$ n (A ∪ B) = n (A) + n (B) = x + y

    b) If n (A) = p, n(B) = q, and n (A ∩ B) = r, show this information in a Venn-diagram and show that n (A ∪ B) = n(A) + n(B) − n(A ∩ B).
    Solution:

    Here, n(A) = p, n(B) = q, and n (A ∩ B) = r
    From Venn-diagram,
    $\quad$ n (A ∪ B) $\quad$ = (p −r) + r + (q − r)
    $\qquad\qquad\qquad$ = p + q − r
    $\qquad\qquad\qquad$ = n (A) + n (B) − n (A ∩ B)
    Hence, n (A ∪ B) = n(A) + n(B) − n(A ∩ B)   Proved
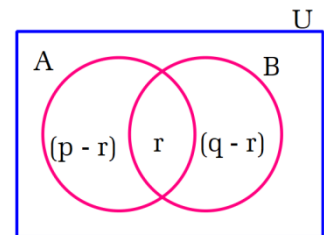
    c) If X and Y are two overlapping subsets of a universal set U, write the relation between n(U), n (X ∪ Y), and n ($\overline{X ∪ Y}$).

    Solution:

    Here, n (U) = n (X ∪ Y) + n ($\overline{X ∪ Y}$).

    d) If A and B are two overlapping subsets of a universal set U, write the relation between n(A), n(A ∩ B) and no (A).
    Solution:
    Here, n_o (A) = n (A) − n(A ∩ B)   or   n (A) = n_o (A) + n(A ∩ B)

2. **From the adjoining Venn-diagram, find the cardinal numbers of the following sets:**
    a) n (U) $\qquad$ **b) n (A)** $\qquad$ **c) n (B)**
    d) n (A ∪ B) $\qquad$ **e) n (A ∩ B)** $\qquad$ **f) n ($\overline{A ∪ B}$)**
    g) n ($\overline{A}$) $\qquad$ **h) n ($\overline{B}$)** $\qquad$ **i) n_o (A)** $\qquad$ **j) n_o (B)**
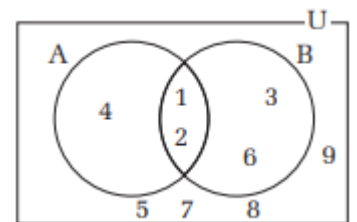
    Solution:
    Here, from Venn-diagram,
    a) n (U) = 9 $\qquad\qquad$ b) n (A) = 3 $\qquad\qquad$ c) n (B) = 4 $\qquad\qquad$ d) n (A ∪ B) = 5
    e) n (A ∩ B)= 2 $\qquad\quad$ f)n ($\overline{A ∪ B}$) = 4 $\qquad$ g)n ($\overline{A}$) = 6 $\qquad$ h) n ($\overline{B}$) = 5
    i) n_o (A) = 1 $\qquad\qquad$ j) n_o (B) = 2

3. **a)** *If n (U) = 65, n (A) = 28, n (B) = 45, and n (A ∩ B) = 20, find*

$\quad$ *(i) n (A ∪ B)* $\qquad$ *(ii) n ($\overline{A \cup B}$)* $\qquad$ *(iii) n_o (A)* $\quad$ *(iv) n_o (B)*

Solution:

$\quad$ Here, n (U) = 65, n (A) = 28, n (B) = 45, and n (A ∩ B) = 20

$\quad$ (i) $\quad$ n (A ∪ B) = n (A) + n (B) – n (A ∩ B) = 28 + 45 – 20 = 53

$\quad$ (ii) $\quad$ n ($\overline{A \cup B}$) = n (U) – n (A ∪ B) = 65 – 53 = 12

$\quad$ (iii) $\quad$ n_o (A) = n (A) – n (A ∩ B) = 28 – 20 = 8

$\quad$ (iv) $\quad$ n_o (B) = n (B) – n (A ∩ B) = 45 – 20 = 25

b) *P and Q are the subsets of a universal set U. If n (P) = 55 %, n (Q) = 50 %, and n*

$\quad$ *($\overline{P \cup Q}$) = 15 %, find:*

$\quad$ *(i) n (P ∪ Q)* $\qquad$ *(ii) n (P ∩ Q)* $\qquad$ *(iii) n (only P)* $\qquad$ *(iv) n (only Q)*

Solution:

$\quad$ Here, n (U) = 100 %, n (P) = 55 %, n (Q) = 50 %, and n ($\overline{P \cup Q}$) = 15 %

$\quad$ (i) $\quad$ n (P ∪ Q) = n (U) – n ($\overline{P \cup Q}$) = 100 – 15 = 85 %

$\quad$ (ii) $\quad$ n (P ∩ Q) = n (P) + n (Q) – n (P ∪ Q) = 55 + 50 – 85 = 20 %

$\quad$ (iii) $\quad$ n (only P) = n (P) – n (P ∩ Q) = 55 – 20 = 35 %

$\quad$ (iv) $\quad$ n (only Q) = n (Q) – n (P ∩ Q) = 50 – 20 = 30 %

c) *X and Y are the subsets of a universal set U. If n (U) = 88, n_o (X) = 35, n_o (Y) = 30, and n (X ∩ Y) = 10, find:*

$\quad$ *(i) n (X)* $\qquad$ *(ii) n (Y)* $\qquad$ *(iii) n (X ∪ Y)* $\qquad$ *(iv) n ($\overline{X \cup Y}$)*

Solution:

$\quad$ Here, n (U) = 88, n_o (X) = 35, n_o (Y) = 30, and n (X ∩ Y) = 10

$\quad$ (i) $\quad$ n (X) = n_o(X) + n (X ∩ Y) $\quad$ = 35 + 10 = 45

$\quad$ (ii) $\quad$ n (Y) = n_o(Y) + n (X ∩ Y) $\quad$ = 30 + 10 = 40

$\quad$ (iii) $\quad$ n (X ∪ Y) = n_o (X) + n_o (Y) – n (X ∩ Y) = 35 + 30 + 10 = 75

$\quad$ (iv) $\quad$ n ($\overline{X \cup Y}$) = n (U) – n (X ∪ Y) = 88 – 75 = 13

Creative section

4. **a)** *In a survey of 600 people in a village of Dhading district, 400 people said they can speak Tamang language, 350 said Nepali language and 200 of them said they can speak both the languages.*

$\quad$ *(i)* $\quad$ *Draw a Venn-diagram to illustrate the above information.*

$\quad$ *(ii)* $\quad$ *How many people can speak Tamang language only?*

$\quad$ *(iii)* $\quad$ *How many people can speak Nepali language only?*

$\quad$ *(iv)* $\quad$ *How many people cannot speak any of two languages?*

Solution:

$\quad$ Let T and N denote the sets of people who can speak Tamang and Nepali languages respectively.

$\quad$ Then, n (U) = 600, n (T) = 400, n (N) = 350 and n (T ∩ N) = 200

$\quad$ Now,

$\quad$ (i) $\quad$ Representing the above information in a Venn-diagram.
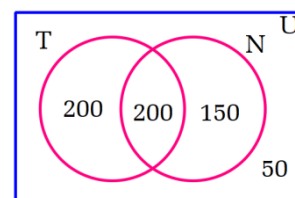
$\quad$ (ii) $\quad$ n_o (T) = n (T) – n (T ∩ N) = 400 – 200 = 200

$\qquad$ ∴ 200 people can speak Tamang language only.

$\quad$ (iii) n_o (N) = n (N) – n (T ∩ N) = 350 – 200 = 150

$\qquad$ ∴ 150 people can speak Nepali language only.

(iv) From Venn-diagram, n ($\overline{T \cup N}$) = 50

Hence, 50 cannot speak any of two languages.

b) **In a survey of 1500 people, 775 of them like Nepal Idol, 975 liked Comedy Champion, and 450 people liked both the shows.**
   (i) **Show the above information in a Venn-diagram.**
   (ii) **How many people did not like both the shows?**

Solution:

Let N and C denote the sets of people who like Nepal Idol and Comedy Champion respectively.
Then, n (U) = 1500, n (N) = 775, n (C) = 975 and n (N ∩ C) = 450
Now,
   (i) Showing the above information in a Venn-diagram.
   (ii) From Venn-diagram, n ($\overline{N \cup C}$) = 200

Hence, 200 people did not like both the shows.
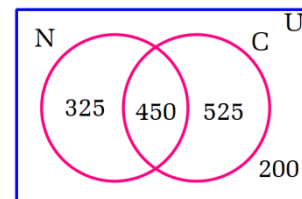
c) **In a group of 250 music lovers, 135 of them like folk songs, and 150 like modern songs. By drawing a Venn-diagram, find:**
   (i) **how many people like both the songs?**
   (ii) **How many people like only folk songs?**

Solution:

Let F and M denote the sets of people who like folk and modern songs respectively.
Then, n (U) = 250, n (F) = 135, n (M) = 150.
Let, n (F ∩ M) = x
Now,
   (i) From a Venn-diagram, (135 – x) + x + (150 – x) = 250
   $$\text{or, } 285 - x = 250$$
   $$\text{or, } x = 35$$
   Thus, 35 people like both the songs.
   (ii) n_o (F) = 135 – x = 135 – 35 = 100
   Hence,100 people like only folk songs.
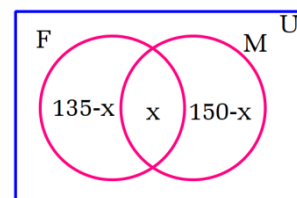
d) **In a survey of a group of farmers, it was found that 80 % farmers have crops farming, 30% farmers have animals farming, and every farmer has at least one farming.**
   (i) **Represent the information in a Venn-diagram.**
   (ii) **What percent of farmers had both farming?**
   (iii) **What percent of farmers had only animals farming?**

Solution:

Let C and A denote the sets of people who have crops and animal farming respectively.
Then, n (U) = 100 (say), n (C) = 80, n (A) = 30 and n ($\overline{C \cup A}$) = 0
Now,
   (i) Representing the above information in a Venn-diagram.
   (ii) From Venn-diagram, (80 – x) + x + (30 – x) = 100
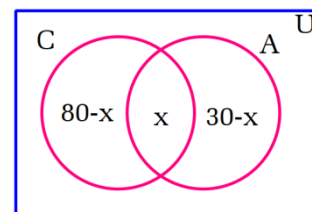 $$\text{or, } 110 - x = 100$$
   $$\text{or, } x = 10$$
   So, 10% people have both the farming.
   (iii) n_o (A) = 30 – x = 30 – 10 = 20
   Hence, 20% people have only animal farming.

5. **a)** *In a group of 500 students, 280 like bananas, 310 like apples, and 55 do not like both the fruits.*

      (i) *Find the number of students who like both the fruits.*

      (ii) *Find the number of students who like only one fruit.*

      (iii) *Show the result in a Venn-diagram.*

Solution:

Let A and B denote the sets of students who like apples and bananas respectively.

Then, n (U) = 500, n (A) = 310, n (B) = 280 and n $(\overline{A \cup B})$ = 55

  (i) We have, n (A ∪ B) = n (U) − n $(\overline{A \cup B})$ = 500 − 55 = 445

    Also, n (A ∩ B) = n (A) + n (B) − n (A ∪ B)

$$= 310 + 280 - 445$$
$$= 145$$

  So, 145 students like both the fruits.

  (ii) The number of students who like only one fruit = n_o (A) + n_o (B)

$$= n (A \cup B) - n (A \cap B)$$
$$= 445 - 145$$
$$= 400$$

  (iii) Showing the above information in a Venn-diagram.

b) *In a survey of 900 students in a school, it was found that 600 students liked tea, 500 liked coffee, and 125 did not like both drinks.*

    (i) *Draw a Venn-diagram to illustrate the above information.*

    (ii) *Find the number of students who like both drinks.*

    (iii) *Find the number of students who like one of these drinks only.*

Solution:

Let T and C denote the sets of students who like tea and coffee respectively.

Then, n (U) = 900, n (T) = 600, n (C) = 500 and n $(\overline{T \cup C})$ = 125

Let, n (T ∩ C) = x

Now,

  (i) Drawing the above information in a Venn-diagram.

  (ii) From Venn-diagram, (600 − x) + x + (500 − x) + 125 = 900

$$\text{or, } 1225 - x = 900$$
$$\text{or, } x = 325$$

  So, 325 students like both drinks.

  (iii) The number of students who like only one drink = n_o (T) + n_o (C)

$$= (600 - x) + (500 - x)$$
$$= (600 - 125) + (500 - 125)$$
$$= 850$$
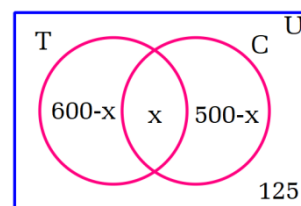
6. **a)** *In a group of 75 students, 20 liked football only, 30 liked cricket only and 18 did not like any of two games?*

    (i) *How many of them liked at least one game?*

    (ii) *Find the number of students who liked both the games.*

    (iii) *How many of them liked football?*

    (iv) *How many of them liked cricket?*

    (v) *Represent the result in a Venn diagram.*

Solution:

Let F and C denote the sets of students who liked football and cricket respectively.

Then, $n(U) = 75$, $n_o(F) = 20$, $n_o(C) = 30$ and $n(\overline{F \cup C}) = 18$

Now,

(i) The number of students who liked at least one game, $n(F \cup C) = n(U) - n(\overline{F \cup C})$

$$= 75 - 18$$
$$= 57$$

(ii) We have, $n(F \cup C) = n_o(F) + n_o(C) + n(F \cap C)$

or, $57 = 20 + 30 + n(F \cap C)$

or, $n(F \cap C) = 7$

∴ 7 students liked both the games.

(iii) The number of students who liked football, $n(F) = n_o(F) + n(F \cap C) = 20 + 7 = 27$

(iv) The number of students who liked cricket, $n(C) = n_o(C) + n(F \cap C) = 30 + 7 = 37$

(v) Representing the result in a Venn diagram
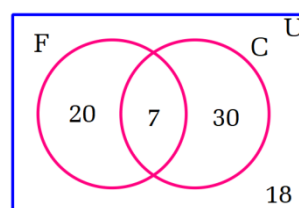
b) *In a survey of 750 tourist who visited Nepal during 'Visit Nepal 2020', it was found that 260 tourists visited Pokhara but not Sauraha, 240 visited Sauraha but not Pokhara and 125 of them did not visit both places.*

(i) Find the number of tourists who visited Pokhara or Sauraha.

(ii) Find the number of tourist who visited Pokhara and Sauraha.

(iii) How many tourists visited Pokhara.

(iv) Show the result in a Venn-diagram.

Solution:

Let P and S denote the sets of tourist who visited Pokhara and Sauraha respectively.

Then, $n(U) = 750$, $n_o(P) = 260$, $n_o(S) = 240$ and $n(\overline{P \cup S}) = 125$

Now,

(i) The number of tourists who visited Pokhara or Sauraha, $n(P \cup S) = n(U) - n(\overline{P \cup S})$

$$= 750 - 125$$
$$= 625$$

(ii) We have, $n(P \cup S) = n_o(P) + n_o(S) + n(P \cap S)$

or, $625 = 260 + 240 + n(P \cap S)$

or, $n(P \cap S) = 125$

∴ 125 tourists visited both Pokhara and Sauraha.

(iii) The number of tourists who visited Pokhara, $n(P) = n_o(P) + n(P \cap S) = 260 + 125 = 385$

(iv) Representing the result in a Venn diagram

Taxation

Competency

- Solving and applying behavioural problems based on arithmetic

Learning Outcomes

- To solve the behavioral problems on taxation, VAT

Level-wise learning objectives

S.N.	Level	Objectives
1.	Knowledge (K)	▪ To define income tax ▪ To identify tax free income ▪ To define VAT ▪ To tell the formulae of finding discount and VAT amounts.
2.	Understanding (U)	▪ To write the income slab under marital status. ▪ To find the social security tax ▪ To calculate discount amount ▪ To find VAT amount
3.	Application (A	▪ To find the income tax ▪ To find the price after VAT
4.	High Ability (HA)	▪ To compare the income taxes paid by an individual and the married couple. ▪ To link various real life/ contemporary problems based on Tax and VAT then solve.

Required Teaching Materials/ Resources

➢ Income tax rates published by IRD, VAT bill, ICT tools etc.

Pre-knowledge:

➢ Check the Pre-knowledge on cost price, selling price, profit and loss, discount and VAT etc.

A.   Income Tax

Teaching Activities

➢ Ask about the yearly income of parents of the student and discuss upon the tax is to be paid to the government.

➢ Divide the students into groups and give them to study the printed form of the present rates of income taxes fixed by Inland Revenue Department (IRD) and discuss upon the following questions

    (i)   What is income tax?

    (ii)   Which authentic body is responsible to collect the tax?

    (iii)   Why should we pay tax to the government?

    (iv)   What do you mean by taxable income?

    (v)   Which incomes are entitles for tax rebate?

    (vi)   What is the rate of social security tax?

➢ Try to clarify about income tax and VAT related problems by browsing the link given in Vedanta ICT corner.

➤ Tell the students to write the important notes and formulae on the colourful chart paper as project work

➤ With some related examples, let the students identify the following formulae then explain them one by one.

      (i)   Taxable income = Yearly income – tax free income

      (ii)   Income tax = rate of tax of taxable income

      (iii)   Tax rate = $\dfrac{\text{Income tax}}{\text{Taxable income}} \times 100\%$

EXERCISE 2.1

General section

1. **a)** **The annual income of a sole proprietor of a grocery shop is Rs 10,00,000. If the tax is exempted up to Rs 4,50,000, what is his/her taxable income?**

Solution:

    Here, taxable income = Rs. 10,00,000 – Rs. 4,50,000 = Rs. 5,50,000

b) **The yearly income of an individual is Rs 4,44,000 with Rs 24,000 remote area allowance. What is his/her taxable income?**

Solution:

    Here, taxable income = Rs. 4,44,000 – Rs. 24,000 = Rs. 4,20,000

c) **The yearly income of an officer is Rs 4,55,880. If he accumulates Rs 45,588 in provident fund and he pays Rs 25,000 as premium of his life insurance in the year, what is his taxable income?**

Solution:

    Here, taxable income = Rs. 4,55,880 – Rs. 45,588 – Rs 25,000 = Rs. 3,85,292

d) **The monthly income of a government servant is Rs 77,280 and he gets the festival expense of one month's salary, what is his taxable income?**

Solution:

    Here, taxable income =12 × Rs. 77,280 + Rs. 77,280 = Rs. 10,04,640

2. **a)** **The monthly salary of an individual is Rs 25,450. If 1% social security tax is charged upto the annual income of Rs 4,00,000, calculate the income tax paid by the individual.**

Solution:

    Here, monthly income = Rs. 25,450

    ∴Yearly income = 12 × Rs 25,450 = Rs. 3,05,400

    Now, income tax need to be paid = 1% of Rs. 3,05,400 = Rs. 3,054

b) **1 % social security tax is charged upto the yearly income of Rs 4,50,000 to a married couple. If the monthly income of a couple is Rs 33,500, how much tax should the couple pay in a year?**

Solution:

    Here, monthly income = Rs. 33,500

    ∴Yearly income = 12 × Rs 33,500 = Rs. 4,02,000

    Now, income tax need to be paid = 1% of Rs. 4, 02,000 = Rs. 4,020

3. *Inland Revenue Department (IRD) has fixed the following rates of income tax for Proprietorship firm. Use it to calculate the income taxes.*

For an individual		For couple	
Income slab	Tax rate	Income slab	Tax rate
Up to Rs 4,00,000	0%	Up to Rs 4,50,000	0%
Rs 4,00,001 to Rs 5,00,000	10%	Rs 4,50,001 to Rs 5,50,000	10%
Rs 5,00,001 to Rs 7,00,000	20%	Rs 5,50,001 to Rs 7,00,000	20%
Rs 7,00,001 to Rs 20,00,000	30%	Rs 7,50,001 to Rs 20,00,000	30%

a) *Mr. Baral has a stationery shop. His annual income is Rs 6,40,000. If he is unmarried, how much income tax should he pay? Find it.*

Solution:

Here,

Annual income of Ram Baral, a stationer = Rs. 6,40,000

= Rs. 4,00,000 + Rs. 1,00,000 + Rs.1,40,000

Now, income tax = 10% of Rs. 1,00,000 + 20% of Rs. 1,40,000

= Rs. 10,000 + Rs. 28,000

= Rs. 38,000

Hence, he should pay Rs. 38,000 income tax.

b) *Mr. Yadav is still unmarried but he is the proprietor of a furniture factory. He earned Rs. 15,00,000 last year, how much income tax did he pay last year?*

Solution:

Here,

Annual income of Mr. Yadav = Rs. 15,00,000

= Rs. 4,00,000 + Rs. 1,00,000 + Rs.2,00,000 + Rs. 8,00,000

Now, income tax = 10% of Rs. 1,00,000 + 20% of Rs. 2,00,000 +30% of Rs. 8,00,000

= Rs. 10,000 + Rs. 40,000 + 2,40,000

= Rs. 2,90,000

Hence, he should pay Rs. 38,000 income tax.

c) *Mrs. Adhikari is the proprietor of boutique training centre. If her annual income is Rs. 6,75,000, how much income tax does she pay?*

Solution:

Here,

Annual income of Mrs. Adhikari = Rs. 6,75,000

= Rs. 4,50,000 + Rs. 1,00,000 + Rs.1,25,000

Now, income tax = 10% of Rs. 1,00,000 + 20% of Rs. 1,25,000

= Rs. 10,000 + Rs. 25,000

= Rs. 35,000

Hence, she pays Rs. 35,000 income tax.

d) *Mr. Manandhar is a married person. He has a registered computer repair service centre. He earned Rs. 9,25,000 in this year. How much tax is charged on his income?*

Solution:

Here,

Annual income = Rs. 9,25,000

= Rs. 4,50,000 + Rs. 1,00,000 + Rs. 2,00,000 + Rs. 1,75,000

Now, income tax = 10% of Rs. 1,00,000 + 20% of Rs. 2,00,000 + 30% of Rs. 1,75,000

= Rs. 10,000 + Rs. 40,000 + Rs. 52,500

= Rs. 1,02,500

Hence, he pays Rs. 1,02,500 income tax.

4. **Study the given income tax rates fixed by IRD and workout the following problems.**

	Assessed as individual			Assessed as couple	
Particulars	Taxable income (Rs)	Tax rate		Taxable income (Rs)	Tax rate
First tax slab	4,00,000	1%		4,50,000	1%
Next	1,00,000 (4,00,001 to 5,00,000)	10%		1,00,000 (4,50,001 to 5,50,000)	10%
Next	2,00,000 (5,00,001 to 7,00,000)	20%		2,00,000 (5,50,001 to 7,50,000)	20%
Next	13,00,000 (7,00,001 to 20,00,000)	30%		12,50,000 (7,50,001 to 20,00,000)	30%
Balance exceeding	20,00,000	36%		20,00,000	36%

a) **The monthly income of an unmarried civil officer is Rs 37,990 and one month's salary is provided as Dashain expense. How much income tax should he/she pay in a year?**

Solution:

Here, yearly income with Dashain expense = 12 × Rs. 37,990 + Rs. 37,990

= Rs. 4,93,870

= Rs. 4,00,000 + Rs. 93,870

Now, the total income tax = 1% of Rs. 4,00,000 + 10% of Rs. 93,870

= Rs. 4,000 + Rs. 9,387

= Rs 13,387

Hence, he/she should pay the income tax of Rs. 13,387 in a year.

b) **The monthly salary of a married couple is Rs 40,500 plus a festival expense of Rs 30,000. Calculate the income tax paid by the couple in a year.**

Solution:

Here, yearly income with festival expense = 12 × Rs. 40,500 + Rs. 30,000

= Rs. 5,16,000

= Rs. 4,50,000 + Rs. 66,000

Now, the total income tax = 1% of Rs. 4,50,000 + 10% of Rs. 66,000

= Rs. 4,500 + Rs. 6,600

= Rs 11,100

Hence, the couple should pay the income tax of Rs. 11,100 in a year.

c) **Mrs. Gurung is a bank Manager in a development bank. Her monthly is Rs 50,000. If her annual income is equivalent to her 15 month's salary, find her income tax in a year.**

Solution:

Here, annual income of Mrs. Gurung = 15 × Rs. 50,000

= Rs. 7,50,000

= Rs. 4,50,000 + Rs. 1,00,000 + Rs. 2,00,000

Now,

Total income tax = 1% of Rs. 4,50,000 + 10% of Rs. 1,00,000 + 20% of Rs. 2,00,000

= Rs. 4,500 + Rs. 10,000 + Rs. 40,000

= Rs 54,500

Hence, she should pay the income tax of Rs. 54,500 in a year.

d) The monthly salary of an individual employee of an INGO is Rs 1,80,000. Calculate the income tax paid by the individual in a year.

Solution:

Here, the annual income of an individual employee = 12 × Rs. 1,80,000

= Rs. 21,60,000

= Rs. 4,00,000 + Rs. 1,00,000 + Rs. 2,00,000 + Rs.13,00,000 + Rs. 1,60,000

Now,

Total income tax = 1% of Rs. 4,00,000 + 10% of Rs. 1,00,000 + 20% of Rs. 2,00,000 + 30% of Rs. 13,00,000 + 36% of Rs. 1,60,000

= Rs. 4,000 + Rs. 10,000 + Rs. 40,000 + Rs. 3,90,000 + Rs. 57,600

= Rs. 5,01,600

Hence, he/she should pay the income tax of Rs. 5,01,600 in a year.

5. a) Mrs. Thakuri deposited Rs. 2,00,000 in her fixed account at a bank for 3 years. The bank pays her the simple interest at the rate of 10% p.a. How much net interest would she get if 5% of interest is charged as income tax?

Solution:

Here, principal (P) = Rs. 2,00,000, time (T) = 3 years and rate (R)=10% p.a.

Now, simple interest (I) $= \dfrac{PTR}{100} = \dfrac{Rs\ 200000 \times 3 \times 10}{100} =$ Rs. 60,000

Also, rate of tax = 5%

∴Tax amount = 5% of Rs. 60,000 = Rs. 3,000

Net interest = Rs. 60,000 – Rs. 3,000 = Rs. 57,000

Hence, she would receive the net interest of Rs. 57,000.

b) Mr. Thapa deposits Rs 50,000 in a bank at the rate of 8% p.a. How much net interest will he get after 4 years if he has to pay 5% of his interest as income tax?

Solution:

Here, principal (P) = Rs. 50,000, time (T) = 4 years and rate (R)= 8% p.a.

Now, simple interest (I) $= \dfrac{PTR}{100} = \dfrac{Rs\ 50000 \times 4 \times 8}{100} =$ Rs. 16,000

Also, rate of tax = 5%

∴Tax amount = 5% of Rs 16,000 = Rs. 800

Net interest = Rs. 16,000 – Rs. 800 = Rs. 15,200

Hence, she would receive the net interest of Rs. 15,200.

d) In the beginning of BS 2076, Dolma deposited Rs 1,20,000 in her account at the rate of 9% p.a. If she paid 5% of her interest as income tax, how much total amount did she receive in the beginning of BS 2079?

Solution:

Here, principal (P) = Rs. 1,20,000, time (T) = 3 years and rate (R)= 9% p.a.

Now, simple interest (I) $= \dfrac{PTR}{100} = \dfrac{Rs\ 120000 \times 3 \times 9}{100} =$ Rs. 32,400

Also, rate of tax = 5%

∴Tax amount = 5% of Rs 32,400= Rs. 1,620

Net interest = Rs. 32,400 – Rs. 1,620 = Rs. 30,780

Again, net amount = Rs. 1,20,000 + Rs. 30,780 = Rs. 1,50,780

Hence, she received the total amount of Rs. 1,50,780.

e) **On the occasion of daughter's 14th birthday, Dharmendra deposits Rs 25,000 in his daughter's bank account at the rate of 6% p.a. If 5% of the interest is charged as income tax, how much amount will she withdraw on her 16th birthday?**

Solution:

Here, principal (P) = Rs. 25,000, time (T) = 2 years and rate (R)= 6% p.a.

Now, simple interest (I) $= \dfrac{PTR}{100} = \dfrac{Rs\ 25000 \times 2 \times 6}{100} = $ Rs. 3,000

Also, rate of tax = 5%

∴Tax amount = 5% of Rs 3,000= Rs. 150

Net interest = Rs. 3,000 – Rs. 150 = Rs. 2,850

Again, net amount = Rs. 25,000 + Rs. 2,850 = Rs. 27,850

Hence, she received the total amount of Rs. 27,850.

6. a) **Mrs. Majhi deposited a certain amount in her bank account at the rate of 6.5% p.a. If she paid 5% of her interest as income tax and received Rs 4940 net interest after 4 years, how much money was deposited by her?**

Solution:

Let, the required sum (P) be Rs x.

Time (T) = 4 years and rate (R) =6.5% p.a.

Now,

Simple interest (I) $= \dfrac{PTR}{100} = \dfrac{x \times 4 \times 6.5}{100} = $ Rs 0.26x

Also, rate of tax = 5%   ∴Tax amount = 5% of Rs 0.26x = Rs 0.013x

According to question, net interest   =Rs 4,940

or, Total interest – tax   = Rs 4,940

or, 0.26x – 0.013x   = Rs 4,940

or, 0.247x   = 4,940

∴ x   = 20,000

Hence, the required sum is Rs 20,000.

b) **Madan Bahadur deposited a sum of money at his bank account at the rate of 10% p.a.. After 5 years, he received Rs 1900, the net interest when 5% of the total interest was charged as income tax. Find, how much sum was deposited by him?**

Solution:

Let, the required sum (P) be Rs x.

Time (T) = 5 years and rate (R) =10% p.a.

Now, simple interest (I) $= \dfrac{PTR}{100} = \dfrac{x \times 5 \times 10}{100} = $ Rs. 0.5x

Also, rate of tax = 5%   ∴Tax amount = 5% of Rs 0.5x = Rs 0.025x

According to question, net interest   =Rs 1,900

or, Total interest – tax = Rs 1,900

or, 0.5x – 0.025x   = Rs 1,900

or, 0.475x   = 1,900

∴ x   = 4,000

Hence, the required sum is Rs. 4,000.

7. **a)** *Mr. Khatiwada is an unmarried secondary level mathematics teacher in a community school. His monthly salary is Rs 39,990 with Rs 2,000 allowance and gets one month's basic salary as festival expense. If 10% and next 13% of his basic salary is deposited in his provident fund and civil investment trust (CIT) respectively, how much income tax should he pay in this year?*

Solution:

Here, monthly basic salary of Mr. Khatiwada = Rs 39,990 – Rs 2,000 = Rs 37,990

Festival expense = Basic salary of one month = Rs 37,990

Monthly provident fund = 10% of Rs 37,990 = Rs 3,799

Monthly deposit at CIT = 13% of Rs 37,990 = Rs 4,938.70

After deducting provident fund and CIT,

his monthly income = Rs 39,990 – Rs 3,799 – Rs Rs 4,938.70 = Rs 31,252.30

∴Taxable income of the year with festival expense = 12 × Rs 31,252.30 + Rs 37,990

$$= Rs\ 4,13,017.60$$
$$= Rs\ 4,00,000 + Rs\ 13,017.60$$

Now, the social security tax for the first Rs 4,00,000 = 1% of Rs 4,00,000 = Rs 4,000

Again, the income tax for Rs Rs 13,017.60 = 10% of Rs 13,017.60

$$= Rs\ 1301.76$$

The total income tax paid by Mr. Khatiwada = Rs 4000 + Rs 1301.76 = Rs 5301.76

Hence, Mr. Khatiwada should pay the income tax of Rs 5,301.76 in a year.

b) *Mrs. Anjali Subba is a medical doctor in a government hospital. Her monthly salary is Rs 50,000 including Rs 2,000 allowance and she receives festival expense equivalent to her one month's basic salary. 10% of her basic salary is deducted as provident fund and she pays Rs 48,500 annually as the premium of her insurance. How much income tax should she pay in a year?*

Solution:

Here, monthly basic salary of Mrs. Anjuli Subba = Rs 50,000 – Rs 2,000 = Rs 48,000

Festival expense = Basic salary of one month = Rs. 48,000

Monthly provident fund = 10% of Rs. 48,000 = Rs 4,800

After deducting provident fund, her monthly income = Rs 50,000 – Rs 4,800

$$= Rs.\ 45,200$$

Premium of insurance = Rs. 48,500

After deducting premium of insurance, taxable income of the year with festival expense

$$= 12 × Rs.\ 45,200 + Rs.\ 48,000 – Rs.\ 48,500$$
$$= Rs.\ 5,41,900$$
$$= Rs.\ 4,50,000 + Rs.\ 91,900$$

Now,

Total income tax = 1% of Rs. 4,50,000 + 10% of Rs.91,900

$$= Rs.\ 4,500 + Rs.\ 9190$$
$$= Rs.\ 13,690$$

Hence, she should pay Rs 13,690 as income tax.

c) *After deducting 10% provident fund, a married person draws Rs 40,500 salary per month and one month's salary as festival expense, the person pays Rs 14,500 annually as the premium of his/her insurance. Calculate the annual income tax paid by the person.*

Here, after deducting 10% provident fund, the monthly salary = Rs. 40,500

Let, the monthly salary of a married person be Rs. x.

Then, x – 10% of x   = Rs 40,500

or, $x - \dfrac{10}{100}x$ = Rs. 40,500

or, $\dfrac{9x}{10}$ = Rs. 40,500

or, x   = Rs. 45,000

Hence, his/her monthly salary is Rs. 45,000

Also, festival expense = Rs. 45,000

After deducting provident fund, the annual income with festival expense

$$= 12 \times \text{Rs. } 40,500 + \text{Rs. } 45,000$$
$$= \text{Rs. } 5,31,000$$

Premium of his/her insurance = Rs. 14,500

∴ After deducting premium of insurance, the taxable income

$$= \text{Rs. } 5,31,000 - \text{Rs. } 14,500$$
$$= \text{Rs. } 5,16,500$$
$$= \text{Rs. } 4,50,000 + \text{Rs. } 66,500$$

Again,

Total income tax = 1% of Rs. 4,50,000 + 10% of Rs. 66,500

$$= \text{Rs. } 4,500 + \text{Rs. } 6,650$$
$$= \text{Rs. } 11,150$$

Hence, the person pays Rs. 11,150 income tax.

d) ***Mr. Sayad Sharma an unmarried employee of a UN Project draws monthly salary of Rs 51,000 after deducting 10% salary in his provident fund and 5% in citizen investment trust. He also receives the festival expense of one month's salary. He pays Rs 22,000 annually as the premium of his life insurance. How much income tax does he pay in a year?***

Solution:

Here,

After deducting 10% provident fund and 5% CIT, the monthly salary = Rs. 51,000

Let, the monthly salary be Rs. x.

Then, x – 10% of x – 5% of x   = Rs 40,500

or, $x - \dfrac{10}{100}x - \dfrac{5}{100}x$ = Rs. 40,500

or, $\dfrac{17x}{20}$ = Rs. 40,500

or, x   = Rs. 60,000

Hence, his monthly salary is Rs. 60,000

Also, festival expense = Rs. 60,000

After deducting provident fund and CIT, the annual income with festival expense

$$= 12 \times \text{Rs. } 51,000 + \text{Rs. } 60,000$$
$$= \text{Rs. } 6,72,000$$

Premium of his/her insurance = Rs. 22,000

∴ After deducting premium of insurance, the taxable income

$$= \text{Rs. } 6,72,000 - \text{Rs. } 22,000$$

= Rs. 6,50,000

= Rs. 4,00,000 + Rs. 1,00,000 + Rs. 1,50,000

Again,

Total income tax = 1% of Rs. 4,00,000 + 10% of Rs. 1,00,000 + 20% of Rs. 1,50,000

= Rs. 4,000 + Rs. 10,000 + Rs. 30,000

= Rs. 44,000

Hence, the he pays Rs. 44,000 income tax.

8. a) ***Mr. and Mrs. Pandey are a married couple. Mr. Pandey is the mayor of a municipality and his monthly salary is Rs 48,000 with Rs 2,000 allowance. Mrs. Pandey is the sole proprietor of a beauty-parlor and her annual income is Rs 6,20,000. Who pays more income tax and by how much?***

Solution:

Here,

For Mr. Pandey, monthly income with allowance = Rs. 48,000

∴His annual income = 12 × Rs. 48,000

= Rs. 5,76,000

= Rs. 4,50,000 + Rs.1,00,000 + Rs. 26,000

Now, income tax = 1% of Rs. 4,50,000 + 10% of Rs.1,00,000 + 20% of Rs. 26,000

= Rs. 4,500 + Rs. 10,000 + Rs. 5,200

= Rs. 19,700

For Mrs. Pandey, yearly income = Rs. 6,20,000

= Rs. 4,50,000 + Rs.1,00,000 + Rs. 70,000

Now, income tax = 0% of Rs. 4,50,000 + 10% of Rs.1,00,000 + 20% of Rs. 70,000

= Rs. 10,000 + Rs. 14,000

= Rs. 24,000

Difference due to their income tax = Rs. 24,000 – Rs. 19,700 = Rs. 4,300

Hence, Mrs. Pandey pays Rs. 4,300 more tax than Mr. Pandey.

b) ***The monthly salary of Ms. Chhiring, an unmarried servant in a bank, is Rs 30,000 and her annual income is equivalent to her salary of 15 months. Similarly, the monthly salary of Sumesh, a married civil servant is Rs 40,000 and his annual income is equivalent to his 13 month's salary including festival expense. Who pays more income tax and by how much?***

Solution:

Here,

For Ms. Chhiring, monthly income = Rs. 30,000

∴His annual income = 15 × Rs. 30,000 = Rs. 4,50,000

= Rs. 4,00,000 + Rs.50,000

Now, income tax = 1% of Rs. 4,00,000 + 10% of Rs.50,000

= Rs. 4,000 + Rs. 5,000

= Rs. 9,000

For Sumesh, yearly income = 13 × Rs. 40,000 = Rs. 5,20,000

= Rs. 4,50,000 + Rs. 70,000

∴Income tax = 1% of Rs. 4,50,000 + 10% of Rs. 70,000

= Rs. 4,500 + Rs. 7,000

= Rs. 11,500

Difference due to their income tax = Rs. 11,500 – Rs. 9,000 = Rs2,500

Hence, Sumesh pays Rs. 2,500 more tax than Ms. Chhiring.

B.   VAT

➢ Marked price (M.P.) and Discount Teaching Activities 1. Recall cost price (C.P.) and selling price (S.P.) of an article.

➢ With examples, discuss on marked price (M.P.) and discount.

➢ Explain discount as the amount of reduction in the marked price of an article.

➢ Paste/show the different types of taxes in the colourful chart paper and explain with appropriate examples.

➢ Make clear VAT as tax levied on purchase of goods or service 6. List the following formulae after discussion

(i)   Discount amount = M.P. – S. P.

(ii)   Discount amount = Discount % of M.P.

(iii)   Rate of discount = $\dfrac{\text{Discount amount}}{\text{M.P.}} \times 100\%$

(iv)   S.P. = M.P. – Discount amount

(v)   S.P. = M.P. – Discount% of M.P. = M.P. (1 – Discount %)

➢ Discuss on the following process of calculating VAT.
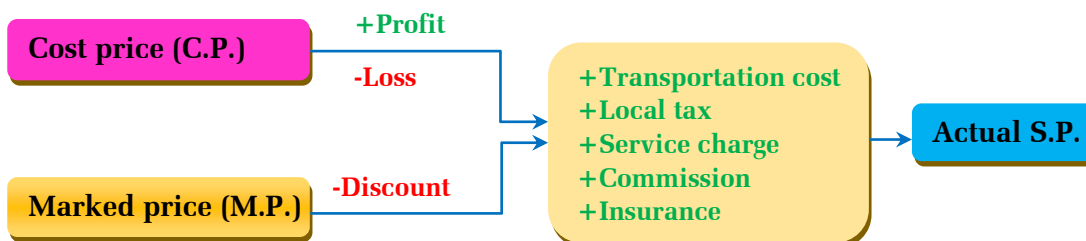
EXERCISE 2.2

General section

1.   a) **If R% be the rate of VAT and Rs x be the selling price, write the formula to find amount of VAT.**

Solution:

Here, VAT amount = VAT% of S.P. = R% of Rx. x

b) **If Rs. x be the selling price and Rs. y be the amount of VAT, write the formula to find VAT percent.**

Solution:

Here, VAT percent = $\dfrac{\text{VAT amount}}{\text{S.P.}} \times 100\% = \dfrac{y}{x} \times 100\%$

c) **If Rs. P be the selling price and R% be the VAT rate, write the formula to find selling price with VAT.**

Solution:

Here, S.P. with VAT = S.P. + VAT% of S.P. = Rs. P + R% of Rs. P

d) **If marked price (M.P.) = Rs x, discount = Rs y and VAT = Rs z, what is the selling price including VAT?**

Solution:

Here, S.P. = M.P. – Discount = Rs. x – Rs. y

∴ S.P. with VAT = S.P. + VAT = Rs. (x – y) + Rs. z = Rs. (x – y + z)

2.   **a)** *Find the selling price with VAT from the table given below.*

S.N.	Particulars	S.P. without VAT	VAT rate	VAT amount
(i)	Mobile set	Rs.22,000	13%	…………..
(ii)	Camera	Rs. 35,000	13%	…………..
(iii)	Television	Rs. 40,000	10%	…………..

Solution:

Here,

(i) For mobile set, S.P. with VAT = VAT% of S.P. = 13% of Rs. 22,000 = Rs. 2,860

(ii) For camera, S.P. with VAT   = VAT% of S.P. = 13% of Rs. 35,000 = Rs. 4,550

(iii) For television, S.P. with VAT = VAT% of S.P. = 10% of Rs. 40,000 = Rs. 4,000

b) *Find the selling price without VAT from the table given below.*

S.N.	Particulars	VAT amount	VAT rate	S.P. without VAT
(i)	Radio	Rs. 585	13%	…………..
(ii)	Bicycle	Rs. 975	13%	…………..
(iii)	Laptop	Rs. 9,900	15%	…………..

Solution:

Here,

(i)   Let, S.P. without VAT of a radio be Rs. x

Now, VAT amount   = VAT% of S.P.

or,   Rs. 585   = 13% of x

or,   0.13x   = Rs. 585

or,   x   = Rs. 4,500

Hence, the S.P. without VAT of the radio is Rs. 4,500.

(ii)   Let, S.P. without VAT of a bicycle be Rs. x

Now, VAT amount   = VAT% of S.P.

or,   Rs. 975   = 13% of x

or,   0.13x   = Rs. 975

or,   x   = Rs. 7,500

Hence, the S.P. without VAT of the bicycle is Rs. 7,500.

(iii)   Let, S.P. without VAT of a laptop be Rs. x

Now, VAT amount   = VAT% of S.P.

or,   Rs. 9,900   = 15% of x

or,   0.15x   = Rs. 9,900

or,   x   = Rs. 66,000

Hence, the S.P. without VAT of the laptop is Rs. 66,000.

3.   **a)** *The selling price of a watch is Rs 3,000. What will be the VAT amount on it at the rate of 13%?*

Solution:

Here, S.P. of a watch = Rs. 3,000

Now,   VAT amount = VAT% of S.P.

= 13% of Rs. 3,000

= Rs. 390

Hence, the required VAT amount is Rs. 390

b) Calculate the VAT amount on a tablet costing Rs 15,000 at the rate of 13%.

Solution:

Here, S.P. of a tablet = Rs. 15,000

Now,　　VAT amount　= VAT% of S.P.

　　　　　　　　　= 13% of Rs. 15,000

　　　　　　　　　= Rs. 1,950

Hence, the required VAT amount is Rs. 1,950.

c) The catalogue price of a refrigerator is Rs 28,500. How much amount of VAT is levied on it at the rate of 13%?

Solution:

Here, M.P. of a refrigerator = S.P. of a refrigerator = Rs. 28,500

Now,　　VAT amount　= VAT% of S.P.

　　　　　　　　　= 13% of Rs. 28,500

　　　　　　　　　= Rs. 3,705

Hence, the required VAT amount is Rs. 3,705.

4. a) The cost of a fan is Rs 1,600. If Mrs. Khadka purchased it with 13% VAT, how much did she pay for it?

Solution:

Here, S.P. of a fan = Rs. 1,600

Now,　　S.P. with VAT = S.P. + VAT% of S.P.

　　　　　　　　　= Rs. 1,600 + 13% of Rs. 1,600

　　　　　　　　　= Rs. 1,808

Hence, Mrs. Khadka paid Rs. 1,808 for the fan.

b) The selling price of a radio is Rs 4,000. How much should a customer pay for it with 13% value added tax?

Solution:

Here, S.P. of a radio = Rs. 4,000

Now,　　S.P. with VAT = S.P. + VAT% of S.P.

　　　　　　　　　= Rs. 4,000 + 13% of Rs. 4,000

　　　　　　　　　= Rs. 4,520

Hence, the customer should pay Rs. 4,520 for the radio.

c) The marked price of a pen-drive is Rs 700 and the shopkeeper levies 13% VAT on it. If you give a 1,000 rupee note, what change will the shopkeeper return to you?

Solution:

Here, M.P. of a pen-drive = S.P. of a pen-drive = Rs. 700

Now,　　S.P. with VAT = S.P. + VAT% of S.P.

　　　　　　　　　= Rs. 700 + 13% of Rs. 700

　　　　　　　　　= Rs. 791

Again, change = Rs. 1,000 – Rs. 791 = Rs. 209

Hence, the shopkeeper will return Rs. 209

d) A family had dinner in a restaurant. If the cost of the dinner was Rs 2,100, how much did the family pay with 10% service charge and 13% VAT?

Solution:

Here, cost of dinner = Rs. 2,100

Service charge = 10%

VAT rate = 13%

Now, the cost of dinner with service charge = Rs. 2,100 + 10% of Rs. 2,100

= Rs. 2,310

Again, the cost of the dinner with VAT = Rs. 2,310 + 13% of Rs. 2,310

= Rs. 2,610.30

Hence, the family should pay Rs. 2,610.30 for the dinner.

5. **a)** **The cost of a rice cooker with 13% VAT is Rs 4,068. Find its cost without VAT.**

Solution:

Here, S.P. of a cooker with 13% VAT = Rs. 4,068

Let, S.P. without VAT = Rs. x

Now, S.P. with VAT = S.P. + VAT% of S.P.

or, Rs. 4,068 = x + 13% of x

or, Rs. 4,068 = 1.13 x

or, x = Rs. 3,600

Hence, the cost of the cooker without VAT is Rs. 3,600

b) **Mr. Magar purchased a mobile set for Rs 11,155 with 15% VAT inclusive. Find the cost of the mobile without VAT and also calculate the VAT amount.**

Solution:

Here, S.P. of a mobile set with 15% VAT = Rs. 11,155

Let, S.P. without VAT = Rs. x

Now, S.P. with VAT = S.P. + VAT% of S.P.

or, Rs. 11,155 = x + 15% of x

or, Rs. 11,155 = 1.15 x

or, x = Rs. 9,700

Hence, the cost of the mobile without VAT is Rs. 9,700

Again, VAT amount = 15% of Rs. 9,700 = Rs. 1,455

c) **Mrs. Maharjan bought a refrigerator for Rs 26,442 with 13% VAT. How much did she pay for the VAT?**

Solution:

Here, S.P. of a refrigerator with 13% VAT = Rs. 26,442

Let, S.P. without VAT = Rs. x

Now, S.P. with VAT = S.P. + VAT% of S.P.

or, Rs. 26,442 = x + 13% of x

or, Rs. 26,442 = 1.13 x

or, x = Rs. 23,400

Again, VAT amount = 13% of Rs. 23,400 = Rs. 3,042

Hence, she has to pay Rs. 3,042 for VAT.

6. **a)** **If the cost of a watch with VAT is Rs 5,130 and without VAT is Rs 4,500, find the VAT rate.**

Solution:

Here, S.P. of a watch with VAT = Rs. 5,130

S.P. without VAT = Rs. 4,500

Now, VAT amount = S.P. with VAT – S.P. without VAT

$$= Rs. 5,130 - Rs. 4,500$$
$$= Rs. 630$$

Again, rate of VAT $= \dfrac{\text{VAT amount}}{\text{S.P. without VAT}} \times 100\%$

$$= \dfrac{Rs. 630}{Rs. 4500} \times 100\%$$

$$= 14\%$$

Hence, the required VAT rate is 14%.

b) *Malvika purchased a fancy bag for Rs 7,119 with VAT. If its cost without VAT is Rs 6,300, calculate the rate of VAT.*

Solution:

Here,   S.P. of a fancy bag with VAT = Rs. 7,119

     S.P. without VAT   = Rs. 6,300

Now, VAT amount   = S.P. with VAT – S.P. without VAT

         = Rs. 7,119 – Rs. 6,300

         = Rs. 819

Again, rate of VAT $= \dfrac{\text{VAT amount}}{\text{S.P. without VAT}} \times 100\%$

$$= \dfrac{Rs. 819}{Rs. 6300} \times 100\%$$

$$= 13\%$$

Hence, the required VAT rate is 13%.

c) *If the cost of a computer with VAT is Rs 67,800 and without VAT is Rs 60,000, find the VAT rate.*

Solution:

Here,   S.P. of a computer with VAT   = Rs. 67,800

     S.P. without VAT   = Rs. 60,000

Now, VAT amount   = S.P. with VAT – S.P. without VAT

         = Rs. 67,800 – Rs. 60,000

         = Rs. 7,800

Again, rate of VAT $= \dfrac{\text{VAT amount}}{\text{S.P. without VAT}} \times 100\%$

$$= \dfrac{Rs. 7800}{Rs. 60000} \times 100\%$$

$$= 13\%$$

Hence, the required VAT rate is 13%.

Creative Section-A

7. a) *Find the selling price of the following appliances with VAT.*

(i)

M.P. =Rs 25,000
Discount rate = 10%
VAT rate = 13%

(ii)

M.P. =Rs 40,000
Discount rate = 15%
VAT rate = 13%

(iii)

M.P. =Rs 85,000
Discount rate = 14%
VAT rate = 13%

(iv)

M.P. =Rs 1,20,000
Discount rate = 14%
VAT rate = 13%

Solution:

(i) Here, M.P. of a cycle   = Rs. 25,000
   Discount rate   = 10%
   VAT rate   = 13%
   Now, S.P. after discount   = M.P. – Discount % of M.P.
      = Rs. 25,000 – 10% of Rs. 25,000
      = Rs. 22,500
   Again, S.P. with VAT   = S.P. + VAT% of S.P.
      = Rs. 22,500 + 13% of Rs. 22,500
      = Rs. 25,425
   Hence, the selling price of the cycle with VAT is Rs. 25,425

(ii) Here, M.P. of a refrigerator   = Rs. 40,000
   Discount rate   = 15%
   VAT rate   = 13%
   Now, S.P. after discount   = M.P. – Discount % of M.P.
      = Rs. 40,000 – 15% of Rs. 40,000
      = Rs. 34,000
   Again, S.P. with VAT = S.P. + VAT% of S.P.
      = Rs. 34,000 + 13% of Rs. 34,000
      = Rs. 38,420
   Hence, the selling price of the refrigerator with VAT is 38,420.

(iii) Here, M.P. of a laptop = Rs. 85,000
   Discount rate = 14%
   VAT rate = 13%
   Now, S.P. after discount = M.P. – Discount % of M.P.
      = Rs. 85,000 – 14% of Rs. 85,000
      = Rs. 73,100
   Again, S.P. with VAT = S.P. + VAT% of S.P.
      = Rs. 73,100 + 13% of Rs. 73,100
      = Rs. 82,603
   Hence, the selling price of the laptop with VAT is Rs. 82,603.

(iv) Here, M.P. of a camera = Rs. 1,20,000
   Discount rate = 14%
   VAT rate = 13%
   Now, S.P. after discount = M.P. – Discount % of M.P.
      = Rs. 1,20,000 – 14% of Rs. 1,20,000
      = Rs. 1,03,200
   Again, S.P. with VAT = S.P. + VAT% of S.P.
      = Rs. 1,03,200 + 13% of Rs. 1,03,200
      = Rs. 1,16,616
   Hence, the selling price of the camera with VAT is Rs. 1,16,616

b) *The marked price of a bike helmet is Rs 3,000 and 10 % discount is allowed on it. Find its cost with 13 % VAT.*

Solution:

Here, M.P. of a bike helmet = Rs. 3,000

Discount rate = 10%

VAT rate = 13%

Now, S.P. after discount = M.P. – Discount % of M.P.

= Rs. 3,000 – 10% of Rs. 3,000

= Rs. 2,700

Again, S.P. with VAT = S.P. + VAT% of S.P.

= Rs. 2,700 + 13% of Rs. 2,700

= Rs. 3,051

Hence, the selling price of the helmet with VAT is Rs. 3,051.

c) **The price of a blanket is marked as Rs 5,500. If the shopkeeper allows 20 % discount and adds 13 % VAT, how much does a customer pay for the blanket?**

Solution:

Here, M.P. of a blanket = Rs. 5,500

Discount rate = 20%

VAT rate = 13%

Now, S.P. after discount = M.P. – Discount % of M.P.

= Rs. 5,500 – 20% of Rs. 5,500

= Rs. 4,400

Again, S.P. with VAT = S.P. + VAT% of S.P.

= Rs. 4,400 + 13% of Rs. 4,400

= Rs. 4,972

Hence, the customer should pay Rs. 4,972 for the blanket.

e) **A trader bought a motorbike for Rs 2,40,000 and fixed its price 20 % above the cost price. Then, he allowed 10 % discount and sold to a customer. How much did the customer pay for it with 13 % VAT?**

Solution:

Here, C.P. of a motorbike = Rs. 2,40,000

∴ M.P. of a bike = Rs. 2,40,000 + 20% of Rs. 2,40,000

= Rs. 2,88,000

Discount rate = 10%

VAT rate = 13%

Now, S.P. after discount = M.P. – Discount % of M.P.

= Rs. 2,88,000 – 10% of Rs. 2,88,000

= Rs. 2,59,200

Again, S.P. with VAT = S.P. + VAT% of S.P.

= Rs. 2,59,200 + 13% of Rs. 2,59,200

= Rs. 2,92,896

Hence, the customer should pay Rs. 2,92,896 for the bike.

8. a) **A shopkeeper bought a television for Rs 16,000 and sold at a profit of 20% to a customer with 13% VAT. How much did the customer pay for the television?**

Solution:

Here, C.P. of a television = Rs. 16,000

Profit percent = 20%

Now,   S.P. of a television   = C.P. + profit% of C.P.

= Rs. 16,000 + 20% of Rs. 16,000

= Rs. 19,200

VAT rate = 13%

Again, S.P. with VAT   = S.P. + VAT% of S.P.

= Rs. 19,200 + 13% of Rs. 19,200

= Rs. 21,696

Hence, the customer should pay Rs. 21,696for the television.

b) *Mrs. Lama marked the price of cosmetic item 25% above its cost price. If the cost price of the cosmetic item was Rs 4,400, at price did she sell it with 13% VAT?*

Solution:

Here,   C.P. of cosmetic item = Rs. 4,400

Now,   M.P. of the cosmetic item   = S.P. of the cosmetic item

= Rs. 4,400 + 25% of Rs. 4,400

= Rs. 5,500

VAT rate   = 13%

Again, S.P. with VAT   = S.P. + VAT% of S.P.

= Rs. 5,500 + 13% of Rs. 5,500

= Rs. 6,215

Hence, the customer should pay Rs. 6,215 for the cosmetic item.

c) *Mr. Sharma bought a computer for Rs 50,000 and fixed its price 15% above the cost price. How much did the customer pay for the computer including 13% value added tax?*

Solution:

Here,   C.P. of computer   = Rs. 50,000

Now,   M.P. of the computer = S.P. of the computer

= Rs. 50,000 + 15% of Rs. 50,000

= Rs. 57,500

VAT rate   = 13%

Again, S.P. with VAT   = S.P. + VAT% of S.P.

= Rs. 57,500 + 13% of Rs. 57,500

= Rs. 64,975

Hence, the customer should pay Rs. 64,975 for the computer.

9.   a) *Mrs. Kandel went to a restaurant with her family. They had three plates of Mo.Mo at Rs 120 per plate, one plate chicken chilly at Rs 220 per plate, and three bottles of cold drink at Rs 40 per bottle. If 13% VAT is levied on the bill after adding 10% service charge on the bill, how much did she pay to clear the bill?*

Solution:

Here, original bill   = 3 × Rs. 120 + Rs. 220 + 3 × Rs. 40 = Rs. 700

Service charge   = 10%

VAT rate   = 13%

Now, the cost of the dinner with service charge   = S.P. + 10% of S.P.

= Rs. 700 + 10% of Rs. 700

= Rs. 770

Again,

The cost of the dinner with service charge and VAT = Rs. 770 + 13% of Rs. 770

= Rs 870.10

Therefore, the family should paid Rs. 870.10

b) **A group of three friends had two plates of chicken chilly, two plates of French fry, two plates of Mo:Mo and a few glasses of fresh juice in a restaurant. If the cost of these items amounts to Rs 900, how much should they pay with 10 % service charge and 13 % VAT to clear the bill?**

Solution:

Here, original bill = Rs. 900

Service charge = 10%

VAT rate = 13%

Now, the cost of the dinner with service charge = S.P. + 10% of S.P.

= Rs. 900 + 10% of Rs. 900

= Rs. 990

Again,

The cost of the dinner with service charge and VAT = Rs. 990 + 13% of Rs. 990

= Rs. 1,118.70

Therefore, they should paid Rs. 1,118.70

10. a) **A retailer allows 15 % discount on the marked price of an electric fan. If a customer pays Rs 2,244 with 10 % VAT, find the marked price of the fan.**

Solution:

Here, rate of discount = 15%

VAT rate = 10%

S.P. of an electric fan with VAT = Rs. 2,244

Let, M.P. of the electric fan be Rs. x

Now, S.P. after discount = M.P. – Discount % of M.P.

= Rs. x – 15% of Rs. x

= Rs. 0.85x

Again, S.P. with VAT = S.P. + VAT% of S.P.

or, Rs. 2,244 = 0.85x + 10% of 0.85x

or, 2,244 = 0.935x

or, x = 2400

Hence, the marked price of the fan is Rs. 2,400.

b) **Allowing 16 % discount on the marked price of a television and levying 13 % VAT, a buyer has to pay Rs 18,984 to buy it. Find the marked price of the television.**

Solution:

Here, rate of discount = 16%

VAT rate = 13%

S.P. of television with VAT = Rs. 18,984

Let, M.P. of the television be Rs. x

Now, S.P. after discount = M.P. – Discount % of M.P.

= Rs. x – 16% of Rs. x

= Rs. 0.84x

Again, S.P. with VAT = S.P. + VAT% of S.P.

or, Rs. 18,984 = 0.84x + 13% of 0.84x

or, 18,984 = 0.9492x

or, x = 20000

Hence, the marked price of the television is Rs. 20,000.

c) **Allowing 15% discount and including same percentage of VAT, the laptop was sold at Rs 64,515. Find the marked price of the laptop.**

Solution:

Here,   rate of discount = 15%

VAT rate = 15%

S.P. of a laptop with VAT = Rs. 64,515

Let, M.P. of the laptop be Rs. x

Now, S.P. after discount   = M.P. – Discount % of M.P.

= Rs. x – 15% of Rs. x

= Rs. 0.85x

Again, S.P. with VAT   = S.P. + VAT% of S.P.

or, Rs. 64,515 = 0.85x + 15% of 0.85x

or, 64,515   = 0.9775x

or, x   = 66,000

Hence, the marked price of the laptop is Rs. 66,000.

11. a) **After allowing 5 % discount on the marked price of a gift item, 10 % VAT is charged on it. Now, its price became Rs 1,672. How much amount was given in the discount?**

Solution:

Here,   rate of discount = 5%

VAT rate   = 10%

S.P. of a gift item with VAT = Rs. 1,672

Let, M.P. of the gift item be Rs. x

Now, S.P. after discount   = M.P. – Discount % of M.P.

= Rs. x – 5% of Rs. x

= Rs. 0.95x

Also, S.P. with VAT   = S.P. + VAT% of S.P.

or, Rs. 1672   = 0.95x + 10% of 0.95x

or, 1672   = 1.045x

or, x   = 1600

Hence, the marked price of the gift item is Rs. 1,600.

Again, discount amount = 5% of Rs. 1,600 = Rs.80.

b) **Mrs. Gurung sold her goods for Rs 16,950 allowing 25 % discount and then levied on 13 % VAT, what was the amount of discount?**

Solution:

Here,   rate of discount = 25%

VAT rate = 13%

S.P. of goods with VAT = Rs. 16,950

Let, M.P. of the gift item be Rs. x

Now, S.P. after discount   = M.P. – Discount % of M.P.

= Rs. x – 25% of Rs. x

= Rs. 0.75x

Also, S.P. with VAT   = S.P. + VAT% of S.P.

or, Rs. 16,950   = 0.75x + 13% of 0.75x

or, 16,950   = 0.8475x

or, x   = 20,000

Hence, the marked price of the goods is Rs. 20,000.
Again, discount amount = 25% of Rs. 20,000 = Rs.5,000.

11. a) *After allowing 20 % discount on the marked price of a computer, 15 % VAT was levied on it. If its price becomes Rs 26,496, what amount was levied in the VAT?*

Solution:

Here,   rate of discount = 20%

VAT rate   = 15%

S.P. of a computer with VAT = Rs. 26,496

Let, M.P. of the computer be Rs. x

Now, S.P. after discount   = M.P. – Discount % of M.P.

= Rs. x – 20% of Rs. x

= Rs. 0.8x

Also, S.P. with VAT   = S.P. + VAT% of S.P.

or, Rs. 26,496 = 0.8x + 15% of 0.8x

or, 26,496   = 0.92x

or, x   = 28,800

∴ S.P. without VAT = Rs. 0.8 × 28,800 = Rs. 23,040

Again, VAT amount = 15% of Rs. 23,040 = Rs.3,456.

b) *After allowing 10 % discount on the marked price of an iPod and levying 13 % value added tax, the price of the iPod becomes Rs 7,119. Find the value added tax.*

Solution:

Here,   rate of discount = 10%

VAT rate = 13%

S.P. of an iPod with VAT = Rs. 7,119

Let, M.P. of the computer be Rs. x

Now, S.P. after discount   = M.P. – Discount % of M.P.

= Rs. x – 10% of Rs. x

= Rs. 0.9x

Also, S.P. with VAT   = S.P. + VAT% of S.P.

or, Rs. 7,119   = 0.9x + 13% of 0.9x

or, 7,119   = 1.017x

or, x   = 7,000

∴ S.P. without VAT = Rs. 0.9 × 7,000 = Rs. 6,3000

Again, VAT amount = 13% of Rs. 6300 = Rs.819.

c) *A tourist paid Rs 5,610 for a carved window made up of wood with a discount of 15% including 10% value added tax (VAT). How much does he get back while leaving Nepal?*

Solution:

Here,   rate of discount = 15%

VAT rate = 10%

S.P. of a carved window with VAT = Rs. 5,610

Let, M.P. of the carved window be Rs. x

Now, S.P. after discount   = M.P. – Discount % of M.P.

= Rs. x – 15% of Rs. x

= Rs. 0.85x

Also, S.P. with VAT $\qquad$ = S.P. + VAT% of S.P.

or, Rs. 5,610 $\qquad$ = 0.85x + 10% of 0.85x

or, 5,610 $\qquad$ = 0.935x

or, x $\qquad$ = 6,000

∴S.P. without VAT = Rs. 0.85× 6,000 = Rs. 5,100

Again, VAT amount = 10% of Rs. 5,100 = Rs.510

Hence, the tourist gets back Rs. 510 while leaving Nepal.

12. a) *A mobile price is tagged Rs 5,000. If a customer gets 12% discount and adding certain percent VAT reaches as Rs 4,972, find out the VAT percentage.*

Solution:

Here,   M.P. of a mobile = Rs. 5,000

Rate of discount = 12%

S.P. of the mobile with VAT = Rs. 4,972

VAT rate=?

Now, S.P. after discount $\qquad$ = M.P. – Discount % of M.P.

= Rs. 5,000 – 12% of Rs. 5,000

= Rs. 4,400

Also, VAT amount $\qquad$ = S.P. with VAT – S.P. after discount

= Rs. 4,972 – Rs. 4,400

= Rs. 572

Again, VAT percent $\qquad = \dfrac{\text{VAT amount}}{\text{S.P. after discount}} \times 100\%$

$= \dfrac{\text{Rs. 572}}{\text{Rs. 4400}} \times 100\%$

= 13%

Hence, the required VAT rate is 13%.

b) *The marked price of a bag is Rs 2,000. The price of the bag becomes Rs 1,921 after 15% discount and adding VAT amount. Find the rate of VAT.*

Solution:

Here,   M.P. of a bag = Rs. 2,000

Rate of discount = 15%

S.P. of the bag with VAT = Rs. 1,921

VAT rate=?

Now, S.P. after discount $\qquad$ = M.P. – Discount % of M.P.

= Rs. 2,000 – 15% of Rs. 2,000

= Rs. 1,700

Also, VAT amount $\qquad$ = S.P. with VAT – S.P. after discount

= Rs. 1,921 – Rs. 1,700

= Rs. 221

Again, VAT percent $\qquad = \dfrac{\text{VAT amount}}{\text{S.P. after discount}} \times 100\%$

$= \dfrac{\text{Rs. 221}}{\text{Rs. 1700}} \times 100\%$

= 13%

Hence, the required VAT rate is 13%.

13. a) *Mrs Karki purchased a sari for Rs 8,000 and sold it for Rs 11,300 with 13% VAT. Find her profit or loss percent.*

Solution:

Here,   C.P. of a sari = Rs. 8,000

S.P. with 13% VAT = Rs. 11,300

Let, S.P. excluding VAT be Rs. x.

Then, S.P. with VAT   = S.P. + VAT% of S.P.

　　　　or, Rs. 11,300 = x + 13% of x

　　　　or, Rs. 11,300 = 1.13x

　　　　or, x   = 10,000

∴ S.P. of the sari is Rs. 10,000

Now, profit amount   = S.P. – C.P.

　　　　　　　　　　= Rs. 10,000 – Rs. 8,000

　　　　　　　　　　= Rs. 2,000

Again, profit percent $= \dfrac{\text{Profit amount}}{\text{C.P.}} \times 100\%$

$= \dfrac{\text{Rs. 2000}}{\text{Rs. 8000}} \times 100\%$

$= 25\%$

Hence, Mrs. Karki makes 25% profit.

b) *A supplier bought a scanner machine for Rs 35,000 and sold it for Rs 47,460 with 13% VAT. Find the profit or loss percent of the supplier.*

Solution:

Here,   C.P. of a scanner machine = Rs. 35,000

　　　　S.P. with 13% VAT = Rs. 47,460

Let, S.P. excluding VAT be Rs. x.

Then, S.P. with VAT   = S.P. + VAT% of S.P.

　　　　or, Rs. 47,460 = x + 13% of x

　　　　or, Rs. 47,460 = 1.13x

　　　　or, x   = 42,000

∴ S.P. of the scanner machine is Rs. 42,000

Now, profit amount   = S.P. – C.P.

　　　　　　　　　　= Rs. 42,000 – Rs. 35,000

　　　　　　　　　　= Rs. 7,000

Again, profit percent $= \dfrac{\text{Profit amount}}{\text{C.P.}} \times 100\%$

$= \dfrac{\text{Rs. 7000}}{\text{Rs. 35000}} \times 100\%$

$= 20\%$

Hence, the supplier makes 20% profit.

Creative Section-B

15. a) *After allowing 15% discount on the marked price of a camera, 13% VAT was levied and sold it. If the selling price of the camera with VAT is Rs 4,420 more than its price after discount, find the marked price of the camera.*

Solution:

Here,   rate of discount = 15%

          VAT rate = 13%

S.P. with VAT – S.P. after discount = Rs. 4,420

Let, M.P. of the camera be Rs. x

Now, S.P. after discount   = M.P. – Discount % of M.P.

                                  = Rs. x – 15% of Rs. x

                                  = Rs. 0.85x

Again, S.P. with VAT = S.P. + VAT% of S.P.

                                  = 0.85x + 13% of 0.85x

                                  = 0.9605x

According to question,

S.P. with VAT – S.P. after discount   = Rs. 4,420

  or, 0.9605x – 0.85x   = 4,420

  or, 0.1105x   = 4,420

  or, x   = 40,000

Hence, the marked price of the camera is Rs. 40,000.

b) **A projector was sold after allowing 10% discount on the marked price and levying 13% VAT. If the selling price of the projector after discount is Rs 5,850, less than its selling price with VAT, find the marked price of the projector.**

Solution:

Here,   rate of discount = 10%

          VAT rate = 13%

S.P. with VAT – S.P. after discount = Rs. 5,850

Let, M.P. of the projector be Rs. x

Now, S.P. after discount   = M.P. – Discount % of M.P.

                                  = Rs. x – 10% of Rs. x

                                  = Rs. 0.9x

Again, S.P. with VAT = S.P. + VAT% of S.P.

                                  = 0.9x + 13% of 0.9x

                                  = 0. 1.017 x

According to question,

S.P. with VAT – S.P. after discount   = Rs. 5,850

  or, 0. 1.017x – 0.9x   = 5,850

  or, 0.117x   = 5,850

  or, x   = 50,000

Hence, the marked price of the projector is Rs. 50,000.

16. a)   **The marked price of a digital watch is Rs 6,000. After allowing 10% discount and including same percentage of value added tax, the watch is sold. By how much percent is the VAT amount less than discount amount?**

Solution:

Here, M.P. of a digital watch = Rs. 6,000

Rate of discount = 10%

VAT rate = 10%

Now, discount amount = 10% of Rs. 6,000 = Rs. 600

Also S.P. after discount   = M.P. – Discount

                                  = Rs. 6,000 – Rs. 600

                                  = Rs. 5,400

Again, VAT amount   = VAT% of S.P.

                                  = 10% of Rs. 5,400

                                  = Rs. 540

Difference between discount and VAT amount = Rs. 600 – Rs. 540 = Rs. 60

Hence, the VAT amount is less than discount amount by $\frac{Rs\ 60}{Rs.600} \times 100\% = 10\%$

b) *The marked price of a guitar is Rs 5,500. After allowing 10% discount and levying same percentage of VAT, the guitar is sold. By how much percent is the VAT amount less than discount amount?*

Solution:

Here, M.P. of a digital watch = Rs. 5,500

Rate of discount = 10%

VAT rate = 10%

Now, discount amount = 10% of Rs. 5,500 = Rs. 550

Also S.P. after discount   = M.P. – Discount

                                  = Rs. 5,500 – Rs. 550

                                  = Rs. 4,950

Again, VAT amount   = VAT% of S.P.

                                  = 10% of Rs. 4,950

                                  = Rs. 495

Difference between discount and VAT amount = Rs. 550 – Rs. 495 = Rs. 55

Hence, the VAT amount is less than discount amount by $\frac{Rs\ 55}{Rs.550} \times 100\% = 10\%$

17. a) *A wholesaler sold a photocopy machine for Rs 48,000 to a retailer. The retailer spent Rs 2,000 for transportation and Rs 1,500 for the local tax. If the retailer sold it at a profit of Rs 4,500 to a customer, how much did the customer pay for it with 13% VAT?*

Solution:

Here,

For wholesaler,   S.P. of a photocopy machine = Rs. 48,000

For retailer,   C.P. of the photocopy machine = Rs. 48,000

                        Transportation cost = Rs. 2,000 and local tax = Rs. 1,500

Now,

C.P. with transportation cost and local tax   = Rs. 48,000 + Rs. 2,000 + Rs. 1,500

                                                 = Rs. 51,500

Profit = Rs. 4,500

∴S.P. of the photocopy machine = Rs. 51,500 + Rs. 4,500 = Rs. 56,000

Again, S.P. with 13% VAT   = Rs. 56,000 + 13% of Rs. 56,000

                                   = Rs. 63,280

Hence, the customer paid the machine for Rs. 63,280.

b) *The Buddha supplier sold a digital T-shirt printer for Rs 3,00,000 to Everest supplier. The Everest supplier spent Rs 5,500 for transportation and Rs 2,500 for the local tax and sold at a profit of 10% to a customer. How much did the customer pay for the printers with 13% VAT?*

Solution:

Here,

For Buddha supplier, S.P. of a T-shirt printer = Rs. 3,00,000

For Everest supplier, C.P. of the photocopy machine = Rs. 3,00,000

Transportation cost = Rs. 5,500 and local tax = Rs. 2,500

Now,

C.P. with transportation cost and local tax = Rs. 3,00,000 + Rs. 5,500 + Rs. 2,500

= Rs. 3,08,000

Also, profit = 10% of Rs. 3,08,000 = Rs. 30,800

$\therefore$ S.P. of the printer = Rs. 3,08,000 + Rs. 30,800 = Rs. 3,38,800

Again, S.P. with 13% VAT = Rs. 3,38,800 + 13% of Rs. 3,38,800

= Rs. 3,82,844

Hence, the customer paid the printer for Rs 3,82,844.

c) *A wholesaler purchased a washing machine for Rs 60,000 and sold it to a retailer at 10% profit. The retailer spent Rs 2,400 for transportation and Rs 1,600 for local tax. Then she sold it to a customer at 12% profit. How much did the customer pay for it with 13% VAT?*

Solution:

Here,

For wholesaler, C.P. of a washing machine = Rs. 60,000

S.P. of a washing machine = C.P. + profit % of C.P.

= Rs. 60,000 + 10% of Rs. 60,000

= Rs. 66,000

For retailer, C.P. of the washing machine = Rs. 66,000

Transportation cost = Rs. 2,400 and local tax = Rs. 1,600

Now,

C.P. with transportation cost and local tax = Rs. 66,000 + Rs. 2,400 + Rs. 1,600

= Rs. 70,000

Also, S.P. of the washing machine = Rs. 70,000 + 12% of Rs. 70,000

= Rs. 78,400

Again, S.P. with 13% VAT = Rs. 78,400 + 13% of Rs. 78,400

= Rs. 88,592

Hence, the customer paid the washing machine for Rs 88,592.

18. a) *A retailer allowed 4% discount on his goods to make 20% profit and sold a refrigerator for Rs 10,848 with 13% VAT. By how much is the discount to be increased so that he can gain only 15%?*

Solution:

Here, rate of discount = 4%

VAT rate = 13%

S.P. with VAT = Rs. 10,848

Let, M.P. of the refrigerator be Rs. x

Now, S.P. after discount = Rs. x – 4% of Rs. = Rs. 0.96x

Also,   S.P. with VAT = S.P. + VAT% of S.P.

   or, Rs. 10,848 = 0.96x + 13% of 0.96x

   or, 10,848   = 1.0848 x

   or, x   = 10,000

∴M.P. of the refrigerator = Rs. 10,000 and S.P. = 0.96 × Rs. 10,000 = Rs. 9,600

Again, profit percent = 20%

Let C.P. of the refrigerator be Rs. y.

Then,   S.P.   = C.P. + profit % of C.P.

   or, Rs. 9,600   = y + 20% of y

   or, Rs. 9,600   = 1.2y

   or, y   = 8,000

∴C.P. of the refrigerator is Rs. 8,000.

And, new S.P. = C.P. + 15% of C.P. = Rs. 8,000 + 15% of Rs. 8,000 = Rs. 9,200

New discount amount = M.P. – new S.P. = Rs. 10,000 – Rs. 9,200 = Rs. 800

Then, new discount percent $= \dfrac{\text{New discount}}{\text{M.P.}} \times 100\% = \dfrac{800}{10000} \times 100\% = 8\%$

∴Increment in discount percent = 8% – 4% = 4%

Hence, the discount is to be increased by 4%.

b)   **A supplier sold a scanner machine for Rs 41,400 with 15% VAT after allowing 10% discount on it's marked price and gained 20%. By how much is the discount percent to be reduced to increase the profit by 4%?**

Solution:

Here,   rate of discount = 10%

           VAT rate = 15%

S.P. of scanner machine with VAT = Rs. 41,400

Let, M.P. of the scanner be Rs. x

Now, S.P. after discount = Rs. x – 10% of Rs. = Rs. 0.9x

Also,   S.P. with VAT = S.P. + VAT% of S.P.

   or, Rs. 41,400 = 0.9x + 15% of 0.9x

   or, 41,400   = 1.035 x

   or, x   = 40,000

∴M.P. of the scanner = Rs. 40,000 and S.P. = 0.9 × Rs. 40,000 = Rs. 36,000

Again, profit percent = 20%

Let C.P. of the scanner be Rs. y.

Then,   S.P.   = C.P. + profit % of C.P.

   or, Rs. 36,000 = y + 20% of y

   or, Rs. 36,000 = 1.2y

   or, y   = 30,000

∴C.P. of the scanner is Rs. 30,000.

And, profit percent = 20% + 4% = 24%

New S.P. = C.P. + 24% of C.P. = Rs. 30,000 + 24% of Rs. 30,000 = Rs. 37,200

New discount amount = M.P. – new S.P. = Rs. 40,000 – Rs. 37,200= Rs. 2,800

Then, new discount percent $= \dfrac{\text{New discount}}{\text{M.P.}} \times 100\% = \dfrac{2800}{40000} \times 100\% = 7\%$

$\therefore$ Reduction in discount percent = 10% – 7% = 3%

Hence, the discount is to be decreased by 3%.

c) *A retailer hired a room in a shopping mall at Rs 45,000 rent per month and started a business of garments. He spent Rs 20,00,000 to purchase different garment items in the first phase and marked the price of each item 30% above the cost price. Then, he allowed 10% discount on each item and sold to customers. His monthly miscellaneous expenditure was Rs 15,000 and the items of worth 10% of the investment remained as stocks after two months. Find his net profit or loss percent.*

Solution:

Here, the amount of investment = Rs. 20,00,000

Stocks after two months = 10% of Rs. 20,00,000 = Rs. 2,00,000

$\therefore$ The investment excluding stocks = Rs. 20,00,000 – Rs. 2,00,000 = Rs. 18,00,000 Now,

M. P. of the items = 130% of Rs. 18,00,000 = Rs. 23,40,000

Discount percent = 10%

$\therefore$ S.P. of the items = 90% of M.P. = 90% of Rs.23,40,000= Rs. 2,106,000

$\therefore$ Gross profit = Rs. 2,106,000 – Rs. 18,00,000 = Rs. 3,06,000

Again, the rent of room in 2 months = 2 × Rs. 45,000 = Rs. 90,000

Miscellaneous expenditure in 2 months = 2 × Rs. 15,000 = Rs. 30,000

$\therefore$ Total expenditure   = Rs. 90,000 + Rs. 30,000

                 = Rs. 1,20,000

Now, net profit = Gross profit – total expenditure

                = Rs. 3,06,000 – Rs. 1,20,000 = Rs. 1,86,000

Then, net profit percent $= \dfrac{\text{net profit}}{\text{investment}} \times 100\% = \dfrac{186000}{1800000} \times 100\% = 10.33\%$ Hence, her net profit percent is 10.33%.