MOCK TEST -2078 (Only from limited chapters)

Optional-I (Mathematics)

Time: 3 hours **Full marks: 100**

सबै प्रश्नहरुको उत्तर दिनुहोस् । Attempt all the questions:

Group-A [5× (1+1) = 6] (Test your knowledge level)

1. (a) फलन $f(x) = \cos x$ को पेरियड लेख्नुहोस् ।

Write down the period of the function $f(x) = \cos x$.

(b) फलनको विपरित फलन परिभाषित हुने अवस्था लेख्नुहोस् ।

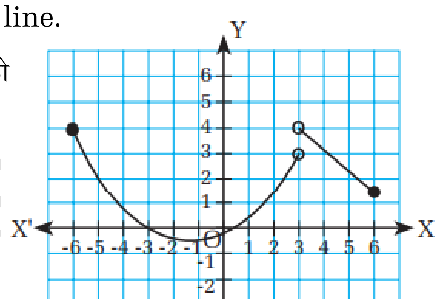
Write down the condition of existence of inverse of a function.

2. (a) संख्या रेखामा अविच्छिन्नता हुने संख्याहरूको समूह लेख्नुहोस् ।

Write a set of numbers which is continuous in number line.

(b) सँगै रहेको फलन f को ग्राफबाट $x = 3$ मा फलनको निरन्तरता वा विच्छिन्नताको उल्लेख गर्नुहोस् ।

From the graph of a function f given aside, state the continuity or discontinuity of the function at $x = 3$.



3. (a) सिङ्गुलर मेट्रिक्स परिभाषित गर्नुहोस् ।

Define singular matrix.

(b) यदि $A = \begin{pmatrix} p & q \\ r & s \end{pmatrix}$ भए, $|A|$ को मान के हुन्छ?

If matrix $A = \begin{pmatrix} p & q \\ r & s \end{pmatrix}$, what is the value of $|A|$?

4. (a) यदि दुई वटा रेखाहरूको झुकाव क्रमशः m_1 र m_2 छन् र तिनीहरूविचको कोण θ भए $\tan\theta$ को मान पत्ता लगाउने सुत्र लेख्नुहोस् ।

If the slopes of two straight lines are m_1 and m_2 respectively and θ be the angle between them, write the formula for $\tan\theta$.

(b) समिकरण $ax^2 + 2hxy + by^2 = 0$ ले प्रतिनिधित्व गर्ने जोडा रेखाहरु एक अर्कामा लम्ब हुने अवस्था लेख्नुहोस ।

Write down the condition of pair of lines represented by $ax^2 + 2hxy + by^2 = 0$ being perpendicular to each other.

5. (a) $\cos 2A$ को सुत्र $\tan A$ को स्वरुपमा लेख्नुहोस् ।

Write down the formula of $\cos 2A$ in terms of $\tan A$.

(b) दुई भेक्टरहरु $\vec{a}$ र $\vec{b}$ विचको कोण θ भए स्केलर गुणनफल लेख्नुहोस् ।

If θ be the angle between two vectors $\vec{a}$ and $\vec{b}$, write down the scalar product between them

Group-B (13 × 2) = 26) (Test your understanding level)

6. (a) यदि $f = \{(1, 3), (0, 0), (-1, -3)\}$ र $g = \{(0, 2), (-3, -1), (3, 5)\}$ भए gof लाई मिलान चित्रमा व्यक्त गरी क्रमजोडाको रुपमा लेख्नुहोस् ।

If $f = \{(1, 3), (0, 0), (-1, -3)\}$ and $g = \{(0, 2), (-3, -1), (3, 5)\}$, write gof in ordered pair form by representing in a mapping diagram.

(b) यदि फलन $f: R \to R$ लाई $f(x) = 9x + 2$ हुनेगरी परिभाषित गरिएको छ भने $f^{-1}(20)$ को मान पत्ता लगाउनुहोस् ।

If $f: R \to R$ is defined by $f(x) = 9x + 2$, find $f^{-1}(20)$.

(c) यदि फलन $f: R \to R$ लाई $f(x) = 2x - 1$ हुनेगरी परिभाषित गरिएको छ भने $ff(-1)$ को मान पत्ता लगाउनुहोस् ।

If $f: R \to R$ is defined by $f(x) = 2x - 1$, find $ff(-1)$.

7. (a) यदि मेट्रिक्स $A = \begin{pmatrix} 5 & 4 \\ x & 6 \end{pmatrix}$ को डिटरमिन्यान्ट 2 भए x को मान निकाल्नुहोस् ।

If the determinant of the matrix $A = \begin{pmatrix} 5 & 4 \\ x & 6 \end{pmatrix}$ is 2, find the value of x.

(b) यदि मेट्रिक्सहरू $\begin{pmatrix} 3 & 1 \\ 5 & x \end{pmatrix}$ र $\begin{pmatrix} 2 & -1 \\ -5 & y \end{pmatrix}$ एक अर्कामा विपरित मेट्रिक्स भए x र y का मान निकाल्नुहोस् ।

If the matrices $\begin{pmatrix} 3 & 1 \\ 5 & x \end{pmatrix}$ and $\begin{pmatrix} 2 & -1 \\ -5 & y \end{pmatrix}$ are inverse to each other, find the values of x and y.

8. (a) यदि रेखा $4x - y = 1$ संग समानान्तर हुने रेखा बिन्दुहरु (2, k) र (3, -1) भएर जान्छ भने k को मान पत्ता लगाउनुहोस् ।

If a line parallel to the line $4x - y = 1$ passes through the points $(2, k)$ and $(3, -1)$, find the value of k.

(b) समिकरण $2x^2 + 7xy + 3y^2 = 0$ ले प्रतिनिधित्व गर्ने जोडा रेखाहरूबिचको न्यूनकोण पत्ता लगाउनुहोस्। Find the acute angle between the pair of lines represented by $2x^2 + 7xy + 3y^2 = 0$.

9. (a) प्रमाणित गर्नुहोस् । Prove that: $\dfrac{\sin 2A - \sin A}{1 - \cos A + \cos 2A} = \tan A$

(b) यदि $\sin\theta = \frac{1}{4}$ भए $\cos 2\theta$ को मान निकाल्नुहोस्। If $\sin\theta = \frac{1}{4}$, find the value of $\cos 2\theta$.

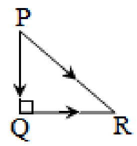
(c) प्रमाणित गर्नुहोस् । Prove that: $\sqrt{2 + 2\cos 2\alpha} = 2\cos\alpha$

10. (a) भेक्टरहरु $\vec{a} = \vec{i} + 3\vec{j}$ र $\vec{b} = 2\vec{i} + \vec{j}$ बिचको कोण पत्ता लगाउनुहोस् ।

Find the angle between the vectors $\vec{a} = \vec{i} + 3\vec{j}$ and $\vec{b} = 2\vec{i} + \vec{j}$.

(b) दिईएको समकोणी त्रिभुज PQR मा $\angle PQR = 90^\circ$ छ । भेक्टर विधिबाट $PR^2 = PQ^2 + QR^2$ हुन्छ भनी प्रमाणित गर्नुहोस् ।

In the given right angled triangle PQR, $\angle PQR = 90^\circ$, prove by vector method that $PR^2 = PQ^2 + QR^2$.



(c) एउटा बर्गिकृत तथ्याङ्कमा माथल्लो चतुर्थांस 60 र चतुर्थांसीय भिन्नताको गुणाङ्क 0.5 भए उक्त तथ्याङ्कको भिन्नता पत्ता लगाउनुहोस् ।

In a grouped data, upper quartile is 60 and coefficient of quartile deviation is 0.5, find the quartile deviation.

Group-C (11 × 4 = 44) (Test your application level)

11. $f(x) = ax + 9$ र $g(x) = 3x + 8$ वास्तविक सङ्ख्याहरुको समुहमा परिभाषित फलनहरु हुन् । यदि $f^{-1}(10) = g^{-1}(11)$ भए a को मान पत्ता लगाउनुहोस् ।

Given that two real valued functions f and g are defined as $f(x) = ax + 9$ and $g(x) = 3x + 8$. If $f^{-1}(10) = g^{-1}(11)$, find the value of a.

12. एउटा फलन $f(x)$ वास्तविक सङ्ख्याहरुको समुहमा $f(x) = 2x + 3$ हुने गरी परिभाषित छ।

A real valued function $f(x)$ is defined by $f(x) = 2x + 3$.

(i) $f(1.9), f(1.99), f(2.1), f(2.01)$ र $f(2)$ का मानहरु पत्ता लगाउनुहोस् ।

Find the values of $f(1.9), f(1.99), f(2.1), f(2.01)$ and $f(2)$.

(ii) के यो फलन $x = 2$ मा अविच्छिन्न छ?

Is this function continuous at $x = 2$?

13. तलका युगपत रेखीय समिकरणहरुलाई मेट्रिक्स बिधिबाट हल गर्नुहोस् ।

Solve the following system of equations by matrix method: $2x + 3y - 7 = 0$, $5y - 4x + 3 = 0$

14. यदि समिकरण $3x^2 + 8xy + my^2 = 0$ ले प्रतिनिधित्व गर्ने जोडा रेखाहरु एक आपसमा लम्ब छन् भने ती रेखाहरुको छुट्टा-छुट्टै समिकरणहरु निकाल्नुहोस् ।

If two straight lines represented by $3x^2 + 8xy + my^2 = 0$ are perpendicular to each other, find the separate equation of the lines.

15. रेखा $4x - 3y + 12 = 0$ संग समानान्तर हुने र बिन्दु (1, 2) बाट जाने रेखाको समिकरण पत्ता लगाउनुहोस्।
Find the equation of line parallel to the line $4x - 3y + 12 = 0$ and passing through (1, 2).

16. प्रमाणित गर्नुहोस्। Prove that: $cosec10^o - \sqrt{3}sec10^o = 4$

17. प्रमाणित गर्नुहोस्। Prove that: $\frac{cos^3A - cos3A}{\sin^3A + sin3A} = \tan A$

18. मानौँ E ले केन्द्रविन्दु (0, 0) र नापो 2 को आधारमा हुने विस्तारीकरण र R ले उद्गम विन्दु (0, 0) को वरिपरि $+90°$ मा हुने परिक्रमण जनाउँदछ। शिर्षविन्दुहरू P(2, 3), Q(6, 7) र R(0, 3) हुने ΔPQR लाई EOR अनुसार स्थानान्तरण गर्नुहोस्। ΔPQR र यसका आकृतिहरूलाई एउटै ग्राफमा देखाउनुहोस्।
Let E denote enlargement with centre (0, 0) and scale factor 2 and R denote the rotation of $+90°$ about the origin. Find the image of ΔPQR with vertices P(2, 3), Q(6, 7) and R(0, 3) under EOR . Then, draw the ΔPQR and its mages on the same graph paper.

19. एकाई वर्गलाई समानान्तर चतुभुज $\begin{pmatrix} 0 & 3 & 5 & 2 \\ 0 & 1 & 2 & 1 \end{pmatrix}$ मा स्थानान्तर गर्ने 2×2 स्थानान्तरण मेट्रिक्स पत्ता लगाउनुहोस्।
Find the 2×2 matrix which transforms the unit square into the parallelogram $\begin{pmatrix} 0 & 3 & 5 & 2 \\ 0 & 1 & 2 & 1 \end{pmatrix}$.

20. दिईएको तथ्याङ्कको आधारमा मध्यिकाबाट मध्यक भिन्नता पत्ता लगाउनुहोस्।
Find the mean deviation of the data given below from median.

प्राताङ्क (Marks)	0-10	30-40	20-30	40-50	10-20
विद्यार्थीको संख्या (No. of students)	5	10	15	6	8

21. दिइएको तथ्यांकबाट स्तरीय भिन्नता र विचरणशिलताको गुणाङ्क पत्ता लगाउनुहोस्।
Find the standard deviation and coefficient of variation of the data given below.

उमेर वर्षमा (Age in years)	0-4	4-8	8-12	12-16	16-20	20-24
मानिसको संख्या (No. of persons)	7	7	10	15	7	6

Group-D (4 × 5 = 20) (Test your HOTS)

22. एउटा रेफ्रिजेरेटरमा राखिएको खानामा ब्याक्टेरियाहरूको सङ्ख्यालाई, N(T)=20T2 - 80T + 500, $(2 \le T \le 14)$ को रुपमा ब्यक्त गर्न सकिन्छ जहाँ, T खानाको तापक्रमलाई जनाउँछ र $T(t) = 4t + 2$, $(0 \le t \le 3)$; जहाँ t ले घण्टामा हुने समयलाई जनाउँदछ।
The number of bacteria in food kept in a refrigerator is expressed by N(T)=20T2 - 80T + 500, $(2 \le T \le 14)$, where T denotes the temperature and $T(t) = 4t + 2$, $(0 \le t \le 3)$, where t represents the time in hour.
(a) (NoT) (t) पत्ता लगाउनुहोस्। Find (NoT) (t)
(b) फ्रिजमा राखेको उक्त खानामा 2 घण्टामा कति ब्याक्टेरिया हुन्छन्? What is the number of bacteria in 2 hours?
(c) कति घण्टाम उक्त खानामा 3300 ब्याक्टेरिया हुन्छन्? In how many hours does the number of bacteria in the food reach 3300?

23. समिकरण $ax^2 + 2hxy + by^2 = 0$ ले प्रतिनिधित्व गर्ने रेखाहरू विचको कोण $\tan^{-1}\left(\pm\frac{2\sqrt{h^2 - ab}}{a+b}\right)$ हुन्छ भनी प्रमाणित गर्नुहोस्।
Prove that the angle between the pair of lines represented by $ax^2 + 2hxy + by^2 = 0$ is given by $\tan^{-1}\left(\pm\frac{2\sqrt{h^2 - ab}}{a+b}\right)$.

24. कुनै चतुर्भुजको भुजाहरुको मध्यविन्दुहरु क्रमशः जोड्ने रेखाखण्डहरुले सधै समानान्तर चतुर्भुज बनाउँदछ भनी भेक्टर विधिबाट प्रमाणित गर्नुहोस्।

Prove by vector method that the line segments joining the middle points of the sides of a quadrilateral, taken in order, always form a parallelogram.

25. शिर्षविन्दुहरु A (3, -1), B (1, -3) र C (5, -3) भएका त्रिभुज ABC लाई x-अक्षमा परावर्तन गरी प्रतिबिम्ब A′, B′ र C′ का निर्देशांकहरु पत्ता लगाउनुहोस् र उक्त प्रतिबिम्बलाई उद्गम विन्दु (0, 0) को वरिपरि -270° द्वारा परिक्रमण गर्दा आउने प्रतिबिम्ब A″, B″ र C″ का निर्देशांकहरु पत्ता लगाउनुहोस्। दुवै त्रिभुजहरुलाई एउटै लेखाचित्रमा प्रस्तुत गर्नुहोस्। साथै यी दुबै स्थानान्तरणहरूको संयुक्त स्थानान्तरणले जनाउने एकल स्थानान्तरण पत्ता लगाउनुहोस्।

Reflect the ΔABC having the vertices A (3, -1), B (1, -3) and C (5, -3) about x-axis and find the coordinates of A', B' and C'. Then, rotate the image so obtained through -270° about origin and find the coordinates of A'', B'' and C''. Present both the triangles on the same graph paper. Also, establish the single transformation which represents the combination of the above two transformations.

*** THE END ***

100/100

MOCK TEST -2078

A+

Group-A [5× (1+1) = 6]

1. (a) **Solution:**

The period of the function $f(x) = \cos x$ is $2\pi^c$. 1

(b) **Solution:**

For the existence of inverse of a function, the function must be one to one and onto. 1

2. (a) **Solution:**

The set of real numbers is continuous in number line. 1

(b) **Solution:**

The function f is discontinuous at $x = 3$ because there is a jump in the graph at $x = 3$. 1

3. (a) **Solution:**

A square matrix having determinant zero is called a singular matrix. 1 1

(b) **Solution:**

$|A| = \begin{vmatrix} p & q \\ r & s \end{vmatrix} = ps - qr$ 1

4. (a) **Solution:**

The angle between the lines is given by $\tan\theta = \pm\frac{m_1 - m_2}{1 + m_1.m_2}$

(b) **Solution:**

The condition of pair of lines represented by $ax^2 + 2hxy + by^2 = 0$ being perpendicular to each other is $a + b = 0$. 1

5. (a) **Solution:**

The formula of cos2A in terms of tanA is $\cos 2A = \frac{1-\tan^2 A}{1+\tan^2 A}$ 1

(b) **Solution:**

The scalar product between $\vec{a}$ and $\vec{b}$ is defined by $\vec{a}.\vec{b} = |\vec{a}||\vec{b}|.\cos\theta$ 1

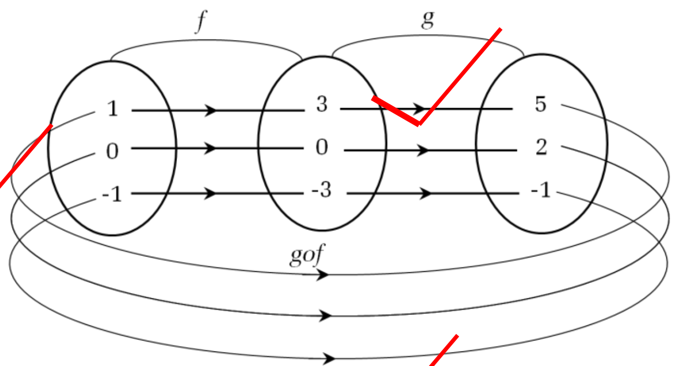
Group-B (13 × 2) = 26)

6. (a) **Solution:**

Here,

The given functions are $f = \{(1, 3), (0, 0), (-1, -3)\}$ and $g = \{(0, 2), (-3, -1), (3, 5)\}$.

Now, representing the function gof in a arrow diagram,



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From arrow diagram, $gof = \{(1, 5), (0, 2), (-1, -1)\}$.

(d) **Solution:**

Here,

The given function is $f(x) = 9x + 2$

For inverse of the function, suppose $f(x) = y$

Then, $y = 9x + 2$

Interchanging the role of x and y, we get

2

$x = 9y + 2$

or, $9y = x - 2$

or, $y = \frac{x-2}{9}$

$\therefore f^{-1}(x) = \frac{x-2}{9}$

Again, $f^{-1}(20) = \frac{20-2}{9} = 2.$

(e) **Solution:**

Here,

The given function is $f(x) = 2x - 1$

Now, $ff(-1) = f[2\times(-1) - 1]$ $[\because f(x) = 2x - 1]$

$= f(-2 - 1)$

$= f(-3)$

$= 2\times(-3) - 1$ $[\because f(x) = 2x - 1]$

$= -6 - 1$

$= -7$

2

Hence, the value of $ff(-1)$ is -7.

7. (a) **Solution:**

Here,

The given matrix $A = \begin{pmatrix} 5 & 4 \\ x & 6 \end{pmatrix}$

Now, determinant of matrix A = 2

or, $\begin{vmatrix} 5 & 4 \\ x & 6 \end{vmatrix} = 2$

or, $30 - 4x = 2$

or, $28 = 4x$

2

or, x $= 7$

Hence, the value of x is 7.

(b) **Solution:**

Let, $A = \begin{pmatrix} 3 & 1 \\ 5 & x \end{pmatrix}$ and $B = \begin{pmatrix} 2 & -1 \\ -5 & y \end{pmatrix}$

Since, the matrices A and B are inverse to each other.

So, $AB = I$

or, $\begin{pmatrix} 3 & 1 \\ 5 & x \end{pmatrix}\begin{pmatrix} 2 & -1 \\ -5 & y \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

or, $\begin{pmatrix} 6-5 & -3+y \\ 10-5x & -5+xy \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

or, $\begin{pmatrix} 1 & -3+y \\ 10-5x & -5+xy \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

Equating corresponding elements, we get

$10 - 5x = 0$

or, $10 = 5x$

or, $x = 2$

And, $-3 + y = 0$

or, $y = 3$

Hence, the value of $x = 2$ and $y = 3$.

2

8. (a) **Solution:**

Here,

The slope of the line $4x - y = 1$ is $m_1 = -\frac{\text{Coeff.of } x}{\text{Coeff.of } y} = -\frac{4}{-1} = 4$

The slope of the line joining the points $(2, k)$ and $(3, -1)$ is $m_2 = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-1-k}{3-2} = -1 - k$

Since, the lines are parallel to each other.

So, $m_1 = m_2$

or, $4 = -1 - k$

or, $k = -5$

Hence, the required value of k is -5.

2

(b) **Solution:**

Here,

The given equation of the pair of lines is $2x^2 + 7xy + 3y^2 = 0$.

Comparing it with $ax^2 + 2hxy + by^2 = 0$, we get

$a = 2,\ 2h = 7\ \therefore h = \frac{7}{2}$ and $b = 3$

Let, θ be the angle between the pair of lines.

Then, $\tan\theta = \pm\frac{2\sqrt{h^2 - ab}}{a+b}$

$= \pm\frac{2\sqrt{\left(\frac{7}{2}\right)^2 - 2\times 3}}{2+3}$

$= \pm\frac{2\sqrt{\frac{49}{4} - 6}}{5}$

$= \pm \frac{2\sqrt{\frac{49-24}{4}}}{5}$

$= \pm \frac{2\sqrt{\frac{25}{4}}}{5}$

$= \pm \frac{2\times\frac{5}{2}}{5}$

$= \pm 1$

For acute angle, taking (+) ve sign, we get

$\tan\theta = 1$

or, $\tan\theta = \tan 45^\circ$

or, $\theta = 45^\circ$

Hence, the acute angle between the pair of lines is 45°.

2

9. (a) **Solution:**

Here, L.H.S. $= \frac{\sin 2A - \sin A}{1 - \cos A + \cos 2A}$

$= \frac{2\sin A.\cos A - \sin A}{1 - \cos A + 2\cos^2 A - 1}$

$= \frac{\sin A\,(2\cos A - 1)}{\cos A\,(2\cos A - 1)}$

$= \tan A$

$=$ R.H.S.

Hence, proved

2

(b) **Solution:**

Here, $\sin\theta = \frac{1}{4}$, $\cos 2\theta = ?$

We know, $\cos 2\theta = 1 - 2\sin^2\theta$

$= 1 - 2 \times \left(\frac{1}{4}\right)^2$

$= 1 - 2 \times \frac{1}{16}$

$= 1 - \frac{1}{8}$

$= \frac{7}{8}$

Hence, the value of $\cos 2\theta$ is $\frac{7}{8}$.

2

(c) **Solution:**

Here, L.H.S. $= \sqrt{2 + 2\cos 2\alpha}$

$= \sqrt{2 + 2(2\cos^2\alpha - 1)}$

$= \sqrt{2 + 4\cos^2\alpha - 2}$

$= \sqrt{4\cos^2\alpha}$

$= 2\cos\alpha$

$=$ R.H.S.

Hence, proved

2

10. (a) **Solution:**

Here, $\vec{a} = \vec{i} + 3\vec{j} = \begin{pmatrix}1\\3\end{pmatrix}$ and $\vec{b} = 2\vec{i} + \vec{j} = \begin{pmatrix}2\\1\end{pmatrix}$

Now, $\vec{a}.\vec{b} = \begin{pmatrix}1\\3\end{pmatrix} . \begin{pmatrix}2\\1\end{pmatrix} = 2 + 3 = 5,$

$|\vec{a}| = \sqrt{1^2 + 3^2} = \sqrt{10}$ and

$|\vec{b}| = \sqrt{2^2 + 1^2} = \sqrt{5}$

Let, θ be the angle between $\vec{a}$ and $\vec{b}$.

Then, $\cos\theta = \dfrac{\vec{a}.\vec{b}}{|\vec{a}||\vec{b}|}$

2

or, $\cos\theta = \dfrac{5}{\sqrt{10}\times\sqrt{5}}$

or, $\cos\theta = \dfrac{5}{\sqrt{50}}$

or, $\cos\theta = \dfrac{5}{5\sqrt{2}}$

or, $\cos\theta = \dfrac{1}{\sqrt{2}}$

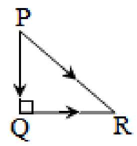
or, $\cos\theta = \cos 45^\circ$

or, $\theta = 45^\circ$

Hence, the angle between the vectors is 45°.

(d) **Solution:**

Here, in ΔPQR; $\angle PQR = 90^\circ$. $\therefore \overrightarrow{PQ}.\overrightarrow{QR} = 0$



Now, $\overrightarrow{PQ} + \overrightarrow{QR} = \overrightarrow{PR}$ [By triangle law of vector addition]

Squaring on both sides, we get

$$(\overrightarrow{PQ} + \overrightarrow{QR})^2 = (\overrightarrow{PR})^2$$

or, $PQ^2 + 2.\overrightarrow{PQ}.\overrightarrow{QR} + QR^2 = PR^2$

or, $PQ^2 + 2 \times 0 + QR^2 = PR^2$

2

or, $PQ^2 + QR^2 = PR^2$

Hence, $PR^2 = PQ^2 + QR^2$ proved

(e) **Solution:**

Here, $Q_3 = 60$, Coefficient of Q.D. $= 0.5$, Q.D. $=?$

Now, coefficient of Q.D. $= \dfrac{Q_3 - Q_1}{Q_3 + Q_1}$

or, $0.5 = \dfrac{60 - Q_1}{60 + Q_1}$

or, $30 + 0.5Q_1 = 60 - Q_1$

or, $1.5\,Q_1 = 30$

or, $Q_1 = 20$

Hence, Q.D. $= \dfrac{Q_3 - Q_1}{2}$

2

$= \dfrac{60 - 20}{2}$

$= \dfrac{40}{2}$

$= 20$

Group-C (11 × 4 = 44)

11. **Solution:**

Here,

The given functions are is $f(x) = ax + 9$ and $g(x) = 3x + 8$

Let, $f(x) = y$ then $y = ax + 9$

Interchanging the role of x and y, we get

$x = ay + 9$

or, $ay = x - 9$

or, $y = \frac{x-9}{a}$

$\therefore f^{-1}(x) = \frac{x-9}{a}$

Also,

Let, $g(x) = y$ then $y = 3x + 8$

Interchanging the role of x and y, we get

$x = 3y + 8$

or, $3y = x - 8$

or, $y = \frac{x-8}{3}$

$\therefore g^{-1}(x) = \frac{x-8}{3}$

Again,

According to question, $f^{-1}(10) = g^{-1}(11)$

or, $\frac{10-9}{a} = \frac{11-8}{3}$

or, $\frac{1}{a} = 1$

or, $a = 1$

Hence, the value of 'a' is 1.

4

12. **Solution:**

Here,

The given $f(x) = 2x + 3$

Now,

(i) $f(1.9) = 2\times1.9 + 3 = 6.8$

$f(1.99) = 2\times1.99 + 3 = 6.98$

As x tends to 2^-, $f(x)$ approaches to 7. So, left hand limit of $f(x) = 7$

$f(2.1) = 2\times2.1 + 3 = 7.2$

$f(2.01) = 2\times2.01 + 3 = 7.02$

As x tends to 2^+, $f(x)$ approaches to 7. So, right hand limit of $f(x) = 7$

4

Again, $f(2) = 2\times2 + 3 = 7$

(ii) Since, left hand limit = right hand limit = functional value of $f(x)$ at $x = 2$. Hence, the function f (x) is continuous at x = 2.

13. **Solution:**

Here,

The given equations are; $2x + 3y - 7 = 0$ or, $2x + 3y = 7$ … (i)

and $5y - 4x + 3 = 0$ or, $-4x + 5y = -3$ … (ii)

Expressing equations (i) and (ii) in matrix form. We get

$$\begin{pmatrix} 2 & 3 \\ -4 & 5 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 7 \\ -3 \end{pmatrix}$$

or, AX = B where $A = \begin{pmatrix} 2 & 3 \\ -4 & 5 \end{pmatrix}$, $B = \begin{pmatrix} 7 \\ -3 \end{pmatrix}$ and $X = \begin{pmatrix} x \\ y \end{pmatrix}$

$\therefore X = A^{-1}B$ ... (iii)

Now, determinant of $A = \begin{vmatrix} 2 & 3 \\ -4 & 5 \end{vmatrix} = 10 + 12 = 22$

Since, $|A| \neq 0$ So, A^{-1} exists and the given system has a unique solution.

Again, $A^{-1} = \frac{1}{|A|}$Ad joint of A

$= \frac{1}{22}\begin{pmatrix} 5 & -3 \\ 4 & 2 \end{pmatrix}$

Putting the value of A -1 in equation (iii), we get

4

$\begin{pmatrix} x \\ y \end{pmatrix} = \frac{1}{22}\begin{pmatrix} 5 & -3 \\ 4 & 2 \end{pmatrix}\begin{pmatrix} 7 \\ -3 \end{pmatrix}$

or, $\begin{pmatrix} x \\ y \end{pmatrix} = \frac{1}{22}\begin{pmatrix} 35 + 9 \\ 28 - 6 \end{pmatrix}$

or, $\begin{pmatrix} x \\ y \end{pmatrix} = \frac{1}{22}\begin{pmatrix} 44 \\ 22 \end{pmatrix}$

or, $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$

Equating the corresponding elements, we get x = 2 and y = 1

Hence, x =2 and y = 1.

14. **Solution:**

Here,

The given equation is $3x^2 + 8xy + my^2 = 0$

Comparing it with $ax^2 + 2hxy + by^2 = 0$, we get

$a = 3, 2h = 8 \therefore h = 4$ and $b = m$

Since, the lines are perpendicular to each other.

So, a + b = 0

or, 3 + m = 0

or, m = -3

Again, the equation becomes $3x^2 + 8xy - 3y^2 = 0$

or, $3x^2 + (9 - 1)\, xy - 3y^2 = 0$

or, $3x^2 + 9xy - xy - 3y^2 = 0$

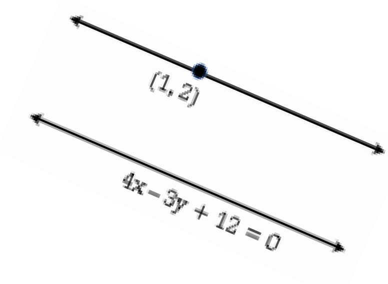
or, $3x\,(x + 3y) - y\,(x + 3y) = 0$

or, $(x + 3y)\,(3x - y) = 0$

4

Hence, the separate equations of lines are x + 3y = 0 and 3x – y = 0.

15. **Solution:**



Here, the slope of the line 4x – 3y + 12 = 0 is $m_1 = -\frac{\text{Coeff.of } x}{\text{Coeff.of } y}$

$= -\frac{4}{-3}$

$= \frac{4}{3}$

Let, m_2 be the slope of the required line.

Since, the lines are parallel to each other.

So, $m_1 = m_2$

or, $\frac{4}{3} = m_2$

Passing point $(x_1, y_1) = (1, 2)$

Now, the equation of line having slope $\frac{4}{3}$ and passing through (1, 2) is given by

$y - y_1 = m_2\,(x - x_1)$

or, $y - 2 = \frac{4}{3}\,(x - 1)$

4

or, $4x - 4 = 3y - 6$

or, $4x - 3y + 2 = 0$ which is required equation.

16. **Solution:**

Here, L.H.S. $= \operatorname{cosec}10^o - \sqrt{3}\sec10^o$

$= \dfrac{1}{\sin10^o} - \dfrac{\sqrt{3}}{\cos10^o}$

$= \dfrac{\cos10^o - \sqrt{3}\sin10^o}{\sin10^o.\cos10^o}$

$= \dfrac{2\times\frac{1}{2}(\cos10^o - \sqrt{3}\sin10^o)}{\frac{1}{2}\times2\sin10^o.\cos10^o}$

$= \dfrac{2(\frac{1}{2}\cos10^o - \frac{\sqrt{3}}{2}\sin10^o)}{\frac{1}{2}\times\sin2(10^o)}$

$= \dfrac{4(\sin30^o.\cos10^o - \cos30^o.\sin10^o)}{\sin20^o}$

$= \dfrac{4\sin(30^o - 10^o)}{\sin20^o}$

$= \dfrac{4\sin 20^o}{\sin20^o}$

$= 4$

= R.H.S.

4

Hence, proved

17. **Solution:**

Here, L.H.S. $= \dfrac{\cos^3A - \cos3A}{\sin^3A + \sin3A}$

$= \dfrac{\cos^3A - (4\cos^3A - 3\cos A)}{\sin^3A + 3\sin A - 4\sin^3A}$

$= \dfrac{\cos^3A - 4\cos^3A + 3\cos A}{\sin^3A + 3\sin A - 4\sin^3A}$

$= \dfrac{3\cos A - 3\cos^3A}{3\sin A - 3\sin^3A}$

$= \dfrac{3\cos A(1 - \cos^2A)}{3\sin A(1 - \sin^2A)}$

$= \dfrac{\cos A\times\sin^2A}{\sin A\times\cos^2A}$

$= \dfrac{\sin A}{\cos A}$

$= \tan A$

= R.H.S.

4

Hence, proved

18. **Solution:**

Here, E = the enlargement with centre (0, 0) and scale factor 2 = E [O, 2]

R = the rotation of $+90^0$ about the origin = R [$+90^o$, (0, 0)]

EOR = R is followed by E

The coordinates of vertices of ΔPQR are P(2, 3), Q(6, 7) and R(0, 3)

Now, rotating ΔPQR through $+90^0$ about centre at origin, we get

$$P(x, y) \xrightarrow{R[+90^0,\ (0,\ 0)]} P'(-x, -y)$$
$$P(2, 3) \longrightarrow P'(-3, 2)$$
$$Q(6, 7) \longrightarrow Q'(-7, 6)$$
$$R(0, 3) \longrightarrow R'(-3, 0)$$

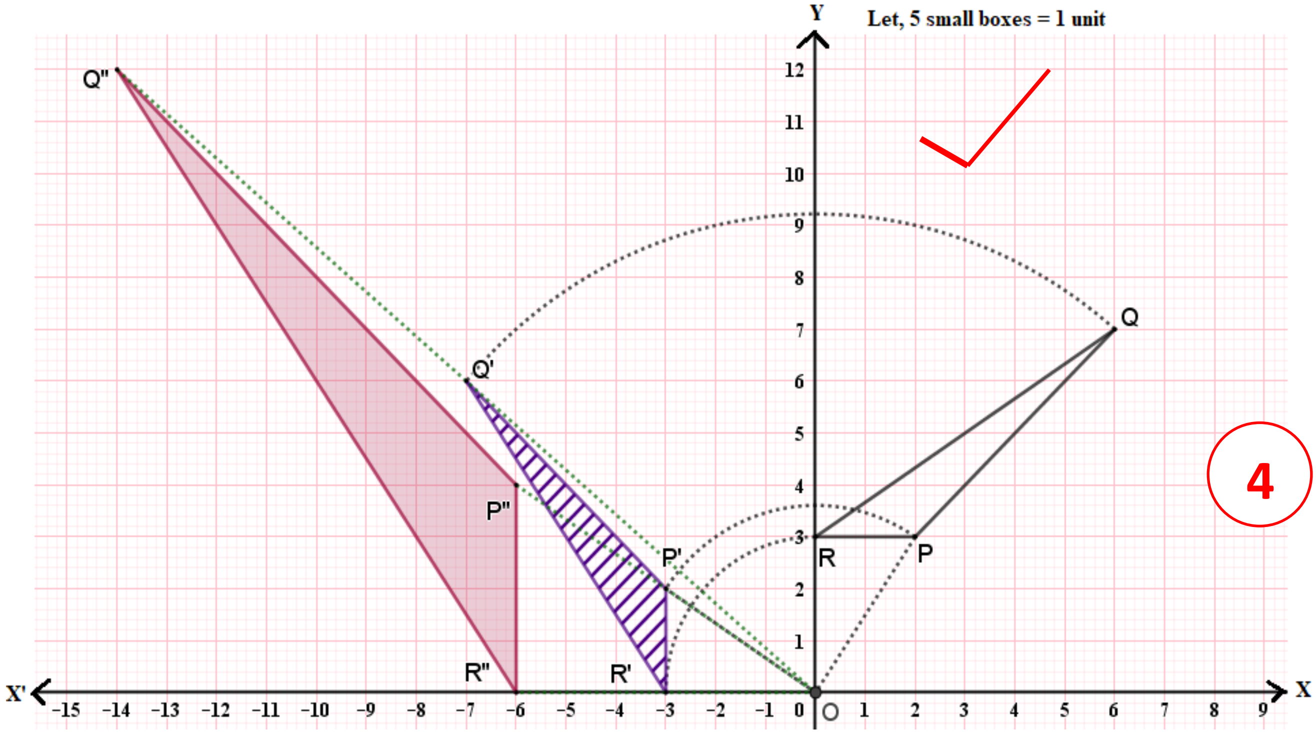
Thus, P′ (-3, 2), Q′ (-7, 6) and R′ (-3, 0) are the vertices of image ΔP′Q′R′.

Again, enlarging ΔP’Q’R’ by enlargement E [(0, 0), 2], we get

$$P(x, y) \xrightarrow{E[(0,\ 0);\ k]} P'(kx, ky)$$
$$P'(-3, 2) \xrightarrow{E[(0,\ 0);\ 2]} P''(-6, 4)$$
$$Q'(-7, 6) \longrightarrow Q''(-14, 12)$$
$$R'(-3, 0) \longrightarrow R''(-6, 0)$$

Thus, P″ (-6, 4), Q″ (-14, 12) and R″ (-6, 0) are the vertices of image ΔP″Q″R″.

At last, representing the Δ PQR and its images on the same graph paper



19. **Solution:**

Here,

Object = unit square $\begin{pmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{pmatrix}$ and image = parallelogram$\begin{pmatrix} 0 & 3 & 5 & 2 \\ 0 & 1 & 2 & 1 \end{pmatrix}$

Let, 2×2 transformation matrix be $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$

Now, Image = T.M. × Object

or, $\begin{pmatrix} 0 & 3 & 5 & 2 \\ 0 & 1 & 2 & 1 \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \times \begin{pmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{pmatrix}$

or, $\begin{pmatrix} 0 & 3 & 5 & 2 \\ 0 & 1 & 2 & 1 \end{pmatrix} = \begin{pmatrix} 0+0 & a+0 & a+b & 0+b \\ 0+0 & c+0 & c+d & 0+d \end{pmatrix}$

or, $\begin{pmatrix} 0 & 3 & 5 & 2 \\ 0 & 1 & 2 & 1 \end{pmatrix} = \begin{pmatrix} 0 & a & a+b & b \\ 0 & c & c+d & d \end{pmatrix}$

Equating the corresponding elements, we get

$$a = 3, b = 2, c = 1 \text{ and } d - 1$$

Hence, required transformation matrix is $\begin{pmatrix} 3 & 2 \\ 1 & 1 \end{pmatrix}$.

4

20. **Solution:**

Here,

Computation of the mean deviation from the median

Marks	m	No. of students (f)	c. f.	|m – Md |	f|m – Md |
0-10	5	5	5	21	105
10-20	15	8	13	11	88
20-30	25	15	28	1	15
30-40	35	10	38	9	90
40-50	45	6	44	19	114
		N = 44			Σf|m – Md| = 412

Now,

Position of median $= \left(\frac{N}{2}\right)^{th}$ class $= \left(\frac{44}{2}\right)^{th}$ class $= 22^{nd}$ class

From the c.f. column, the c.f. just greater than or equal to 22 is 28 and its corresponding class is (20-30) $\therefore$ Median class = (20-30) where L = 20, c.f = 13, f = 15 and i = 10.

Also, median $= L + \left(\frac{\frac{N}{2} - cf}{f}\right) \times i = 20 + \left(\frac{22 - 13}{15}\right) \times 10 = 26$

4

Again,

M.D. from median $= \frac{\Sigma f|m - Md|}{N} = \frac{412}{44} = 9.36$

Hence, the mean deviation from median is 9.36.

21. **Solution:**

Here,

Computation of the standard deviation from the median:

Ages	No. of persons (f)	m	fm	m^2	fm^2
0-4	7	2	14	4	28
4-8	7	6	42	36	252
8-12	10	10	100	100	1000
12-16	15	14	210	196	2940
16-20	7	18	126	324	2268
20-24	6	22	132	484	2904
	N = 52		Σfm = 624		$\Sigma fm^2 = 9392$

Now, standard deviation (σ) $= \sqrt{\frac{\Sigma fm^2}{N} - \left(\frac{\Sigma fm}{N}\right)^2}$

$$= \sqrt{\frac{9392}{52} - \left(\frac{624}{52}\right)^2}$$

$= \sqrt{180.62 - 144}$

$= \sqrt{36.62}$

$= 6.05$

4

Again, mean $(\overline{X}) = \frac{\sum fm}{N} = \frac{624}{52} = 12$

Coefficient of variation $= \frac{\sigma}{\overline{x}} \times 100\% = \frac{6.05}{12} \times 100\% = 50.42\%$

Group-D (4 × 5 = 20)

22. **Solution**

Here, $N(T)=20T^2 - 80T + 500$, $(2 \le T \le 14)$, and $T(t) = 4t + 2$, $(0 \le t \le 3)$

(a) $(NoT)(t) = N[T(t)]$
$= N(4t + 2)$
$= 20(4t + 2)^2 - 80(4t + 2) + 500$
$= 20(16t^2 + 16t + 4) - 320t - 160 + 500$
$= 320t^2 + 320t + 80 - 320t - 160 + 500$
$= 320t^2 + 420$

$\therefore (NoT)(t) = 320t^2 + 420$

(b) When time (t) = 2 hours then no. of bacteria $(NoT)(2) = 320 \times 2^2 + 420$
$= 1280 + 420$
$= 1700$

Thus, no. of bacteria in 2 hours = 1700

(c) Here, no. of bacteria (N) = 3300, time (t) =?

We have, $N = 320t^2 + 420$

or, $3300 = 320t^2 + 420$

or, $320t^2 = 2880$

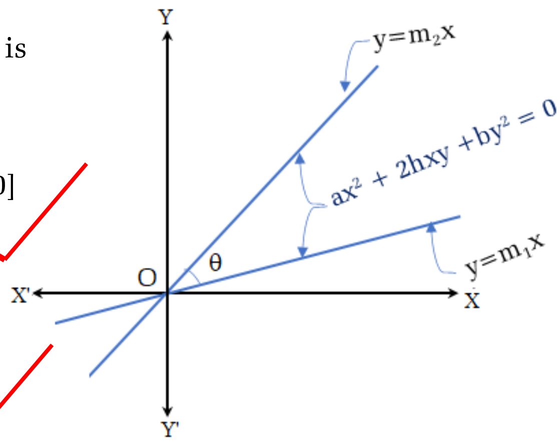
or, $t^2 = 9$

$\therefore t = 3$

5

Thus, required time is 3 hours.

23. **Solution**



Here, the given homogeneous equation of second degree is

$ax^2 + 2hxy + by^2 = 0$

or, $by^2 + 2hxy + ax^2 = 0$

or, $\frac{by^2}{b} + \frac{2hxy}{b} + \frac{ax^2}{b} = \frac{0}{b}$ [Dividing each term by b≠0]

or, $y^2 + \frac{2h}{b}xy + \frac{a}{b}x^2 = 0$ ... (i)

Since, the homogeneous equation $ax^2 + 2hxy + by^2 = 0$ represents a pair of lines which pass through the origin.

Let, $y = m_1x$ and $y = m_2x$ be the equations of lines represented by $ax^2 + 2hxy + by^2 = 0$.

Then, by combining these equations, we get

$(y - m_1x)(y - m_2x) = 0$

or, $y^2 - m_2xy - m_1xy + m_1m_2x^2 = 0$

or, $y^2 - (m_1 + m_2)xy + m_1m_2x^2 = 0$ ... (ii)

Since, the equations (i) and (ii) are identical. So, comparing them, we get

$m_1 + m_2 = -\frac{2h}{b}$ and $m_1m_2 = \frac{a}{b}$

Let, θ be the angle between the pair of lines $y = m_1x$ and $y = m_2x$.

Then, $\tan\theta = \pm\dfrac{m_1-m_2}{1+m_1.m_2}$

or, $\tan\theta = \pm\dfrac{\sqrt{(m_1+m_2)^2-4m_1m_2}}{1+m_1.m_2}$ $\quad[\because (a-b)^2 = (a+b)^2 - 4ab]$

or, $\tan\theta = \pm\dfrac{\sqrt{\left(-\frac{2h}{b}\right)^2-4\times\frac{a}{b}}}{1+\frac{a}{b}}$

or, $\tan\theta = \pm\dfrac{\sqrt{\frac{4h^2}{b^2}-\frac{4a}{b}}}{\frac{b+a}{b}}$

or, $\tan\theta = \pm\dfrac{\sqrt{\frac{4h^2-4ab}{b^2}}}{\frac{b+a}{b}}$

or, $\tan\theta = \pm\dfrac{\frac{2}{b}\sqrt{h^2-ab}}{\frac{b+a}{b}}$

or, $\tan\theta = \pm\dfrac{2\sqrt{h^2-ab}}{a+b}$

5

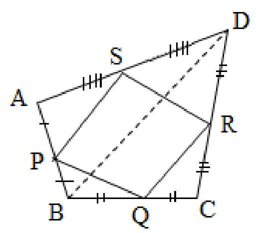
$\therefore \theta = \tan^{-1}\left(\pm\dfrac{2\sqrt{h^2-ab}}{a+b}\right)$

Hence, the angle between the pair of straight lines represented by the homogeneous equation $ax^2 + 2hxy + by^2 = 0$ is $\tan^{-1}\left(\pm\dfrac{2\sqrt{h^2-ab}}{a+b}\right)$.

24. **Solution**

Given: In quadrilateral ABCD; P, Q, R and S are mid-points of sides AB, BC, CD and AD respectively

i.e. $\overrightarrow{PA} = \overrightarrow{BP} = \frac{1}{2}\overrightarrow{BA}$, $\overrightarrow{BQ} = \overrightarrow{QC} = \frac{1}{2}\overrightarrow{BC}$, $\overrightarrow{CR} = \overrightarrow{RD} = \frac{1}{2}\overrightarrow{CD}$ and $\overrightarrow{AS} = \overrightarrow{SD} = \frac{1}{2}\overrightarrow{AD}$



Construction: B and D are joined.

To prove: PQRS is a parallelogram.

Proof:

(i) In ΔABD; $\overrightarrow{PS} = (\overrightarrow{PA} + \overrightarrow{AS})$ [By Δ law of vector addition]

$= \frac{1}{2}\overrightarrow{BA} + \frac{1}{2}\overrightarrow{AD}$ $\quad[\overrightarrow{PA} = \frac{1}{2}\overrightarrow{BA}$ and $\overrightarrow{AS} = \frac{1}{2}\overrightarrow{AD}]$

$= \frac{1}{2}(\overrightarrow{BA} + \overrightarrow{AD})$

$= \frac{1}{2}\overrightarrow{BD}$ [By Δ law of vector addition]

$\therefore$ PS // BD and PS $= \frac{1}{2}$BD.

(ii) In ΔQCR; $\overrightarrow{QR} = (\overrightarrow{QC} + \overrightarrow{CR})$ [By Δ law of vector addition]

$= \frac{1}{2}\ \overrightarrow{BC} + \frac{1}{2}\ \overrightarrow{CD}$ $[\overrightarrow{QC} = \frac{1}{2}\overrightarrow{BC}$ and $\overrightarrow{CR} = \frac{1}{2}\ \overrightarrow{CD}]$

$= \frac{1}{2}\ (\overrightarrow{BC} + \overrightarrow{CD})$

$= \frac{1}{2}\ \overrightarrow{BD}$ [By Δ law of vector addition]

$\therefore$ QR // BD and QR$= \frac{1}{2}$BD

From (i) and (ii), we get PS // QR and PS = QR

Similarly, PQ // SR and PQ = SR

Hence, PQRS is a parallelogram. QED

5

25. **Solution:**

Here,

The coordinates of vertices of ΔABC are A (3, -1), B (1, -3) and C (5, -3).

Now, reflecting ΔABC about x-axis, we get

$P(x, y) \xrightarrow{\text{x-axis}} P'(x, -y)$

$A(3, -1) \longrightarrow A'(3, 1)$

$B(1, -3) \longrightarrow B'(1, 3)$

$C(5, -3) \longrightarrow C'(5, 3)$

Thus, A′ (3, 1), B′ (1, 3) and C′ (5, 3) are the vertices of image ΔA′B′C′.

Again, rotating ΔA'B'C' through -270° about origin, we get

We have,

$P(x, y) \xrightarrow{R[-270^\circ;\ (0, 0)]} P'(-y, x)$

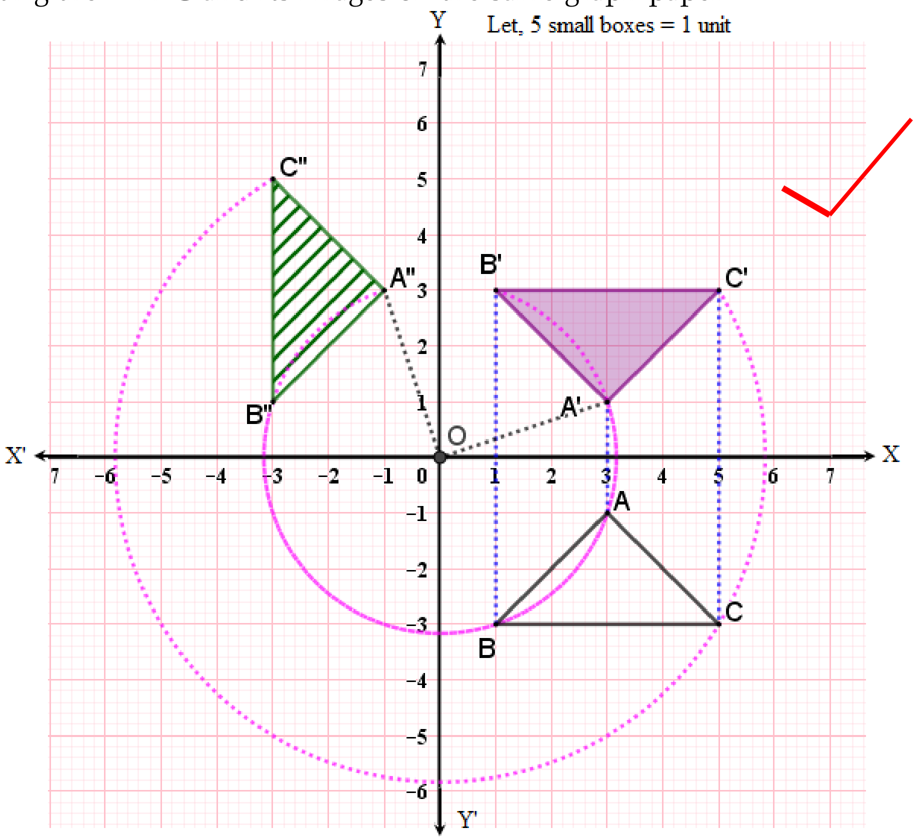
$A'(3, 1) \longrightarrow A''(-1, 3)$

$B'(1, 3) \longrightarrow B''(-3, 1)$

$C'(5, 3) \longrightarrow C'(-3, 5)$

Thus, A″ (-6, 4), B″ (-14, 12) and C″ (-6, 0) are the vertices of image ΔA″B″C″.

Representing the Δ ABC and its images on the same graph paper



To find the single transformation,

$A(3, -1) \longrightarrow A''(-1, 3)$

$B(1, -3) \longrightarrow B''(-3, 1)$

$C(5, -3) \longrightarrow C'(-3, 5)$

$P(x, y) \longrightarrow P'(y, x)$

Hence, the single transformation is the reflection about the line $y = x$.

5

***** THE END *****